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STATIONARY ROTATION STABILITY OF UNBALANCED ROTOR WITH AUTOBALANCING DEVICE WITH LIQUID ON FLEXIBLE SHAFT

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The condition of rotor rotation stability with liquid autobalancing device consisting of chamber, float and incompressible homogeneous liquid filling the space between them has been obtained. The restoring force and forces of internal and external friction take effect on the rotor. The latter depend linearly respectively on strain rate and absolute velocity of rotor connection point to the shaft.

To remove rotating bodies unbalance various autobalansing device with liquid (ADL) are used [1]. When using such systems it is necessary to know critical angular velocities at which stationary rotations stability is disturbed. In a number of papers, for example in [2, 3], the approximate conditions of stability of balanced cylinder steady rotation filled partially with liquid which are difficult to be applied to the systems of ADL are obtained. The immediate investigation of stability of rotors rotation with liquid ADL is not described in literature.

In the proposed paper by analogy with [4] for unbalanced discs at flexible shaft the stability of rotor rotation with liquid ADL without free surface when influenced by forces of internal and external friction is studied. The nature of these forces is stated in detail in [4]. So the forces of external friction are caused by viscous resistance of environment, supports, special dampers and depend on rates of absolute displacements of rotor and shaft points; forces of internal friction are caused by material particles resistance and are taken as proportional to the shaft strain rate to a first approximation. The investigation of the considered forces ratio influence on stability of rotor rotation with ADL is of some interest. The solution of such problem for the unbalanced disc without ADL is given in [4]. Let rotor with ADL is fixed symmetrically with respect to the supports of vertical flexible shaft passing through its geometric centre O_1 (Figure).

ADL consists of the correction chamber – 1 with the height h, the float – 2 and homogeneous incompressible fluid – 3 filling the space between their walls. Centre of rotor mass (point P) is shifted from O_1 to the distance $O_1P=e$. Point is a projection of shift support axis to the plane of motion. At system rotation the shift sags at a place of rotor attaching point per the value O_2O_1 the float for which geometric and material symmetry axes coincide as well in floating gyroscopes [5] is centered on rotation axis O_2 due to pressure forces and fluid flows to the side of sag. Let us suppose that at rotor disturbed motion fluid separation from walls does not occur and float centering remains. In this case fluid layer centre of mass is on the line of O_2O_1 centers in point G. The stated suppositions allow mapping out a hydrodynamic problem.

According to [4] let us introduce in a plane of motion of points O_1, G, P two coordinate systems with common origin in point O_2: fixed O_2xy and mobile $O_2\xi\eta$, axis $O_2\xi$ which is parallel to the segment O_1P. Laws of angular motion of rotor and system $O_2\xi\eta$ are determined by the same rotation angle $\beta(t)$ (t – time) therefore, rotor in the mobile coordinate system can move only steadily. Let us take the coordinates in a fixed system of point O_1 which are denoted by x, y as generalized coordinates. We consider that rotor is influenced from the side of a shaft by elastic force $\bar F_c=-c\overline{O_1O_2}$, forces of internal and external friction $\bar F_k=-k\bar V^r_{O_1}$ and $\bar F_\chi=-\chi\bar V_{O_1}$, applied in the point O_1 proportional correspondingly to a shaft bending $\overline{O_1O_2}=x\cdot\bar i+y\bar j$; rate of point O_1 in the mobile coordinate system $O_2\xi\eta$

$$\bar V^r_{O_1} = (\dot x + y\dot\beta)\bar i + (\dot y - x\dot\beta)\bar j \qquad (1)$$

and absolute velocity

$$\bar V_{O_1} = \dot x\cdot\bar i + \dot y\cdot\bar j\ , \qquad (2)$$

where $\bar i$, $\bar j$ are the orts of rectangular coordinate system O_2xy. Equilibrium condition of the enumerated forces and system inertial forces is written down in the form of

$$-c\overline{O_2O_1} - k\bar V^r_{O_1} - \chi\bar V_{O_1} - m_1\bar a_P - m_2\bar a_G = 0. \qquad (3)$$

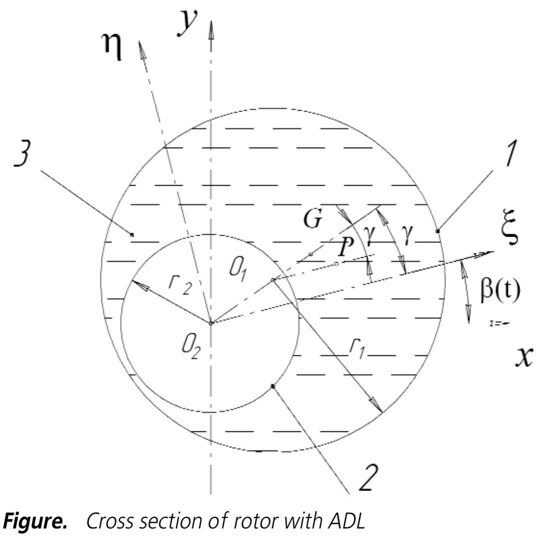

Figure. *Cross section of rotor with ADL*

Therein m_1, m_2 and $\bar a_P$, $\bar a_G$ are masses and acceleration of mass centers of rotor and liquid respectively. The latter are calculated through the coordinates of points P and G in the fixed reference system.

