JOULE HEATING EFFECTS ON QUARTZ PARTICLE MELTING IN HIGH-TEMPERATURE SILICATE MELT

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Abstract. This work is mostly focused on the melting process model simulation of quartz particles having the radius within the range of $10^{-6}$–$10^{-3}$ m. The melting process is simulated accounting for the heat generation at an electric current passage through a quartz particle.

1. Introduction
As a rule, natural sand is presented by silica grains ($\text{SiO}_2$). $\alpha$-Quartz is the most widespread form of silicon dioxide [1]. Among silica-based materials, vitreous silica has a special place. It possesses a number of valuable physicochemical properties, such as heat resistance, refractoriness, chemical and radiation resistance, and transparency within a wide wavelength range. Methods for the production of silica sand-based products differ from those accepted for the common glass technology. This is because the super high viscosity of silica melt even at temperatures exceeding 2000°C and its higher volatility [2-3]. The low-temperature plasma (LTP) used for silica te melting possesses a high energy concentration and 3000–5000°C temperature that will allow achieving the required viscosity of silicate melt and providing its uniform heating [4-7].

This work is mostly focused on the melting process model simulation of quartz particles having the radius within the range of $10^{-6}$–$10^{-3}$ m. The melting process is simulated accounting for the heat generation at an electric current passage through a quartz particle.

2. Experimental
Quartz particles are suggested to be melt in the plasma apparatus designed for the production of high-temperature silicate melts [8]. The flow of plasma-supporting gas in this plasma apparatus serves as a heat-transfer agent generated from a plasma nozzle. The LTP jet enters the melting space of the water-cooled furnace to produce melting of refractory silicate raw material. The batcher and worm feeder designed for the plasma apparatus allow feeding the raw material in the plasma jet area. The powder raw material is introduced in the depth of the previously formed silicate melt and melted over the whole melting space of the furnace owing to Joule heating.

The plasma apparatus specifications [9] allow using it for quartz melting.

2.1. Task description
It is advisable to divide the whole period of the melting process into time intervals of $0 < t \leq t_1$ and $t_1 < t \leq t_2$, where $t_1$ is the time of quartz particle heating up to melting temperature $T_m$; $t_2$ is the time of completion of the melting process; $t_3$ is the time of stabilization of the equilibrium state (the temperature of particles equals to that of silicate melt).

The particle energy at the stage of heating up to melting temperature can be obtained from
The following notations are suggested for equations (1) – (2):

\( t \) is the time; \( T \) is the temperature; \( c \) is the specific heat capacity; \( \rho \) is the density; \( \alpha \) is the heat exchange rate; \( S_{p,0} = 4\pi r_{p,0}^2 \) is the surface area of quartz particle at its initial radius \( r_{p,0} \); \( V_{p,0} = \frac{4}{3}\pi r_{p,0}^3 \) is the quartz particle volume at its initial radius \( r_{p,0} \); \( Q_{el} \) is the heat generated by electric heating.

Indices \( p \), \( am \) and \( i \) denote parameters relating to the quartz particle, ambient air, and initial state, respectively.

The heat exchange rate can be obtained using the Nusselt number:

\[ \alpha = \frac{\lambda_{am}}{r_{p}} Nu. \]

Let us assume that upon the achievement of the melting temperature by quartz particle, the amount of heat transferred to it due to heat exchange and Joule heating, is spent for its melting. Taking the above assumption into account, the volume of the solid (crystalline) portion of quartz particle is changed during the melting stage:

\[ c_p \rho_p V_p \frac{dT_p}{dt} = \alpha S_p (T_{am} - T_p) + Q_{el}. \]  \( T_p(0) = T_i. \)

3. Results and discussion

Equations (1), (3) consider the effect from the electric heating on the whole process. In order to determine the heat generation produced by the electric source, the Joule-Lenz’s law should be used. According to this law, the amount of heat released is proportional to the square of the current such that

\[ Q = I^2 R, \]

where \( I \) is the current in the unit.

Quartz particle releases the amount of heat equaling to

\[ Q_{el} = i^2 r, \]

where \( i \) is the current passing through quartz particle; \( r = \rho_{sp} \frac{l}{S_q} \) is the resistance of quartz particle; \( \rho_{sp} \) is the specific resistance of quartz particle.

Let quartz particle be a cylinder with the diameter equaling to its height, i.e. \( d_{cyl} = l_{cyl} \). At the same time, the diameter of cylinder is selected such that the volume of a cylindrical particle equals to the volume of a spherical particle, i.e. \( \frac{4}{3}\pi r^3 = \frac{4}{3}\pi l_{cyl}^3 \).

The specific resistance \( \rho_{sp} \) ranges from \( 10^{14} \) Ohm·m at 20°C to 0.9 Ohm·m at 1600°C.

Let us analyze the intensity of the current passing through quartz particle at the stage of its heating. Quartz particle is in the silicate melt the specific resistance of which is low and in the order of \( \rho_{sp} = 1 \) Ohm·m. Quartz particle has \( 10^{14} \) Ohm·m specific resistance. Let us assume that the volume fraction ratio of the silicate melt and that of unmelted quartz particles is 10:1 (Fig. 1).
The section of the circuit shown in Figure 1a is presented by a parallel combination of two conductors shown in Figure 1b. The electric current in the section (Fig. 1a) can be obtained from
\[ i = \frac{I(s_q + s_{mt})}{S_f}, \]  
where \( s_q, s_{mt} \), and \( S_f \) are the areas of quartz particle, silicate melt, and furnace, respectively.

