UDC 514.76

ON ONE CLASS OF PAIRS OF *n*-SURFACES IN (*n*+2)-DIMENTIONAL PROJECTIVE SPACE

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Two n-dimensional surfaces in (n+2)-dimensional projective space have been studied. Point correspondence is established between their points. Some geometric images connected to a pair of surfaces are considered. Consideration has a local character everywhere. All used functions are supposed to be analytical.

Introduction

Multidimensional projective geometry has rather long history of its development. In XX century the projective theory of multidimensional surfaces, line networks on them and projective correspondence between surfaces is developed intensively. This work belongs to the same direction and continues the investigations carried out in the previous works [1, 2].

The aim of the work is to study the pair of n-surfaces in (n+2)-dimensional projective space, to find a set of geometry objects connected closely to a pair of surfaces, to ascertain a possibility of attaching to a pair of other n-surfaces possessing properties of pair initial surface.

1. Analytical device

1. Let us consider (n+2)-dimensional projective space P_{n+2}, referred to a projective reference point $R=\{A_I\}$, consisting of analytical points $A_0,A_1,...,A_n,A_{n+1},A_{n+2}$ $(I,J,K,...=0,1,...,n+2)$ with derivation formulas $dA_I=\omega_I^J A_J$, where ω_I^J are the differential forms of Pfaff satisfying $D\omega_I^J=\omega_I^K\Lambda\omega_K^J$ the equation of the structure and a correlation $\omega_I^I=0$.

Let two surfaces S_n^1, S_n^2, and Π be given in space P_{n+2}: $S_n^1\to S_n^2$ is the smooth biunique correspondence between them.

Let us carry out the following partial canonization of the reference point: points A_0 and $A_{n+2}=\Pi(A_0)$ are placed in the proper points of surfaces S_n^1 and S_n^2 of the pair; points $A_1,...,A_n$ – in a tangent n-plane $L_n^1=(A_0,A_1,...,A_n)$ to n-surface S_n^1 in point A_0; points $A_1,...,A_n$ – in a tangent n-plane $L_n^2=(A_{n+2},A_1,...,A_n)$ to n-surface S_n^2 in point A_{n+2}.

Point correspondence Π induces projective correspondence between bundles of tangent directions associated to two proper points A_0 and A_{n+2}. Let us choose pair reference point so that directions A_0A_i $(i,j,k,...=1,2,...,n)$ correspond to directions $A_{n+2}A_i$ in this project. Then the basic equations of our problem have the form

$$\omega_0^{n+1}=0,\ \omega_0^{n+2}=0,\ \omega_{n+2}^{0}=0,\ \omega_{n+2}^{n+1}=0,\ \omega_{\ n+2}^{i}=\omega^{i}. \quad (1)$$

Let us further note $\omega_0^i=\omega^i$ for shot.

Continuing the equations (1), we have

$$\omega_i^{n+1}=\Lambda_{ij}^{n+1}\omega^j,\quad \omega_i^{n+2}=\Lambda_{ij}^{n+2}\omega^j,$$
$$\omega_i^0=\Lambda_{ij}^0\omega^j,\quad \omega_0^0-\omega_{n+2}^{n+2}=0. \quad (2)$$

$$\nabla\Lambda_{ij}^{n+1}+\Lambda_{ij}^{n+1}(\omega_0^0+\omega_{n+1}^{n+1})=\Lambda_{ijk}^{n+1}\omega^k,$$
$$\nabla\Lambda_{ij}^{0}+\Lambda_{ij}^{0}\omega_0^0+\Lambda_{ij}^{n+1}\omega_{n+1}^{0}=\Lambda_{ijk}^{0}\omega^k,$$
$$\nabla\Lambda_{ij}^{n+2}+\Lambda_{ij}^{n+2}(\omega_0^0+\omega_{n+2}^{n+2})+\Lambda_{ij}^{n+1}\omega_{n+1}^{n+2}=\Lambda_{ijk}^{n+2}\omega^k, \quad (3)$$
$$D(\omega_0^0-\omega_{n+2}^{n+2})\equiv 0.$$

Here symbol ∇ denotes the operator of covariant differentiation.

