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The interval-parametric synthesis of a linear controller of speed control system of a descent submersible vehicle

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Abstract. The algorithm for the definition of the interval settings of the linear regulator ensuring its robust stability and admissible oscillation was developed. The algorithm is based and constructed on the sufficient conditions binding the interval coefficient of the characteristic polynomial of the system and its regulator parameters. The application of this algorithm for the definition of the interval coefficient of the transfer function was also considered in the given paper. Performance of the algorithm was tested by construction of localization regions of the roots of the interval polynomial upon the determined intervals of the controller parameters.

1. Introduction

One of the main objectives of the robust control theory is the analysis and synthesis of the linear control systems with unknown parameters which are located in certain ranges. The solution of the problem relating to the analysis of the stability of the specified system is possible on the basis of the necessary and sufficient criterion applying the Kharitonov theorem [1]. The overview of the methods concerning problem solution of control system synthesis with the interval - indefinite parameters is presented in papers [2-5]. Despite the available results, the investigation in the area of the synthesis of the robust control system with interval - indefinite parameters is still crucial. Thereby, the problem concerning the interval - parametric synthesis of the robust regulators where the location of the system poles is in the left semiplane at any variations of the coefficient of the characteristic polynomial of the system is of great importance.

The aim of the given work is to develop the technology intended to determine the interval parameters of the robust regulator which ensure the robust stability of the system at any interval coefficients of the control object.

2. Algebraic conditions for stability and oscillation of the interval polynomial

Let us assume that the automatic control system contains an object $W_{co}\left(s\right)=\dfrac{B(s)}{A(s)}=\dfrac{[b_c]s^c+[b_{c-1}]s^{c-1}+...[b_0]}{[a_j]s^j+[a_{j-1}]s^{j-1}+...[a_0]}$, where $\underline{a_j}\le a_j\le\overline{a_j}$, $\underline{b_c}\le b_c\le\overline{b_c}$ and linear regulator is $W_c\left(s,\vec{k}\right)=K\left(s,\vec{k}\right)/s$, where $\vec{k}$ is the vector of adjustable parameters. Let us write down the characteristic polynomial of the control system

$$P\left(s,\vec{k}\right)=B(s)K\left(s,\vec{k}\right)+sA(s)=([b_c]s^c+[b_{c-1}]s^{c-1}+...[b_0])*([k_m]s^m+[k_{m-1}]s^{m-1}+...+[k_0])+s([a_j]s^j \\ +[a_{j-1}]s^{j-1}+...[a_0])=\sum_{i=0}^{n}[p_i(\vec{k})s^i] \quad (1)$$

Let us introduce the coefficient parameters of stability λ_i [6] formed by the quadruples of the adjacent polynomial coefficients (1): $[\lambda_i]=\dfrac{[p_{i-1}][p_{i+2}]}{[p_i][p_{i+1}]}$, $i=\overline{1,n-2}$. The interval automatic control system is stable if the following conditions are satisfied

$$\overline{\lambda_i}=\frac{\overline{p_{i-1}}\,\overline{p_{i+2}}}{\underline{p_i}\,\underline{p_{i+1}}}\le\lambda^*,\ i=\overline{1,n-2},\ \lambda^*\approx 0.465 \quad (2)$$

Stability domains constructed upon these conditions will be located inside the exact stability region found by any necessary and sufficient condition. Restriction of this field is insignificant and occurs through discarding points, which refer to systems with a minor stability factor.

The inclination of the system to the oscillations is characterized by the index of oscillation δ_z [6-7] inequalities for the determination of unknown parameters of the regulator. The oscillatory character $[\delta_z]$ for the interval system is determined on the basis of the coefficients of the interval characteristic polynomial [1] $[\delta_z]=\dfrac{[p(\overline{k})_z^2]}{[p(\overline{k})_{z-1}][p(\overline{k})_{z+1}]}$, $z=\overline{1,n-1}$. Sufficient condition for a given degree of oscillation is formulated on the basis of oscillation indexes δ_z [6-7]. To locate the roots of the interval characteristic polynomial (1) in a given angular sector $\pm\varphi$, we must select controller settings, which would enable the following conditions

$$\delta_z=\frac{\underline{p(\overline{k})_z^2}}{\overline{p(\overline{k})_{z-1}}\,\overline{p(\overline{k})_{z+1}}}\ge\delta_d,z=\overline{1,n-1}, \quad (3)$$

where δ_d - is an admissible oscillation index.

