LINEAR TIME INVARIANT SYSTEM POLE PLACING BY POLYNOMIAL DIVISION METHOD

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Introduction

Control quality of every system is fully determined by its poles placing, however, to manage the preferred control quality only few of them have to be placed inflexibly. These poles are designated as prevailing or dominant. In order to let dominant poles determine control quality, all other poles should be placed in some region of complex plane away of dominant poles; these poles are called unrestricted.

The main goal of this research is to develop a method of system pole placing by dividing its characteristic polynomial into two multipliers: the first one is supposed to place one real dominant pole or a pair of complex dominant poles; the second one is supposed to place all other poles in a preferred area of a complex plane.

Formulation of the problem

Characteristic polynomial of certain linear time invariant system has the following form:

$$D(s) = \sum_{i=0}^{n} a_i \cdot s^i \,, \qquad (1)$$

where a_i is a characteristic polynomial coefficient fully determined by parameters of systems regulators, n – characteristic polynomials degree.

Obviously, considered system has n poles.

Let us designate the polynomial, which determines a dominate poles placing, as $A(s)$; and the result of dividing (1) by $A(s)$ as $B(s)$; the remainder of such division – as R.

Due to these designations, characteristic equation of considered system has following form:

$$A(s) \cdot B(s) + R = 0 \qquad (2)$$

Now it is possible to formulate the problem of this research: to develop a method of extracting a polynomial $A(s)$, supposed to determine dominant poles placing, from characteristic polynomial; placing unrestricted poles (roots of $B(s)$ polynomial); synthesizing system regulators parameters according to poles placing.

Development of polynomials general form

In order to solve the problem, first of all a general view of $A(s)$, $B(s)$ and R should be developed.

As it is said in the introduction, the considered system can have one real dominant pole or a pair of complex dominant poles, so $A(s)$ has the following general form:

$$A(s) = s - s_0 \,,$$

where s_0 is a real dominant pole of the considered system, in the first case or:

$$A(s) = (s - s_1) \cdot (s - s_2) = s_1{}^2 - (s_1 + s_2) \cdot s + s_1 \cdot s_2 \,,$$

in the second case. Let us designate a sum of two complex poles as x, its production – as y.

Consequently, coefficients of $B(s)$ polynomial can be calculated by the following formulas:

$$b_i = a_{i+1} + a_{i+2} \cdot s_0 \,, \quad i \in [0; n-1]$$

$$b_i = a_{i+2} + x \cdot b_{i+1} - y \cdot b_{i+2}; \quad i \in n-2...0$$

where a_i is a one of $D(s)$ coefficients.

Obviously, the multiplication of these two polynomials must be equal to $D(s)$ characteristics polynomial. According to (2), in order to ensure this equality in systems parameters synthesis it is necessary to remember that R must be equal to zero.

To do so, a general view of R was developed:

$$R = \sum_{i=0}^{m} a_i \cdot s_0^i = D(s_0)\,,$$

for the first case, or

$$R(s) = (a_1 + x \cdot b_0 - y \cdot b_1) \cdot s + a_0 - y \cdot b_0 \,,$$

for the second case.

To place each dominant pole, the synthesized regulator must have one parameter, and another one to place all unrestricted poles. So, to place one real dominant pole a PI-regulator should be used, to place a pair of complex dominant poles - PID-regulator.

Synthesis algorithm

In order to synthesize a proper system regulator, we will have to get through eight steps:

1. Define a desirable stability degree.
2. Define a type of necessary regulator.
3. Add this regulator to the system and calculate its characteristic equation.
4. Calculate coefficients of all polynomials.
5. Choose one of regulators parameters, which is included into coefficients of an R polynomial.
6. Equate R polynomials real and imaginary part to zero and find regulators parameters function of the parameter chosen before.
7. Plot a D-partition of $B(s)$ polynomial in a plane of the main parameter, in order to find a set of values for this parameter.
8. Calculate other parameters.

Example of application

As an example of a newly developed method application, let us examine a closed-loop control system consisting of a control object and a regulator.

A schematic diagram of the considered system is given below (Fig.1).