2. Let point A_{n+1} describe n-surface S_n^2 with tangent n-plane $L_n^3=(A_{n+1},A_1,...,A_n)$. Having set smooth biunique mapping Π_1: $S_1^n\to S_n^3$ we establish projective correspondence between bundles of tangent directions on these surfaces. Let us also demand that direction $A_{n+1}A_i$ corresponds to the direction A_0A_i. Then

$$\omega_{n+1}^{n+2}=0,\quad \omega_{n+1}^{0}=0,\quad \omega_{n+1}^{n+1}-\omega_0^0=0. \quad (4)$$

As $D\omega_{n+1}^0\equiv 0$, $D\omega_{n+1}^{n+2}\equiv 0$, then we can attach the third surface S_n^3 to a pair of surfaces S_n^1 and S_n^2 so that coordinate lines correspond on these surfaces and they have the same first and second normals which are specified in the following manner:

$$L_2=(A_0,A_{n+1},A_{n+2})\ \text{and}\ L_{n-1}=(A_1,A_2,...,A_n).$$

Then equations (1-4) are written down as

$$\omega_0^{n+1}=\omega_0^{n+2}=\omega_{n+1}^0=\omega_{n+1}^{n+2}=\omega_{n+2}^0=\omega_{n+2}^{n+1}=0,$$
$$\omega_{n+1}^{n+1}=\omega_{n+2}^{n+2}=\omega_0^0,\ \omega_i^0=\Lambda_{ij}^0\omega^j,\ \omega_i^{n+1}=\Lambda_{ij}^{n+1}\omega^j,$$
$$\omega_i^{n+2}=\Lambda_{ij}^{n+2}\omega^j,\ \Lambda_{ij}^0=\Lambda_{ji}^0,\ \Lambda_{ij}^\alpha=\Lambda_{ji}^\alpha,$$
$$\nabla\Lambda_{ij}^0+\Lambda_{ij}^0\omega_0^0=\Lambda_{ijk}^0\omega^k,\ \nabla\Lambda_{ij}^{n+2}+2\Lambda_{ij}^{n+2}\omega_0^0=\Lambda_{ijk}^{n+2}\omega^k,$$
$$\nabla\Lambda_{ii}^{n+1}+2\Lambda_{ii}^{n+1}\omega_0^0=\Lambda_{iik}^{n+1}\omega^k\ (\alpha,\beta,\gamma,...=n+1,n+2). \quad (5)$$

3. Let us consider two directions on the surface S_n^1

$$d_i:\omega^1=\omega^2=...=\omega^{i-1}=\omega^{i+1}=...=\omega^n=0 \quad (6)$$

and

$$d_j:\omega^1=\omega^2=...=\omega^{j-1}=\omega^{j+1}=...=\omega^n=0\ (i\neq j). \quad (7)$$

If the condition $d_i[d_jA_0]=0$ (mod L_2) is fulfilled then these directions are referred to as surface conjugate directions and lines (6, 7) are referred to as surface adjoint lines. This implies that the coordinate lines net on surface S_n^1 is the conjugate net, if

$$\Lambda_{ij}^{n+1}=\Lambda_{ij}^{n+2}=0,\ \Lambda_{ii}^{n+1}\Lambda_{jj}^{n+2}-\Lambda_{ii}^{n+2}\Lambda_{jj}^{n+1}\neq 0\ \ (i\neq j). \quad (8)$$

Similarly we determine that coordinate lines on the surfaces S_n^2 and S_n^3 form the conjugate net, if respectively

$$\Lambda_{ij}^0=\Lambda_{ij}^{n+1}=0,\ \Lambda_{ii}^0\Lambda_{jj}^{n+1}-\Lambda_{ii}^{n+1}\Lambda_{jj}^0\neq 0\ \ (i\neq j) \quad (9)$$

and

$$\Lambda_{ij}^0=\Lambda_{ij}^{n+2}=0,\ \Lambda_{ii}^0\Lambda_{jj}^{n+2}-\Lambda_{ii}^{n+2}\Lambda_{jj}^0\neq 0\ (i\neq j). \quad (10)$$

It follows from (8–10).

Theorem 1. If for three surfaces S_n^1, S_n^2, S_n^3 the coordinate net is conjugate on any two of them then it is conjugate as well on the third one.

Let coordinate lines be conjugate on the surface S_n^1 then it follows from the equations (5, 8) that forms ω_i^j ($i\neq j$) are the main and we can assume

$$\omega_i^j=\Lambda_{ik}^j\omega^k\ \ (i\neq j).$$

Closing these equations we obtain

$$d\Lambda_{ik}^j+\Lambda_{ik}^j\omega_0^0-\Lambda_{il}^j\omega_k^l-\Lambda_{it}^l\Lambda_{lk}^j\omega^t+$$
$$+\Lambda_{ik}^0\omega^j+\Lambda_{ik}^{n+1}\omega^j+\Lambda_{ik}^{n+2}\omega^j=\Lambda_{ikl}^j\omega^l. \quad (11)$$

Setting (8) we finish reference point construction (accurate within normalization). If we demand that coordinate net consists of conjugate lines on all three surfaces i. e. conditions (9) attach to conditions (8) then we obtain the particular class of our surface configuration.

