МОДЕЛИРОВАНИЕ В НАУЧНЫХ ИССЛЕДОВАНИЯХ

ANALYSIS OF THE BEHAVIOUR OF THE ABSOLUTE MINIMUM OF MINIMAX NORMALIZATION FACTOR

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ИССЛЕДОВАНИЕ ПОВЕДЕНИЯ АБСОЛЮТНОГО МИНИМУМА МИНИМАКСНОГО НОРМИРОВОЧНОГО МНОЖИТЕЛЯ

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Abstract. В этой статье исследуется поведение минимаксного значения нормировочного множителя в зависимости от кратности периодов поставок и соотношения стоимостей партий поставок в многопродуктовой модели управления запасами с ограничением оборотного капитала для двух видов товаров.

Keywords. Оборотный капитал, нормировочный множитель, многопродуктовая модель, оптимизация, управление.

Introduction. The most important aspect of financial and economic activity of any enterprise is the effective use of material and industrial stocks. The normalization factor *k* is defined as the ratio of the maximum total value of stocks *Ymax* to the sum of their maximum value [1]. The value of the normalization factor determines the size of the working capital invested in the stocks forming the enterprise, and the error in the estimation of the normalization factor, and hence in the amount of working capital can be costly for the company. The purpose of this work is to investigate the behavior of the absolute minimum of minimax normalization factor.

Inventory model, where multiple frequency of the shipments is any number from 2 to ∞. In order to obtain a general formula for calculating minimax normalization factor for the two types of goods with any multiplicity of delivery periods, we introduce a parameter that takes into account the multiplicity of the two periods of the supply of goods $m=\frac{T_1}{T_2}$, where *m* is any integer from 2 to ∞. In general, we believe that the optimality condition is the equality of the two dominant values of local maxima A_1A_4 and B_2B_4. The rest of the local maxima (their number is equal to *m-1*) yield to dominant maxima in value, therefore they are excluded from the analysis. We calculate the values of the dominant local maxima [2]. The first local maximum $A_1A_4 = A_3A_4$, $A_1A_4 = q_1p_1$, $A_3A_4 = A_1A_2$, wherein$A_1A_2 = q_2p_2\frac{\theta_2}{T_2}$. Then $A_1A_4 = q_1p_1 + q_2p_2\frac{\theta_2}{T_2}$. The second local maximumB_2B_4=B_2B_3+B_3B_4. WhereinB_2B_3=$q_1p_1\frac{mT2-\theta2}{mT2}$; B_3B_4=q_2p_2. Then B_2B_4=q_1p_1+$q_2p_2 - \frac{q_1p_1}{m}\frac{\theta2}{T2}$. At the point of minimax local maxima are equal to each other, so we equate values $A_1A_4^* = B_2B_4^*$ and obtain

$$A_1A_4\frac{\theta_2^*}{T_2} = B_2B_4\frac{\theta_2^*}{T_2} = q_1p_1 + q_2p_2\frac{\theta_2^*}{T_2} = q_1p_1 + q_2p_2 - \frac{q_1p_1}{m}\frac{\theta_2^*}{T_2} = \left(\frac{q_1p_1}{m} + q_2p_2\right)\frac{\theta_2^*}{T_2} = q_2p_2.$$

Hence the optimal relative shift of the second delivery of goods is

$$\frac{\theta_2^*}{T_2} = \frac{q_2p_2}{\frac{q_1p_1}{m} + q_2p_2} = \frac{mq_2p_2}{q_1p_1 + mq_2p_2},$$

or via γ_2: $\frac{\theta_2^*}{T_2} = \frac{m\gamma_2}{+m\gamma_2}$. Then $\frac{\theta_1^*}{T_2} = 1 - \frac{\theta_2^*}{T_2} = 1 - \frac{m\gamma_2}{1+m\gamma_2} = \frac{1}{1+m\gamma_2}$.

