Modeling of seismic field in porous medium:
Simulation study of single pore and pore ensemble effects

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Abstract. The article considers the effect of porous media on elastic wave field. Based on numerical modeling, diffraction pattern of the wave propagating through a single pore in carbonates has been produced. Matrix properties (calcite and dolomite) and fluid (water) are modeled based on thin core section image. The qualitative comparison with the available computational data has been performed. Provided that ensemble of pores is involved, the effect of porous medium on seismic field has been studied. For comparison with experimental data the model of porous sintered aluminum Al-6061 has been considered. The processing of numerical modeling results made it possible to estimate average velocities in the model of porous aluminum and compare them with physical modeling data. The provided estimates have indicated qualitative (single pore) and quantitative (ensemble of pores) correlation of simulation and experiment results.

1. Introduction

Porous media and their effect on the elastic wave field has been a current issue for the past few decades. This interest is due to both material engineering objectives and hydrocarbon exploration. These two different knowledge domains are interrelated in physical modeling. For example, in laboratory modeling of seismic waves in porous media several types of solids including aluminum \cite{1, 2}, soda-lime glass \cite{3} can be used as a matrix material. In laboratory experiments both statistical and dynamic characteristics are estimated, P- and S- wave velocities being basic types among them. Considering porous media, the main disadvantage of laboratory models is that it is difficult to measure their primary properties – porosity, while elastic matrix model results in problems associated with experiment producibility.

Computer modeling does not have this disadvantage; however, it requires qualitative calibration of results and quantitative wherever it is possible. The analysis of numerical results in wave field calculation in porous media implies estimation of the wave velocity change. The comparison of these estimates with physical modeling data is referred to as quantitative calibration. For qualitative analysis one may refer to diffraction patterns, wavefront comparison, particle-motion polarization and so on.

In the present research computer modeling of total field of elastic waves (seismic field) has been performed. In solving an inverse problem an explicit second-order conditionally stable scheme is applied \cite{4}, involving a set of modifications \cite{5} to carry out calculations for a large range of wavelengths ($> 100 \lambda$).
2. Problem setting
Modeling of total wave field is performed for two problems, two different medium models and two scenarios.

Problem one- the model of a single large-size pore filled with fluid and propagation of elastic pores is subjected to analysis. In this scenario calculations performed in this model compare the total wave field with calculation results (i.e. comparison of qualitative patterns).

Problem two- plane wave propagation in a porous layer with random pore distribution is considered. Calculations performed in this model describe how velocity of P-wave propagation changes due to porosity. Calculation involves two porosity values for which laboratory modeling data are available.

If a single pore is being considered, calculation results are compared with finite-difference modeling [6]. To calculate wave field induced in the media with ensemble of pores, the model equivalent to porous aluminium (sintered Al-6061) has been produced [2]. The choice of the material is made due to existing quantitative physical modeling data.

Linearized finite-difference method proposed by J Wilkinson is used to solve both problems [4, 5].

3. Numerical modeling
The effect of a single pore on propagating seismic wave was considered. One of the recent publications [6] is devoted to the calculation of seismic field diffraction induced by a single pore for different frequencies. It involves an image of carbonate rock section and simplified matrix of elastic characteristics for analysis.

The problem of plane wave propagation was solved through the model of medium sample introduced from the above-mentioned-research [6]. Figure 1 presents geometry of the first problem, a model containing one large-size pore, incident-wave signal. The signal (elastic wave) is represented by a damped sinusoid with wavelength which is approximately 10 times less than the characteristic “pore thickness” L (shown figure 1). In this case, calculation L is 200 computational cells; wavelength $\lambda = 20$ computational cells.

![Figure 1. P-wave velocity matrix in the sample, m/s (according to [6]), an arrow indicates direction of plane-wave propagation represented by a damped sinusoid.](image)

In this scenario the main peculiarity of diffraction is dissipation of incident wavefront around a pore (figure 2). Pore material (fluid) has substantially lower impedance. Interaction of reflections and refractions caused by a single pore results in distinctive local attenuation of wavefront is observed in the figure. Figure 2 represents a shadow-graph of wave fields. Black and white colors are extrema of...
alternating damped sinusoid. Grey background in figure 2 indicates areas where there are no disturbances.

Figure 2. Full-waveform seismic data (wave pattern) resulting from impulse-pore interaction. An arrow indicates direction of P wave incidence representing a damped sinusoid. L indicates pore location.