2. Focal images

1. Point F_i^j ($i\neq j$) is referred to as pseudo-focus [3] of a straight line A_0A_i if at displacement of point A_0 in a direction A_0A_i a tangent to the line described by the point F_i^j, belongs to hyperplane $L_{n+1}^j=(A_0,A_1,...,A_{j-1},A_{j+1},...,A_{n+1})$.

Let point $F_i^j=x_i^jA_0+A_i$ ($i\neq j$) be pseudo-focus of the straight line A_0A_i then

$$(dF_i^j,A_0,A_1,...,A_{j-1}A_{j+1},...,A_{n+1})|_{\omega^1=\omega^2=...=\omega^{j-1}=\omega^{j+1}=...=\omega^n=0}=0.$$

hence

$$x_i^j=-\Lambda_{ij}^j\ (i\neq j,\text{ on } j-\text{to not summarize}).$$

Thus, (n–1) pseudo-focus exists on each straight line A_0A_i

$$F_i^{1,j}=-\Lambda_{ij}^jA_0+A_i\quad(\text{on}-\text{to not summarize}). \quad (12)$$

Let us denote

$$F_i^{(1)}=-\frac{1}{n-1}\Lambda_{ij}^jA_0+A_i\ \ (\text{on } j-\text{to summarize}). \quad (13)$$

harmonical pole of the point A_0 relative to pseudo-focuses of the straight line A_0A_i.

Pseudo-focuses of the straight lines $A_{n+2}A_i$ and $A_{n+1}A_i$, as well as harmonical poles of the points A_{n+2} and A_{n+1} relative to pseudo-focuses of the straight lines $A_{n+2}A_i$ and $A_{n+1}A_i$ respectively are similarly found in the form

$$F_i^{2,j}=-\Lambda_{ij}^jA_{n+2}+A_i;\quad F_i^{3,j}=-\Lambda_{ij}^jA_{n+1}+A_i$$
$$(j-\text{to not summarize}), \quad (14)$$

$$F_i^{(2)}=-\frac{1}{n-1}\Lambda_{ij}^jA_{n+2}+A_i,\ F_i^{(3)}=-\frac{1}{n-1}\Lambda_{ij}^jA_{n+1}+A_i$$
$$(\text{on } j\text{ -to summarize}). \quad (15)$$

It follows from (12–15).

Theorem 2. If point F_i^1 is a harmonical pole of the point A_0 for straight lines A_0A_i, then these points are also harmonical poles of the points A_{n+2} and A_{n+1} relative to the straight lines A_0A_{n+2} and A_0A_{n+1}, respectively.

Conditions of this have the form $\Lambda_{ij}^j=0$ (on j – to summarize).

If we set $\Lambda_{in}^n=-1$, $\Lambda_{n1}^1=1$, ($i\neq n$, on – not to summarize), then a canonical reference point will be completely constructed. This set is geometrically characterized by the fact that points $F_i^{1,n}=A_0+A_i$ and $F_i^{1,1}=A_0+A_n$, ($i\neq n$) are pseudo-focuses of the straight lines A_0A_i ($i\neq n$) and A_0A_n.

It follows from (14) that in this case points $F_i^{2,n}=A_{n+2}+A_i$ and $F_i^{2,1}=A_{n+2}+A_n$ are pseudo-focuses of the straight lines $A_{n+2}A_i$ ($i\neq n$) and $A_{n+2}A_n$. Points $F_i^{3,n}=A_{n+1}+A_i$ ($i\neq n$) and $F_i^{3,1}=A_{n+1}+A_n$ are pseudo-focuses of straight lines $A_{n+1}A_i$ ($i\neq n$) and $A_{n+1}A_n$.

2. Let us consider the first normal $L_2=(A_0,A_{n+1},A_{n+2})$ of pair surfaces. Point $V=x^0A_0+x^{n+1}A_{n+1}+x^{n+2}A_{n+2}$ is a focus of this plane if $(dM,A_0,A_{n+1},A_{n+2})=0$. Hence $x^0\omega^i+x^{n+1}\omega_{n+1}^i+x^{n+2}\omega_{n+2}^i=0$ and focus line equation have the form $(x^0+x^{n+1}+x^{n+2})=0$.

Thus, focus curve of the plane L_2 degenerates into n-fold straight line.

$$x^0+x^{n+1}+x^{n+2}=0,\quad x^i=0.$$

3. Characteristic element of hyperplane $\Gamma_\alpha^1=(L_n^1,A_\alpha)$ is such linear subspace of this hyperplane the whole first differential neighborhood of which does not go out of this hyperplane.