$$x_P = x + e\cdot\cos\beta,\quad y_P = y + e\cdot\sin\beta,$$
$$x_G = r\cdot x,\quad y_G = r\cdot y, \qquad (4)$$

where $r=r_1^2/(r_1^2-r_2^2)$, r_1 and r_2 are the radii of the chamber and the float respectively. Projecting (3) on the axes x, y subject to (1, 2) and (4) we obtain the differential equations of motion of rotor with ADL

$$\begin{aligned} &m\ddot{x}+(p+\chi)\dot{x}+cx+py\dot{\beta}=m_1e(\dot{\beta}^2\cos\beta+\ddot{\beta}\sin\beta);\\ &m\ddot{y}+(p+\chi)\dot{y}+cy-px\dot{\beta}=m_1e(\dot{\beta}^2\sin\beta-\ddot{\beta}\cos\beta); \end{aligned} \quad (5)$$

in which the notation $m=m_1+r\cdot m_2$ is entered.

Supposing in (5) that $e=0$, $m_2=0$ and $\chi=0$ we have the equation of motion of balanced disk mass centre on an elastic shaft at internal friction influence given in [6]. Comparing the equations in [6] with (5) we correlate the ratio between the coefficient p and coefficient k entering into dependence of stress on deformation of shaft visco-elastic material $\sigma=E\varepsilon+\kappa\dot{\varepsilon}$, $p=c\kappa/E$. Therein there is shaft flexural stiffness for simply supported shaft with the length l with a disk in the middle [6] $c=48EI/l^3$; E is the module of material elasticity; I is the inertia moment of sectional area relative to the centroidal axis of shaft bending perpendicular plane. At $\dot{\beta}=\omega$=const of the equation (5) the calculation is admitted

$$x_c=A\cos(\omega t+\gamma) \quad y_c=A\sin(\omega t+\gamma);$$

$$A=\frac{m_1e\omega^2}{\sqrt{(c-m\omega^2)^2+\chi^2\omega^2}}; \quad \mathrm{tg}\gamma=-\frac{\chi\omega}{c-m\omega^2}; \quad (6)$$

where A is the shaft bending, γ is the motion phase angle (the angle between rotor unbalance vector $\overline{O_1P}$ and bending deflection $\overline{O_2O_1}$).

It is seen from (6) that the given calculation corresponding to the stationary rotation of rotor with FBD does not depend on the forces of internal friction. It is obvious as the shaft deformation does not change at such motion. The calculation (6) is studied in detail in [7].

To study the stability of the rotation under consideration let us enter the deviations $x'=x-x_c$, $y'=y-y_c$. Supposing that angular velocity ω remains constant the disturbed motion equation is obtained from (5) subject to (6).

$$\begin{aligned} &m\ddot{x}'+(p+\chi)\dot{x}'+cx'+p\omega y'=0;\\ &m\ddot{y}'+(p+\chi)\dot{y}'+cy'-p\omega x'=0. \end{aligned} \quad (7)$$

Characteristic equation for the system (7) has the form

$$a_0\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda^1+a_4=0, \quad (8)$$

where $a_0=m^2$, $a_1=2m(p+\chi)$, $a_2=(p+\chi)^2+2cm$, $a_3=2c(p+\chi)$, $a_4=c^2+\chi^2\omega^2$.

Gurvits matrix H of the coefficients of equation (8) according to [8] is written

$$H=\begin{Vmatrix} a_1 & a_3 & 0 & 0\\ a_0 & a_2 & a_4 & 0\\ 0 & a_1 & a_3 & 0\\ 0 & a_0 & a_2 & a_4 \end{Vmatrix}.$$

The conditions of stability are of the form of inequations [8]

$$\Delta_i>0;\ i=1...4, \quad (9)$$

where Δ_i are the main diagonal minors of matrix H.

Inequations (9) are fulfilled if

$$\omega<\omega^*=(1+\frac{\chi}{p})\sqrt{\frac{c}{m_1}}\cdot\sqrt{\frac{1}{\mu}}, \quad \mu=\frac{m}{m_1}>1. \quad (10)$$

At $\mu=1$ we obtain the condition of stability of rotor stationary rotation without ADL given in [4]. Comparing (10) with a congruent inequation in [4] we conclude that the limiting value of angular velocity at stationary rotation of rotor with ADL $\omega^*(\mu)$ is less than for the rotor without ADL. It is shown in [7] that fluid ADL decreases shaft bending and system unbalance at angular velocities

$$\omega>\omega_*=\sqrt{\frac{c}{m_1}}\cdot\sqrt{\frac{2}{1+\mu}}. \quad (11)$$

It follows from (10, 11) that steady stationary rotation of rotor with fluid ADL decreasing vibration is possible at angular velocities meeting the condition $\omega_*<\omega<\omega^*$.

Conclusions

Autobalansing device with liquid connected to the rotor decreases angular critical speed of the shaft the transition through which results in system rotation instability. Stable system operation is realized in the range of angle velocities determined by the parameters of rotor and autobalancing device.

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