Robust stability and robust oscillation of the interval characteristic polynomial can be estimated upon the conditions (2) and (3).

3. The algorithm of the definition of the regulator interval parameters

Suppose that at least two leading coefficients of the characteristic polynomial (1) are known. Let us create the inequality systems to determine the unknown parameters of the regulator using the conditions (2), (3).

$$\begin{cases}\overline{\lambda_i}=\dfrac{\overline{p(\overline{k})_{i-1}}\,\overline{p(\overline{k})_{i+2}}}{\underline{p(\overline{k})_i}\,\underline{p(\overline{k})_{i+1}}}\le\lambda^*,\ i=n-2,n-3,...1\ \ \lambda^*\approx 0,465;\\ \underline{\delta_z}=\dfrac{\underline{p(\overline{k})_z^2}}{\overline{p(\overline{k})_{z-1}}\,\overline{p(\overline{k})_{z+1}}}\ge\delta_d,z=n-1,n-2,...1.\end{cases} \quad (4)$$

We will obtain the equations for the limits of the regulator coefficients $\vec{k}$ on the basis of the system (4).

$$\overline{k_m}=\frac{\underline{a_{z-1}^2}-\delta_d\,\overline{a_{z-2}}\,\overline{a_z}}{\delta_d\,\overline{b_c}\,\overline{a_z}},\ z=g,g-1,...0,g=n-l\,; \quad (5)$$

$$\overline{k_m} = \frac{\overline{a_{i-2}}\,\overline{a_{i+1}} - \lambda(\underline{a_{i-1}}\,\underline{a_i})}{\overline{b_c}\,\overline{a_i}},\ i = g-1, g-2, \ldots 0, g = n-l\,; \tag{6}$$

$$\underline{k_m} = \frac{\overline{a_{i+2}}\,\underline{a_i} - \lambda^{*}\delta_d\,\underline{a_{i+1}}\,\overline{a_{i+1}}}{\overline{a_{i+2}}\,\overline{b_c}},\ i = g, g-1, \ldots 0, g = n-l\,; \tag{7}$$

$$\underline{k_m} = \frac{-\underline{a_i}}{\underline{b_c}},\ i = g, g-1, \ldots 0, g = n-k\,; \tag{8}$$

$$\overline{k_{m-1}} = \frac{(\underline{a_i} + \underline{b_c}\,\underline{k_m})^2 + \delta\overline{a_{i+1}}(\overline{a_{i-1}} + \overline{b_{c-1}}\,\overline{k_m})}{\delta\overline{b_c}\,\overline{a_{i+2}}},\ i = g-1, \ldots 0\,; \tag{9}$$

$$\overline{k_{m-1}} = \frac{\overline{a_{i+2}}(\overline{a_{i-1}} + \overline{b_{c-1}}\,\overline{k_m}) + \lambda\underline{a_{i+1}}(\underline{a_i} + \underline{b_{c-1}}\,\underline{k_m})}{\overline{b_c}\,\overline{a_{i+2}}},\ i = g-1, ..0\,; \tag{10}$$

$$\underline{k_{m-1}} = \frac{-(\underline{b_{c-1}}\,\underline{k_m} + \underline{a_{i-1}}) \pm \sqrt{\delta b_{c-1}\overline{k_{m-1}}(\overline{a_i} + \underline{b_{c-1}}\,\overline{k_m})}}{\underline{b_c}}, i = g-1, \ldots 0\,; \tag{11}$$

$$\underline{k_{m-1}} = \frac{(-\overline{b_{c-1}}\,\overline{k_{m-1}}\,\overline{a_{i+1}} - \lambda((\underline{a_i} + \underline{b_c}\,\underline{k_m})(\underline{a_{i-1}} + \underline{b_{c-1}}\,\underline{k_m}))}{\underline{b_c}\lambda(\underline{a_i} + \underline{b_c}\,\underline{k_m})}, i = g-1, \ldots 0\,. \tag{12}$$

The algorithm intended to determine the limits of the regulator parameters of the control system is constructed on the basis of the equations (5) - (12). These limits ensure the robust stability of the system. The given algorithm includes the following stages:

1. The specification of the coefficients of the control object and the admissible index of oscillation δ_d.
2. The polynomial formation (1).
3. The check of the fulfillment of the problem specification (2), (3) for the known coefficients of the polynomial (1).
4. The calculation of the upper limits of the parameter k_m on the basis of equations (5), (6) and the selection of the maximum value.
5. The calculations of the bottom limit of the parameter k_m on the basis of the equations (7), (8) and the selection of the maximum value.
6. Similar calculation of the limits of other parameters on the basis of the equations (9) – (12).
7. The check of the fulfillment of the problem specification (2)-(3) for the evaluated values of the limits of the regulator parameters.