Let point $M_\alpha=A_0+x^iA^i+t_\alpha A_\alpha$ describe the characteristic element of hyperplane Γ_α then at any ω^i the condition $(dM_\alpha,\Gamma_\alpha^1)=0$ is fulfilled. Hence $x^i\omega_i^\beta+t_\alpha\omega_\alpha^\beta=0$ ($\alpha\neq\beta$, α – it is fixed). As $\omega_\alpha^\beta=0$ at $\alpha\neq\beta$ that $x^i\Lambda_{ij}^\alpha=0$ and as $\det\|\Lambda_{ij}^\alpha\|\neq 0$, $x^i=0$.

Therefore, the straight line A_0A_α is the characteristic element of hyperplane Γ^1_α.

Similarly the straight lines $A_{n+2}A_0$ and $A_{n+2}A_{n+1}$ are the characteristic elements of hyperplanes $\Gamma^2_0=(L^2_n,A_0)$ and $\Gamma^2_{n+1}=(L^2_n,A_{n+1})$ respectively. And straight lines $A_{n+1}A_0$ and $A_{n+1}A_{n+2}$ are the characteristic elements of hyperplanes $\Gamma^3_0=(L^3_n,A_0)$ and $\Gamma^3_{n+1}=(L^3_n,A_{n+2})$ respectively.

4. Let us take straight line A_0A_α and determine focuses of this line. Point $C_\alpha=A_0+t_\alpha A_\alpha$ is a focus of straight line A_0A_α, if $(dC_\alpha,A_0,A_\alpha)=0$. Then we obtain $\omega^i+t_\alpha\omega^i_\alpha=0$ or $(1+t_\alpha)\omega^\alpha=0$, hence $t_\alpha=1$.

Therefore, at any displacement the point $C_\alpha=A_0-A_\alpha$ is a focus of straight line A_0A_α.

Similarly we obtain that point $C=A_{n+1}-A_{n+2}$ is a focus of straight line $A_{n+1}A_{n+2}$.

As $dC_\alpha=(\omega^0_0-\omega^\alpha_\alpha)C_\alpha$ then point C_α is motionless at displacement of points A_0 and A_α on planes S^1_n and S^v_n $(v=2,3)$ and all straight lines A_0A_α connecting proper points of these planes pass through one and the same motionless point C_α.

Point C of straight line $A_{n+1}A_{n+2}$ is similarly characterized.

5. The equation of any hyperplane passing through n-surface L^1_n, may be written down in the form $x_{n+1}x^{n+1}+x_{n+2}x^{n+2}=0$.

Let us assume that this hyperplane is a focal one i.e. it passes through L^1_n and the one infinitely close to it at certain focal displacement. It follows from

$$d(A_0,A_1,\dots,A_n)=(\dots)L^1_n+\omega^\alpha_1(A_0,A_\alpha,A_2,\dots,A_n)+\dots+$$
$$+\omega^\alpha_n(A_0,A_1,\dots,A_{n-1},A_\alpha)$$

that it is possible then and only then when $x_\alpha\omega^\alpha_i=0$ or $x_\alpha\Lambda^\alpha_{ij}\omega^j=0$.

This system has nontrivial solutions in respect to ω^j then and only then when

$$\det\left\|x_\alpha\Lambda^\alpha_{ij}\right\|=0. \qquad (16)$$

Expanding this determinant we obtain the equation of n degree, therefore, in general case there are n focal hyperplanes and n focal displacements on surface S^1_n.

If the coordinate net of lines on the surface S^1_n. is a conjugant net then the equation (16) takes on form

$$(x_{n+1}\Lambda^{n+1}_{11}+x_{n+2}\Lambda^{n+2}_{11})(x_{n+1}\Lambda^{n+1}_{22}+x_{n+2}\Lambda^{n+2}_{22})\dots$$
$$(x_{n+1}\Lambda^{n+1}_{nn}+x_{n+2}\Lambda^{n+2}_{nn})=0.$$

Therefore, on straight line $A_{n+1}A_{n+2}$, we obtain n points each of which determines focal hyperplane and proper focal displacement which is the coordinate one.

If on surfaces S^2_n and S^3_n the coordinate nets are also conjugant then we can find for them similarly the focal hyperplanes and focal displacements.