At a later stage (figure 3 a, b) when a wave has travelled through the pore, reflected off the model boundary and travelled through the pore again, wave pattern is almost random. Small sections with P-wave trains can be distinguished (P indicates these areas in figure 3). As we compare our calculations with the results given in [6], figure 10 from the mentioned publication is presented in figure 3, b.

Figure 3. a – present calculation result, b – calculations from ([6], figure 10). P indicates observable P-wave trains in both calculations/
It is obvious that in both cases a pore is affected by short-period vibrations (black and white colors in the figures are wave extrema); pore margins are clearly traced in seismic profile. In figure 3, an event time corresponds to that of pseudo-random field formation which can be observed in [6]. Thus, solution to the first problem has indicated that a single pore can be displayed in a wave field due to contrasting properties, with apparent diffraction patterns being a constituent feature of the wave field. This is a kind of qualitative test for calculation program which has been further applied to the solution of porous layer problem.

Elastic wave incident in a medium which contains a layer with random arrangement of pores is being considered. Two porosity values (9% and 17%) have been considered, since there were data of laboratory modeling for these values in publications [2]. The problem geometry is shown in figure 4.

Figure 4. Geometry of wave propagation in a porous medium.

A plane compressional wave strikes AB boundary. The medium is homogenous above EF line, with aluminum parameters being Al-6061: Vp=6260 m/s, Vs=3080 m/s, ρ=2700 kg/m³. Below EF line there is random porous medium (11% illustrated in the figure), with fluid characteristics being Vp=331 m/s, Vs=0 m/s, ρ=1.225 kg/m³.

Contrarily to the previous problem, it is necessary to estimate wave extrema movement along the model, therefore, Riker impulse was employed as output signal.

\[
\hat{U}_y \mid_{AB} = F(t) = -2\pi f \sqrt{e} \left( t - t_0 \right) \cdot e^{-2\pi f (t-t_0)^2}
\]

where, f – frequency, t₀ – 1/f – impulse period, \( \hat{U}_y \) - Y-component of drift velocity

Pore-size distribution is presented in figure 5. Minimum size is 1 computational cell, whereas maximum size in case of 17% porosity is 10 cells.
Quantitative testing algorithm for calculating elastic wave propagation in a porous medium has involved solving the problem in experimental analysis, results of which are available [2]. The mentioned experiment presents data on sintered aluminium mesostructure considering two cases: 9 % and 17 % porosity.

Pore size is about 20x10^{-6} m, acoustic frequency is 5MHz, wavelength ≈ 0.1 cm. Thus, pore size is merely 50 times less than the wavelength, therefore, such pores can be described by several computational cells. According to experimental results in [2] on aluminium medium with 9 % porosity, average velocity of compressional wave propagation is \( V_P = 5125 \) m/s, and \( V_P = 4120 \) m/s for 17% porosity sample.

Numerical modeling of plane compressional wave propagation has been carried out regarding these porosity values. During modeling average velocity of P-wave propagation was determined on the basis of the slope of time-distance graph. As matrix is dry in a medium, pores were considered to be filled with air \( k_{air} = 0.00014 \) hPa, \( \rho_{air} = 1.2 \) kg/m³.

The digital images of seismic field throughout the model at two time points can be observed in figure 6. The record indicates that at 40 ms time point incident wave front has reached porous section of the model, while at the time point of 60 ms the wave has propagated through the calculated area. The result of pore ensemble diffraction can be clearly seen around certain pores.
Calculations have shown that in the case of 9\% porosity compressional wave velocity is \(V_P=5123\) m/s, while porosity at 17 \%, compressional wave velocity is \(V_P=4074\) m/s.

4. Conclusion

Thus, the numerical modeling of velocities in porous aluminium, based on the results of above-mentioned procedures differed from velocities calculated in the experiment (medium porosity 9 \%) by 0.04\%; whereas porosity being 17 \%, discrepancy is 1.1 \%.

Based on numerical modeling, the effect of porous medium on seismic field has been studied, with a single pore and pore ensemble effects being involved. In the case of a single pore simulation, calculations correspond to results of researches published by other authors. Model of medium with porous layer being produced, analysis of numerical modeling results made it possible to estimate average velocities and compare them with the data of physical modeling. It has been shown that calculations yield results are sufficiently consistent which, in its turn, is proved by qualitative (single pore simulation) and quantitative (pore ensemble simulation) testing.

References