We calculate the value of the optimal local maximum

$$A_1A_4 = \frac{\theta_2^*}{T_2} = Y_{minmax} = q_1p_1 + q_2p_2\frac{\theta_2^*}{T_2} = q_1p_1 + q_2p_2\frac{mq_2p_2}{q_1p_1 + mq_2p_2} = \frac{(q_1p_1)^2 + m(q_1p_1 \times q_2p_2) + m(q_2p_2)^2}{q_1p_1 + mq_2p_2}.$$

Then the minimax normalization factor for the two products with an arbitrary value of the consignment deliveries and multiple frequency is equal to

$$K_{m;1}^{(2)} = \frac{Y_{\min\max}}{Y_{\Sigma}} = \frac{(q_1p_1)^2 + m(q_1p_1 \times q_2p_2) + m(q_2p_2)^2}{(q_1p_1 + mq_2p_2)(q_1p_1 + q_2p_2)} = \frac{(q_1p_1)^2 + m(q_1p_1 \times q_2p_2) + m(q_2p_2)^2}{(q_1p_1)^2 + (m+1)(q_1p_1 \times q_2p_2) + m(q_2p_2)^2} =$$

$$= 1 - \frac{q_1p_1 \times q_2p_2}{(q_1p_1)^2 + (m+1)q_1p_1 \times q_2p_2 + m(q_2p_2)^2}.$$

You can convert this expression with the Y_2. Then it takes the form

$$K_{m;1}^{(2)} = 1 - \frac{\gamma_2}{1 + (m+1)\gamma_2 + m(\gamma_2)^2}.$$

As seen from the formula obtained, the normalization factor $K_{m;1}^{(2)}$ – is a function of the variables γ_2 and m. We investigate the behavior $K_{m;1}^{(2)}$ changing $\gamma 2$ from 0 to ∞. If $\gamma_2 = 0$, then $K_{m;1}^{(2)} = 1$. If $\gamma_2 \to \infty$, then

$$\lim_{\gamma_2 \to \infty} K_{m;1}^{(2)}(\gamma_2) = \lim_{\gamma_2 \to \infty}\left(1 - \frac{\gamma_2}{1 + (m+1)\gamma_2 + m(\gamma_2)^2}\right) = \lim_{\gamma_2 \to \infty}\left(1 - \frac{\frac{\gamma_2}{\gamma_2}}{\frac{1}{\gamma_2} + \frac{(m+1)\gamma_2}{\gamma_2} + \frac{m(\gamma_2)^2}{\gamma_2}}\right) = \lim_{\gamma_2 \to \infty}\left(1 - \frac{1}{\frac{1}{\gamma_2} + (m+1) + m\gamma_2}\right) = 1.$$

Thus, the left and right ends of the range of values of the normalization factor $K_{m;1}^{(2)}$ takes the value of one. Because of the function is continuous and has no singular points, there is its minimum value in this range. To define it, we must find the partial derivative functions γ_2 and equate it to zero

$$\frac{\partial K_{m;1}^{(2)}}{\partial \gamma_2} = \frac{d}{d\gamma_2}\left(1 - \frac{\gamma_2}{1 + (m+1)\gamma_2 + m(\gamma_2)^2}\right) = \frac{m(\gamma_2)^2 - 1}{\left[1 + (m+1)\gamma_2 + m(\gamma_2)^2\right]^2}.$$

Denominator obviously does not vanish, so we equate to zero the numerator $m(\gamma_2)^2 - 1 = 0$, hence $\gamma_2^{*(1)} = -\frac{1}{\sqrt{m}}$. The first root of this equation is $\gamma_2^{*(1)} = \frac{1}{\sqrt{m}}$,

The second root $\gamma_2^{*(1)} = -\frac{1}{\sqrt{m}}$. It is obvious that the second root is negative and it does not make economic sense. We find the absolute minimum of minimax normalization factor

$$\min_{\gamma_2} K_{m;1}^{(2)} = K_{m;1}^{(2)*}\left(\gamma_2^* = \frac{1}{\sqrt{m}}\right) = 1 - \frac{\gamma_2^*}{1 + (m+1)\gamma_2^* + m(\gamma_2^*)^2} = 1 - \frac{\frac{1}{\sqrt{m}}}{1 + (m+1)\frac{1}{\sqrt{m}} + m\left(\frac{1}{\sqrt{m}}\right)^2} =$$

$$= 1 - \frac{1}{1 + m + 2\sqrt{m}}.$$

Behavior of absolute minimum minimax normalization factor $K_{m;1}^{(2)}$ depending on the ratio of the value of batches of supplies Y_2^*, which is optimal for each possible parameter of multiplicity of periods of supplies of two goods m.

Conclusions. In the course of this work, the behavior of the absolute minimum minimax normalization factor was investigated. The closer the values of m and Y_2^* unit, the lower the value of the minimax normalization factor, the closer it is to the absolute minimum of minimax normalization factor.

References

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