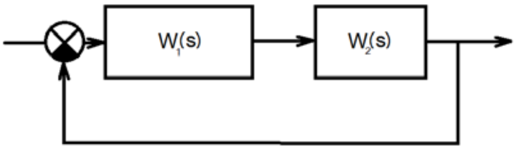

Fig.1 Schematic diagram of the considered system

Here, $W_1(s)$ is a transfer function of a control object, $W_2(s)$ is a transfer function of a regulator. Control object transfer function is given below.

$$W_1(s) = \frac{1}{1.382 \cdot 10^{-5} \cdot s^4 + 2.15 \cdot 10^{-3} \cdot s^3 + 0.083 \cdot s^2 + 1.476 \cdot s + 11.2258}$$

In order to demonstrate features of this method completely, let us examine both cases of application: with one dominant pole and with a pair of complex dominant poles.

According to the synthesis algorithm, let us determine a desirable dominant poles location and stability degree. We assume, that it is necessary to place our dominant poles in points $-2 \pm i \cdot 2$ of complex plane. So, the desirable stability degree is 2. Moreover, we assume, that it is necessary to place all unrestricted poles on the left of the complex plane point with coordinates (-15;0).

Control object transfer function is already given, so let us define a controller type. In order to place two dominant poles in the desired points a regulator with three tunable parameters should be used, for example, PID-regulator (transfer function is given below).

$$W_2(s) = \frac{K_1 \cdot s^2 + K_2 \cdot s + K_3}{s}$$

Characteristic equation of a system is given below.

$$D(s) = 1.382 \cdot 10^{-5} \cdot s^5 + 2.15 \cdot 10^{-3} \cdot s^4 + 0.083 \cdot s^3 +$$
$$+ (K_1 + 1.476) \cdot s^2 + (K_2 + 11.226) \cdot s + K_3$$

From this equation, let us find coefficients of $B(s)$ and R polynomials. The result of these calculations is given below.

$$B(s) = 1.38 \cdot 10^{-5} \cdot s^3 + 2.09 \cdot 10^{-3} \cdot s^2 + 0.0747 \cdot s + K_1 + 1.46$$
$$R(s) = (K_2 - 4 \cdot K_1 + 4.791) \cdot s + K_3 - 8 \cdot K_1 - 11.675$$

From imaginary part of $R(s)$ polynomial a K_2 parameter function of K_1 can be defined; from a real part of this polynomial a K_3 parameter function of K_1 can be defined. With these functions, all regulator parameters can be calculated by value of K_1. The mentioned functions are given below.

$$K_2 = 4 \cdot K_1 - 4.791$$
$$K_3 = 8 \cdot K_1 + 11.675$$

K_1 parameter can be calculated by plotting a D-partition.

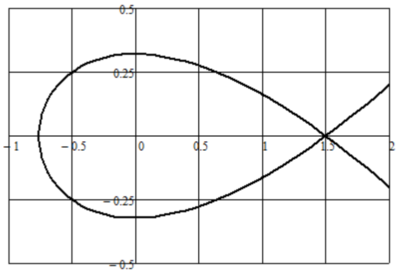

Fig.2 D-partition of B(s) polynomial in a plane of K_1 parameter

From this plot it is obvious, that in order to provide systems stability, K_1 should vary from 0 to 1.5. Let us designate a K_1 parameter equal to 1.4.

According to (5.1.1), K_2 and K_3 will be equal to 0.809 and 22.875.

With these values, poles of the considered system will be placed in points, listed below: $-2 \pm i \cdot 2; -15.289 \pm i \cdot 38.437; -120.977$.

In order to place a single real dominant pole $s_0 = -2$ and place all other poles from the left of the point (-8;0) of the complex plane, the same technique will be used.

Regulators transfer function:

$$W_2(s) = \frac{K_1 + K_2 \cdot s}{s}.$$

Characteristics equation:

$$D(s) = 1.382 \cdot 10^{-5} \cdot s^5 + 2.15 \cdot 10^{-3} \cdot s^4 + 0.083 \cdot s^3 +$$
$$+1.476 \cdot s^2 + (K_2 + 11.226) \cdot s + K_1$$

$B(s)$ polynomial:

$$B(s) = 0.00215 \cdot s^3 + 0.0789 \cdot s^2 + 1.318 \cdot s + K_2 + 8.589$$

The value of K_2 parameter found by D-partition: K_2=3; value of K_1 parameter found from the equality of R to zero: K_1=23.179.

With these value of regulators parameters, system poles were placed in points listed below: -2; -25.5026; -108.2028; $-9.925 \pm i \cdot 14.3377$.

Conclusion

This newly developed method is fully applicable for linear time-invariant control system synthesis; it can be easily used as a base for scientific software.

In addition, it can be easily expanded to robust system synthesis.

References

1. Bessekersky V.A., Popov E.P. Control system theory. - Moscow: Professiya, 2007 - 752 p.