3. Field of invariant hyperquadric

Let us attach a field of hyperquadric to n-surface

$$C^\alpha_{00}(x^0)^2+C^\alpha_{ij}x^ix^j+2C^\alpha_{0i}x^0x^i+$$
$$+2C^\alpha_{i\beta}x^ix^\beta+2C^\alpha_{0\beta}x^0x^\beta+C^\alpha_{\beta\gamma}x^\beta x^\gamma=0. \qquad (17)$$

Coefficients C^α_{IJ} of hyperquadric fields satisfy the differential equations

$$dC^\alpha_{IJ}=C^\alpha_{LJ}\omega^L_I+C^\alpha_{IL}\omega^L_J+C^\alpha_{IJi}\omega^i. \qquad (18)$$

Let us demand that the field hyperquadrics corresponding to the point A_0, touch the surface S^1_n in this point i.e. they contain this point A_0 and points A_0+dA_0 of a tangent of n-surface. This demand results in the following equations

$$C^\alpha_{00}=0,\quad C^\alpha_{0i}=0.$$

Let us normalize coefficients C^α_{IJ}, assuming $C^\alpha_{0\beta}=\delta^\alpha_\beta$.

Let us also demand that our hyperquadrics are not only tangent to n-surface S^1_n, but the osculating ones as well i. e. points $A_0+dA_0+½d^2A_0$ lie on each hyperquadric accurate within the values of the second order infinitesimal. This demand results in condition $C^\alpha_{ij}=-\Lambda^\alpha_{ij}$.

Therefore, the equation of the field of osculating hyperquadrics takes on form

$$\Lambda^\alpha_{ij}x^ix^j-2x^0x^\alpha+2C^\alpha_{i\beta}x^ix^\beta+C^\alpha_{\beta\gamma}x^\beta x^\gamma=0. \qquad (19)$$

We obtain from the equations (18)

$$dC^\alpha_{ij}=\Lambda^\alpha_{kj}\omega^k_i+\Lambda^\alpha_{ik}\omega^k_j+C^\alpha_{\beta j}\omega^\beta_i+C^\alpha_{i\beta}\omega^\beta_j+C^\alpha_{ijk}\omega^k,$$
$$dC^\alpha_{i\beta}=\delta^\alpha_\beta\omega^0_i+C^\alpha_{\gamma\beta}\omega^\gamma_i+C^\alpha_{j\beta}\omega^j_i+$$
$$+\Lambda^\alpha_{ij}\omega^j_\beta+C^\alpha_{i\gamma}\omega^\gamma_\beta+C^\alpha_{i\gamma k}\omega^k,$$
$$dC^\alpha_{\beta\gamma}=\delta^\alpha_\beta\omega^0_\gamma+C^\alpha_{\varepsilon\gamma}\omega^\varepsilon_\beta+C^\alpha_{i\gamma}\omega^i_\beta+$$
$$+\delta^\alpha_\gamma\omega^0_\beta+C^\alpha_{\beta i}\omega^i_\gamma+C^\alpha_{\beta\varepsilon}\omega^\varepsilon_\gamma+C^\alpha_{\beta\gamma k}\omega^k. \qquad (20)$$

In order to satisfy these conditions of hyperquadric invariance let us consider a number of geometric objects connected invariantly with n-surface S^1_n. Let us assume that the osculating plane of the second order of n-surface fills in the whole space then the relative invariant $I=I(\Lambda^\alpha_{ij})$ of the surface exists [4]. It represents a homogeneous polynomial of degree $4n$ relative to Λ^α_{ij}.

Let us assume

$$V^{ij}_\alpha=\frac{\partial\ln I(\Lambda^\alpha_{ij}}{\partial\Lambda^\alpha_{ij}}$$ and $V^{ij}_\alpha\Lambda^\alpha_{kj}=2\delta^i_k$, $V^{ij}_\alpha\Lambda^\beta_{kj}=2\delta^\alpha_\beta$.

Using the equation (3), we obtain

$$\nabla V^{ij}_\alpha=V^{ij}_\alpha\omega^0_0+V^{ij}_{\alpha k}\omega^k,$$
$$\nabla V^{ij}_{\alpha k}=V^{ij}_\alpha\omega^0_k+V^{il}_\alpha\delta^j_k\omega^0_l+V^{lj}_\alpha\delta^i_k\omega^0_l-V^{ij}_\alpha\Lambda^\beta_{kl}\omega^l_\alpha-$$
$$-V^{ij}_\alpha\Lambda^\gamma_{lk}\omega^l_\gamma+V^{jl}_\alpha\Lambda^\gamma_{lk}\omega^i_\gamma+V^{ij}_{\alpha kl}\omega^l. \qquad (21)$$