4. Application of the algorithm in the interval-parametric synthesis of the robust controller

Let us consider the operability of the reduced algorithm by the example of interval-parametric synthesis of speed control system of a descent submersible (DS) vehicle. The description of the given system is presented in work [8]. The block diagram of the system constructed on the basis of mathematical description [8] is illustrated in Figure 1.

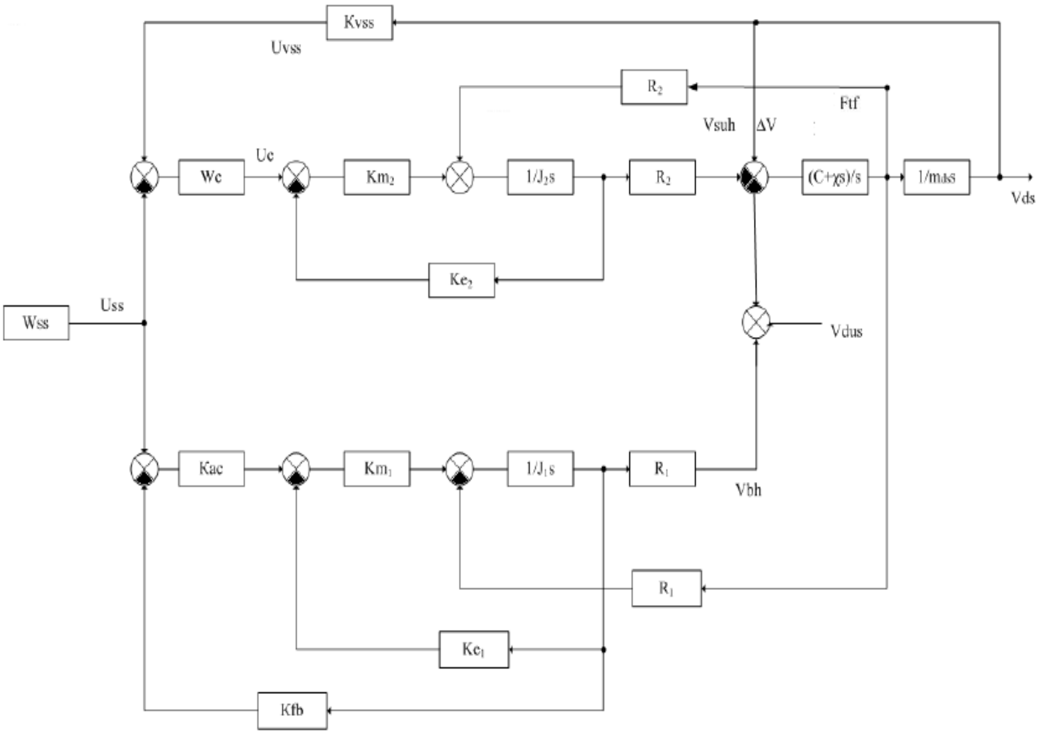


Figure 1. Structure diagram of the speed control system of a DS.

The system has the following constant parameters [8]: *Kvss =0,2[Vs/m]* – transfer constant of the vertical speed sensor of DS; $Km_1=Km_2=0,3[Nm/A]$ – transfer constant of the engines; $Ke_1=Ke_2=1[Vs/rad]$ – transfer constants of counter-emf of the engines of boat hoist (BH) and shock-absorbing hoist SAH; $J_1=100\ [kgm^2]$, $J_2=0,5[kgm^2]$ – are relatively the moments of inertia of BH and SAH; $R_1=0,2[m]$ and $R_2=0,1[m]$ – are relatively the drum radii of BH and SAH; $K_{fb}=0,21[Vs/rad]$ – feedback coefficient on speed of BH. The length of the rope and the DS mass are considered as the interval parameters of the system. Speed loop of BH has a typical structure including speed feedback and proportional speed controller $K_{ac}=20000$. The aim of this loop is to provide the required dynamic of BH in the descent and ascent mode of DS by means of the selection of the parameters settings *Kac* and *Kfb* (Figure 1).