For the parallel combination the following condition is met:
\[ i_1 + i_2 = i, \]  
where \( i_1 \) is the current in quartz particle; \( i_2 \) is the current in the silicate melt. At the same time, by Ohm’s law:
\[ u = i_r = i_1r_1 = i_2r_2, \]
\[ r = \frac{r_1r_2}{r_1 + r_2}, \]
where \( r \) is the total resistance; \( r_1 \) and \( r_2 \) are resistances of quartz particle and silicate melt, respectively.

From (5), the values of \( i_1 \) and \( i_2 \) can be obtained:
\[ i_1 = \frac{i}{r_1}, \quad i_2 = \frac{i}{r_2}. \]

Resistances \( r_1 \) and \( r_2 \) can be obtained from:
\[ r_1 = \rho_{sp} \frac{l_q}{s_q}, \quad r_2 = \rho_{sp2} \frac{l_{mt}}{s_{mt}}, \]
where \( \rho_{sp} = 10^{14} \text{Ohm} \cdot \text{m}; \rho_{sp2} = 1 \text{Ohm} \cdot \text{m}; l_q = 1 \text{ mm}; l_{mt} = 1 \text{ mm}; s_q = \frac{\pi l_q^2}{4}, s_{mt} = 10s_q. \]

Then \( r_1 = \frac{4 \cdot 10^{17}}{\pi}, r_2 = \frac{10^2}{\pi}, r = \frac{r_1r_2}{r_1 + r_2}. \)

The ratio between current values \( i_1 \) and \( i_2 \) can be found as follows:
\[ \frac{i_1}{i_2} = \frac{r_2}{r_1} = \frac{1}{4} \cdot 10^{-15}. \]  

Suppose that the current in the unit is 400 A; then equation (5) can be used to determine \( i = 0.216 \) A. And equations (6) and (8) determine \( i = \frac{0.216}{1 + 4 \cdot 10^{-15}} = 5.4 \cdot 10^{-17} A. \)

Substituting this value in \( Q_{el} = i^2 r \) we get \( Q_{el} = 37.1 \cdot 10^{-17} \text{ J/s}. \)

However, the specific resistance of quartz particle decreases with the increase of temperature. Therefore, the model simulation is used such that \( Q_{el} = 2 \cdot 10^{-2} \text{ J/s}. \) Calculations are made for the case of the quartz particle heating and melting in the silicate melt accounting for the heat exchange rate that
depends on the radius of the solid (crystalline) quartz particle portion at its melting stage. Phase transformations are neglected.

Figure 2. Dependence $\Delta t_{21}$ and the particle initial radius $r_{0,p}$

Figure 3. Dependence $t_2$ and the particle initial radius $r_{0,p}$

Figure 2 contains the plot of the dependence between the melting time $\Delta t_{21} = t_2 - t_1$ and the quartz particle radius accounting for Joule heating and the dependence between the heat exchange rate and the radius of the solid (crystalline) quartz particle portion at its melting stage. Figure 3 contains the
plot of the dependence between time $t_2$ and the quartz particle radius accounting for Joule heating and the dependence between the heat exchange rate and the radius of the solid (crystalline) quartz particle portion at its melting stage. Results of calculation of $\Delta t_{21}$ and $t_2$ are also given in the Table below.

### Table. Dependence of $\Delta t_{21}$ and $t_2$ on the quartz particle radius.

<table>
<thead>
<tr>
<th>$r_{0,p}$ [m]</th>
<th>$10^{-6}$</th>
<th>$2 \cdot 10^{-6}$</th>
<th>$5 \cdot 10^{-6}$</th>
<th>$10^{-5}$</th>
<th>$2 \cdot 10^{-5}$</th>
<th>$5 \cdot 10^{-5}$</th>
<th>$10^{-4}$</th>
<th>$2 \cdot 10^{-4}$</th>
<th>$5 \cdot 10^{-4}$</th>
<th>$10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t_{21}$ [s]</td>
<td>$10^{-6}$</td>
<td>$8 \cdot 10^{-6}$</td>
<td>$2.5 \cdot 10^{-5}$</td>
<td>$10^{-4}$</td>
<td>$8 \cdot 10^{-4}$</td>
<td>$2.5 \cdot 10^{-3}$</td>
<td>$10^{-2}$</td>
<td>$8 \cdot 10^{-2}$</td>
<td>$0.25$</td>
<td>$1.02$</td>
</tr>
<tr>
<td>$t_2$, [s]</td>
<td>$2 \cdot 10^{-4}$</td>
<td>$4 \cdot 10^{-6}$</td>
<td>$5 \cdot 10^{-4}$</td>
<td>$2 \cdot 10^{-4}$</td>
<td>$4 \cdot 10^{-4}$</td>
<td>$5 \cdot 10^{-3}$</td>
<td>$2 \cdot 10^{-2}$</td>
<td>$4 \cdot 10^{-2}$</td>
<td>$0.5$</td>
<td>$2.0$</td>
</tr>
</tbody>
</table>

### 4. Conclusion

The results showed that electric heating had no a significant effect on the time period of quartz particle heating and melting. After the heating, the melted layer around the crystalline particle was rapidly heated up to the melting temperature due to the electric current that accelerated the melting process. The research was financed by the Ministry of Education and Science of the Russian Federation (State Order N 11.351.2014/K).

### References