Let us assume

$$M^i_\alpha=\frac{(n+4)^2}{(n+2)^2}V^{ij}_{\alpha j}+\Lambda^\beta_{jkl}\left(V^{jk}_\alpha V^{il}_\beta+\frac{(n+4)^2}{(n+2)^2}V^{kl}_\beta V^{ji}_\alpha\right),$$

then

$$\nabla M^i_\alpha=-K^{i\beta}_{\alpha j}\omega^j_\beta+M^j_{\alpha i}\omega^i,$$

where

$$K_{\alpha j}^{i\beta} = \left(\frac{(n+4)^2}{n+2} - 2n \right) \delta_j^i \delta_\alpha^\beta - 2\Lambda_{lk}^\beta \Lambda_{js}^\gamma V_\beta^{sl} V_\gamma^{ik},$$

and

$$\nabla K_{\alpha j}^{i\beta} = K_{\alpha jl}^{i\beta} \omega^l.$$

As $\det\|K_{\alpha j}^{i\beta}\| \neq 0$ then we can introduce an inverse tensor $\tilde{K}_{\alpha j}^{i\beta}$ by the formulas $K_{\alpha j}^{i\beta}\tilde{K}_{\varepsilon i}^{s\alpha} = \delta_i^s \delta_\varepsilon^\beta$, $K_{\alpha j}^{i\gamma}\tilde{K}_{\gamma t}^{j\varepsilon} = \delta_t^i \delta_\alpha^\varepsilon$, the components of which satisfy the equations $\nabla \tilde{K}_{\alpha j}^{i\beta} = \tilde{K}_{\alpha jt}^{i\beta}\omega^t$.

Let us consider the values

$$l_\alpha^j = M_\beta^i \tilde{K}_{\alpha i}^{j\beta}, \quad \nabla l_\alpha^j + \omega_\alpha^j = l_{\alpha i}^j \omega^i,$$

$$l_i^0 = \frac{n+4}{2(n+2)} \Lambda_{ij}^\alpha l_\alpha^j - \frac{1}{2(n+2)} V_{\alpha j}^{lj} \Lambda_{il}^\alpha,$$

$$\nabla l_i^0 + l_i^0 \omega_o^0 + \omega_i^0 = l_{ij}^0 \omega^j,$$

$$\nabla l_{ij}^0 = -2 l_{ij}^0 \omega_0^0 - l_i^0 \omega_j^o - l_j^0 \omega_i^0 + \Lambda_{ij}^\alpha l_k^\alpha \omega_\alpha^k - \Lambda_{ij}^\alpha \omega_\alpha^0 + l_{ijk}^0 \omega^k.$$

The introduced quasi-tensor l_i^0 defines the invariant normal of the second sort $x^0 - l_i^0 x^i = 0$; $x^\alpha = 0$ of n-surface S_n^1.

Let us consider a set of values

$$l_\alpha^0 = \frac{1}{n}(V_\alpha^{ij} l_{ij}^0 - V_\alpha^{ij} l_i^0 l_j^0) + l_\alpha^i l_i^0,$$

$$\nabla l_\alpha^0 = -l_\alpha^0 \omega_0^0 - l_\alpha^i \omega_i^0 - \omega_\alpha^0 + l_{\alpha k}^0 \omega^k.$$

The geometric object (l_α^0, l_α^i) is the object of equipment of n-surface S_n^1. It determines the equipped plane $x^0 - l_\alpha^0 x^\alpha = 0$; $x^i - l_\alpha^i x^\alpha = 0$ and normal of the first sort $x^i - l_\alpha^i x^\alpha = 0$ of this plane.

If we now assume

$$C_{i\beta}^\alpha = \Lambda_{ij}^\alpha l_\beta^j - \delta_\beta^\alpha l_i^0;$$

$$C_{\beta\gamma}^\alpha = \Lambda_{ij}^\alpha l_\beta^i l_\gamma^j - l_{(\beta}^0 \delta_{\gamma)}^\alpha + C_{i(\beta}^\alpha l_{\gamma)}^i,$$

in equation (19) of osculating hyperquadrics then all the equations of invariance (20) are fulfilled.

Thus, we obtain a field of invariant hyperquadrics attached to the surface S_n^1

$$\Lambda_{ij}^\alpha x^i x^j - 2x^i x^\alpha + 2(\Lambda_{ij}^\alpha l_\beta^j - \delta_\beta^\alpha l_i^0) x^i x^\beta + $$
$$+(\Lambda_{ij}^\alpha l_\beta^i l_\gamma^j - l_{(\beta}^0 \delta_{\gamma)}^\alpha + C_{i(\beta}^\alpha l_{\gamma)}^i) x^\beta x^\gamma = 0. \quad (21)$$