The length of the cable from the interval *[2; 20] m* changes with the change of the depth of the descent of submersible vehicle and hence its parameters are equal to æ= *(6·104/l)*, *C=(6·106/l)*. Weight m_{ds} of the descent submersible vehicle can also undergo some changes by the lifting of various objects from the bottom and is within the range *[300; 350] kg*.

The specified factors lead to the variation of dynamic properties of speed control systems of the descent submersible vehicle that has negative impact on the damping of oscillation of the descent submersible vehicle and can cause the system performance loss. In connection with above mentioned factors it is necessary to impart robust properties to the system which ensure the permissible operation quality at any possible variations of unstable parameters.

As a result of block diagram transformation shown in Figure 1 the transfer function of open-loop system is as follows:

$$W_{ol}(s)=\frac{(b_0+b_1s+b_2s^2)(k_1+k_2s)}{s(a_4s^4+a_3s^3+a_2s^2+a_1s^1+a_0)},$$

where

$[a_4]=[600000;875000],[a_3]=[28921800;\ 35447625]\ [a_2]=[2394400080;\ 2738425575]$

$[a_1]=[25429505400;\ 29034005400.0000]$, $a_0=2160540000$, $b_0=43210800$, $b_1=4032108$, $b_2=36000$.

We need to determine the setting intervals of PI-controller: k_1, k_2 which provide that the automatic control system is robustly stable and conditions for oscillation at given δ_d=1,4.

We write the interval characteristic polynomial of this system, as follows

$$P(s,\vec{k})=[p_5]s^5+[p_4]s^4+[p_3(k_2)]s^3+[p_2(k_1,k_2)]s^2+p_1(k_1,k_2)s+p_0(k_1), \tag{13}$$

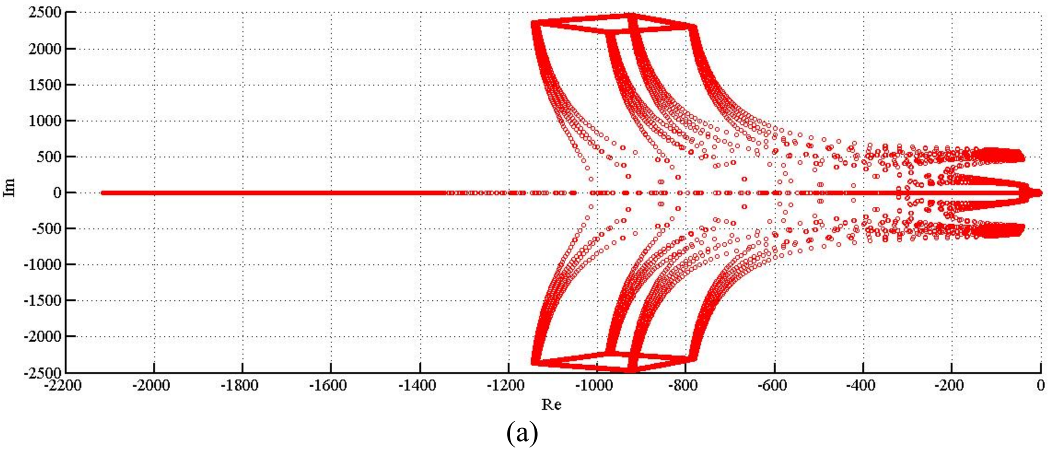
where $[p_5]=[a_4]$; $[p_4]=[a_3]$; $[p_3]=[a_2]+[k_2]b_2$; $p_2=[a_1]+([k_2]b_1+[k_1]b_2)$; $p_1=[a_0]+([k_2]b_0+[k_1]b_1)$; $p_0=[k_1]b_0$.