If a reference point is canonized then the equation (21) takes on form

$$\Lambda_{11}^\alpha (x^1)^2 + \Lambda_{22}^\alpha (x^2)^2 + \ldots + \Lambda_{nn}^\alpha (x^n)^2 - 2x^0 x^\alpha = 0. \quad (22)$$

4. Invariant projective transformations

1. Let us consider the equation of the 1-family

$$\omega^i = t^i \Theta; D\Theta = \Theta \wedge \Theta_1; \quad dt^i - t^i \omega_0^0 + t^j \omega_j^i = t^i{}_j \omega^j. \quad (23)$$

The line described by a point A_0 corresponds to it on n-surface S_n^1. Point $X = x^i A_i$ belonging to the second normal L_{n-1} of n-surface S_n^1, along (23) describes the line with a tangent $TX(t)$. We have

$$\frac{dX}{\Theta} = (\ldots)^i A_i + x^i \omega_i^0 A_0 + x^j \omega_i^\alpha A_\alpha =$$
$$= (\ldots)^i A_i + \Lambda_{ij}^0 x^i t^j A_0 + \Lambda_{ij}^\alpha x^i t^j A_\alpha.$$

Linear space strained on L_{n-1} and $TX(t)$ crosses with $L_2^1 = (A_0 A_{n+1} A_{n+2})$ in point

$$Y = (L_{n-1}, TY(t)) \cap L_1^1 = \Lambda_{ij}^0 x^i t^j A_0 + \Lambda_{ij}^\alpha x^i t^j A_\alpha.$$

Along (23) point Y describes the line with a tangent $TY(t)$. We have

$$\frac{dY}{\Theta} = (\ldots)^0 A_0 + (\ldots)^\alpha A_\alpha + (\Lambda_{ij}^0 x^i t^j t^k \delta_k^l + \Lambda_{ij}^\alpha \Lambda_{\alpha k}^l x^i t^j t^k) A_l.$$

Linear space strained on L_2^1 and $TX(t)$, crosses with L_{n-1} in point

$$\tilde{X} = (L_2^1, TX(t)) \cap L_{n-1} = \tilde{x}^i A_i;$$
$$\tilde{x}^i = \Lambda_{ij}^0 x^i t^j t^k \delta_k^l + \Lambda_{ij}^\alpha \Lambda_{\alpha k}^l x^i t^j t^k. \quad (24)$$

The relation (24) determines projective transformation of $(n-1)$-surface L_{n-1} into itself which is defined by matrix (Π_i^j): $\Pi_i^j = (\Lambda_{il}^0 \delta_k^j + \Lambda_{il}^\alpha \Lambda_{\alpha k}^j) t^l t^k$. This transformation is the transformation W, if $\Pi_i^i = 0$ [5].

Thus, in $(n-1)$-surface L_{n-1} we obtain quadric, to each point of which the transformation W of plane L_{n-1} into itself corresponds. This quadric has the equation

$$(\Lambda_{ij}^0 + \Lambda_{ik}^\alpha \Lambda_{\alpha j}^k) x^i x^j = 0; x^\alpha = 0; \; x^0 = 0. \quad (25)$$

Cone

$$(\Lambda_{ij}^0 + \Lambda_{ik}^\alpha \Lambda_{\alpha j}^k) x^i x^j = 0; \quad x^\alpha = 0. \quad (26)$$

corresponds to quadric (25) in n-plane L_n^1.

If the equations (8, 9) are fulfilled then the equation (26) has the form

$$(\Lambda_{ii}^0 + \Lambda_{ii}^{n+1} + \Lambda_{ii}^{n+2})(x^i)^2 = 0; \quad x^\alpha = 0. \quad (27)$$

2. Let us consider point $X = x^0 A_0 + x^\alpha A_\alpha$ belonging to 2-plane L_2. Along (23)

$$\frac{dX}{\Theta} = (\ldots)^0 A_0 + (\ldots)^\alpha A_\alpha + (x^0 \omega_0^k + x^\alpha \omega_\alpha^k) A_k,$$

then

$$Y = (L_2^1, TX(t)) \cap L_{n-1} = (x^0 \omega_0^k + x^\alpha \omega_\alpha^k) A_k = y^k A_k.$$

We obtain

$$\tilde{X} = (L_{n-1}, TX(t)) \cap L_2^1 = \tilde{x}^0 A_0 + \tilde{x}^\alpha A_\alpha,$$

from

$$\frac{dY}{\Theta} = (\ldots)^i A_i + y^k \omega_k^0 A_0 + y^k \omega_k^\alpha A_\alpha,$$

where

$$x^0 = (x^0 \omega^i + x^\alpha \omega_\alpha^i) \omega_i^0; \tilde{x}^\beta = (x^0 \omega^i + x^\alpha \omega_\alpha^i) \omega_i^\beta. \quad (28)$$