To solve this task, we elaborate the system in the form (4):

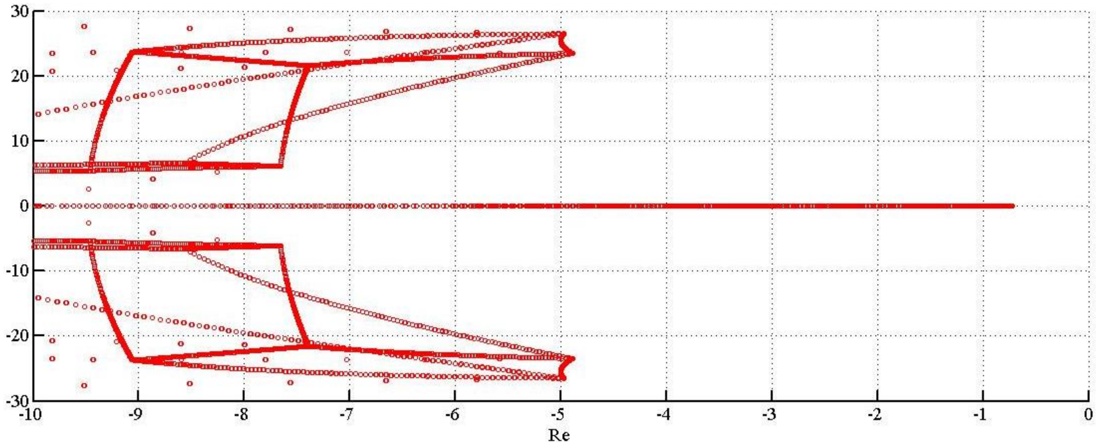
$$\begin{cases}
\overline{\lambda_1}=\dfrac{b_0\overline{k_0}(\overline{a_2}+b_2\overline{k_1})}{(\underline{a_0}+b_0\underline{k_1}+b_1\underline{k_0})(\underline{a_1}+b_1\underline{k_1}+b_2\underline{k_0})}\le 0,465;\\
\overline{\lambda_2}=\dfrac{(\overline{a_0}+b_0\overline{k_1}+b_1\overline{k_0})\overline{a_3}}{(\underline{a_1}+b_1\underline{k_1}+b_2\underline{k_0})(\underline{a_2}+b_2\underline{k_1})}\le 0,465;\\
\overline{\lambda_3}=\dfrac{(\overline{a_1}+b_1\overline{k_1}+b_2\overline{k_0})\overline{a_4}}{(\underline{a_2}+b_2\underline{k_1})\underline{a_3}}\le 0,465;\\
\underline{\delta_1}=\dfrac{(\underline{a_0}+b_0\underline{k_1}+b_1\underline{k_0})^2}{b_0\overline{k_0}(\overline{a_1}+b_1\overline{k_1}+b_2\overline{k_0})}\ge 1,4;\\
\underline{\delta_2}=\dfrac{(\underline{a_1}+b_1\underline{k_1}+b_2\underline{k_0})^2}{(\overline{a_0}+b_0\overline{k_1}+b_1\overline{k_0})\,(\overline{a_2}+b_2\overline{k_1})}\ge 1,4;\\
\underline{\delta_3}=\dfrac{(\underline{a_2}+b_2\underline{k_1})^2}{(\overline{a_1}+b_1\overline{k_1}+b_2\overline{k_0})\overline{a_3}}\ge 1,4;\\
\underline{\delta_4}=\dfrac{(\underline{a_3})^2}{(\overline{a_2}+b_2\overline{k_1})\overline{a_4}}\ge 1,4;
\end{cases}$$

Solving this system and using an algorithm to determine the unknown parameters of the regulator $[k_0]=[1.385\times10^6;1.111\times10^8]$ $[k_1]=[2.74\times10^5;1.824\times10^7]$

This result is checked by constructing the localization regions of the roots the interval polynomial (13) (figure 2).



(a)



(b)

Figure 2. (a) Regions of root localization; (b) regions of root localization on a larger scale.

Thus, the system with the interval objects under consideration will be robust – stable at any parameters of PI-regulator from the computed and specified ranges.

5. Conclusion

The algorithm intended to determine the limit of parameters of the regulator ensuring the robust stability of the system with the interval control objects based on the robust modification of the coefficient method was developed in the given paper. The efficiency of this algorithm was checked on the interval-parametric synthesis of the robust PI- regulator and was confirmed by the graphs of the areas of the location of roots of the characteristic polynomial.

Acknowledgments

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