Therefore (28) defines projective transformation of 2-plane L_2 into itself specified by matrix

$$(\tilde{\Pi}_{\alpha_1}^{\beta_1}), (\alpha_1, \beta_1 = 0, n+1, n+2) : \tilde{\Pi}_0^0 = \omega_i^0 \omega^i =$$
$$= \Lambda_{ij}^0 t^i t^j; \tilde{\Pi}_\alpha^0 = \omega_\alpha^i \omega_i^0 = \Lambda_{ij}^0 t^i t^j;$$
$$\tilde{\Pi}_0^\alpha = \omega_i^\alpha \omega^i = \Lambda_{ij}^\alpha t^i t^j; \tilde{\Pi}_\alpha^\beta = \omega_i^\beta \omega_\alpha^i = \Lambda_{ij}^\beta t^i t^j.$$

This transformation is transformation W, if $(\Lambda_{ij}^0 + \Lambda_{ij}^{n+1} + \Lambda_{ij}^{n+2}) t^i t^j = 0$.

Therefore, we obtain cone

$$(\Lambda_{ij}^0 + \Lambda_{ij}^{n+1} + \Lambda_{ij}^{n+2}) x^i x^j = 0; \quad x^\alpha = 0, \quad (29)$$

in n-plane L_n^1. 1-families (23) giving transformations W of 2-plane L_2^1 into themselves correspond to its generators.

If the reference point is canonized then the equations (29) are reduced to the equations (27).

3. Let point $X=x^\alpha A_\alpha$, belonging to equipping straight line $l_1=(A_{n+1},A_{n+2})$ of n-surface S_n^1 be given. Reasoning similarly, we obtain transformation $\tilde{x}^\alpha=x^\beta\omega_\beta^i\omega_i^\alpha$ of straight line into itself which is defined by matrix (Π_α^β): $\Pi_\alpha^\beta=\Lambda_{\alpha j}^i\Lambda_{ik}^\beta t^k t^k$.

If this is transformation W, then we obtain cone $(\Lambda_{ij}^{n+1}+\Lambda_{ij}^{n+2})x^ix^j=0$; $x^\alpha=0$ in n-plane L_n^1. 1-families (23) giving transformations W of straight line l_1 correspond to its generators. If the reference point is canonical then the equations take on form

$$(\Lambda_{ii}^{n+1}+\Lambda_{ii}^{n+2})(x^i)^2=0;\ x^\alpha=0.$$

If the reference point is canonized (accurate within normalization), then the similar constructions with the same properties may be also obtained for surfaces S_n^2 and S_n^3.

Conclusion

It was shown that the third surface possessing the same properties that the initial surfaces S_n^1 and S_n^2 may be attached to a pair of n-surfaces. The canonical reference point was constructed provided that the coordinate net of lines is conjugate on these surfaces. The private class of a pair of n-surfaces when the coordinate net is conjugate on all surfaces was noted. A series of invariant geometric images for each of surfaces of constructed triplet of n-surfaces was found.

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Received on 17.04.2006

UDC 541.127

DETERMINATION OF AMBIGUITY ELLIPSE PARAMETERS AT TWO DIMENSIONS USING GENERALIZED METHOD OF UNCERTAINTY CENTRE

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Definition of empirical dependence parameters at two dimensions using ambiguity ellipse algorithm in a generalized method of uncertainty centre has been considered. The algorithm of optimal parameters definition is offered.

To ascertain the laws of some phenomena the experimental investigations are carried out. During them the values of one or another physicochemical magnitude are measured. At physical experiment processing the empirical models or formula are often used. They include experimentally inaccurately measured magnitudes; inaccuracy is as a rule taken into account in output variables. The case when the output and input variables of the model are timed measured, especially in the case of developing concrete techniques of physical experiment data analysis, is insufficiently represented in scientific literature and is urgent. Statement and investigation of problem solvability of parameters uncertainty set immersion of two-dimensional linear parameter-oriented model at accurate measurement of input variables and inaccurate measurement of output variables were studied in detail in papers [1–4].

Algorithm of parameter uncertainty set immersion of two-dimensional linear dependence into ambiguity ellipse at interval assignment of input and output variables was examined in [5–8]. In these papers inaccuracies which influenced the result of the algorithm operation were made when ambiguity ellipse parameters being determined by generalized method of uncertainty centre.

Let us examine the algorithm [5–8] more detailed. At two dimensions the experimental points should meet the system of interval equations

$$[y]_1=[a]+[b][x]_1,$$
$$[y]_2=[a]+[b][x]_2.$$