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GENERALIZED STATIC CHARACTERISTICS OF ELECTRIC POWER SUBSYSTEMS AND THEIR STEEPNESS FACTORS

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Representation of parts of electric power systems by generalized static characteristics has been considered. The design procedure of steepness factors of generalized static characteristics depending on features of equivalent subsystems is discussed. The generalized static characteristics and their steepness factors give the equivalent information on power subsystems and can be used at estimation of static aperiodic stability of power supply systems.

Steepness factors and load and generator effects are widely used in mode calculations and estimation of static stability of electric systems [1]. Static characteristics and steepness factors not only of separate elements of the systems but of their complex combinations forming subsystem are of practical interest. The generalized static characteristics and their steepness factors give the equivalent information on subsystem state. Let us singled out the subsystems in which there are no slack nodes by active power and subsystems containing slack nodes of active power depending on conditions accepted at equivalent. In this paper the calculation expressions for determining steepness factors of static characteristics of complex electric power subsystems are introduced for the first time.

Let us turn to subsystem without slack nodes by active power. Let us arrange to call a part of a system having connection with the main or adjacent part in a single node i , the limited part (Figure). Let us suppose that in a certain initial steady-state condition in the main part of electric system the stationary disturbance occurred. It resulted in change of module ΔU_i and phase $\Delta \delta_i$ of node voltage to which the limited subsystem abuts. Circuit layout, composition of equipment (including consumers), transformation ratios, settings of the first and secondary regulators are stable in the given subsystem. The two latter conditions are adequate to constancy of source active power. Let us analyze the changes which occur in the singled out subsystem at alternate and independent occurrence of disturbances ΔU_i and $\Delta \delta_i$.

Under the action of applied disturbance ΔU_i ($\delta_i = \text{const}$) the subsystem operating conditions are de-

formed. Active power of power sources is constant and load node power (according to static characteristics) and loss in network elements change. Unbalance of active (ΔP) and reactive (ΔQ) powers which tends to the node i is formed in subsystem.

At stationary increment of phase $\Delta \delta_i$ ($U_i = \text{const}$) the subsystem moves as a unit relative to synchronous axis of node i . Stress vectors of all nodes in subsystem change by the same angle as phase increment in common angle i . Reference angles of stress vectors in subsystem are constant. Its mode is constant as well that indicates the absence of subsystem reaction on the specified disturbance $\Delta \delta_i$. As the limited subsystem has the only connection with the main part then functional dependences

$$P_{ic} = P(U_i); Q_{ic} = Q(U_i) \quad (1)$$

are the full mode equivalent of subsystem.

At known reset conditions of the system the generalized static characteristics are found out by calculations of a number of steady-state conditions of equivalent subsystem at stress module variation of equivalent node. Equivalent node is accepted as a balancing one by active and reactive powers. Calculations allow introducing dependences (1) in Table, diagram or analytical at proper results processing.

The subsystem diagram is given in the form of passive multiport (n -pole network). Two-terminal networks substituting generators, sources of reactive power and load are connected to its vertices. Let us suppose that elements of two-terminal networks are introduced by their static characteristics by voltage.

Let us consider the calculation of steepness factors α_{ic} and β_{ic} of generalized static characteristics of the equivalent subsystem (Figure)

$$\alpha_{ic} = \partial P_{ic} / \partial U_i; \beta_{ic} = \partial Q_{ic} / \partial U_i$$

on the basis of the increment equations given below in which α_i, β_i are the steepness factors of static characteristics of two-terminal networks:

$$\begin{bmatrix} \frac{\partial P_1}{\partial U_1} - \alpha_{p1} & \frac{\partial P_1}{\partial U_i} & \dots & \frac{\partial P_1}{\partial U_n} & \frac{\partial P_1}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial P_1}{\partial \delta_n} \\ \frac{\partial P_i}{\partial U_1} & \frac{\partial P_i}{\partial U_i} & \dots & \frac{\partial P_i}{\partial U_n} & \frac{\partial P_i}{\partial \delta_1} & \dots & -1 & \dots & \frac{\partial P_i}{\partial \delta_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial U_1} & \frac{\partial P_n}{\partial U_i} & \dots & \frac{\partial P_n}{\partial U_n} - \alpha_{pn} & \frac{\partial P_n}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial P_n}{\partial \delta_n} \\ \hline \frac{\partial Q_1}{\partial U_1} - \beta_{q1} & \frac{\partial Q_1}{\partial U_i} & \dots & \frac{\partial Q_1}{\partial U_n} & \frac{\partial Q_1}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial Q_1}{\partial \delta_n} \\ \frac{\partial Q_i}{\partial U_1} & \frac{\partial Q_i}{\partial U_i} & \dots & \frac{\partial Q_i}{\partial U_n} & \frac{\partial Q_i}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial Q_i}{\partial \delta_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial U_1} & \frac{\partial Q_n}{\partial U_i} & \dots & \frac{\partial Q_n}{\partial U_n} - \beta_{qn} & \frac{\partial Q_n}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix} \times \begin{bmatrix} \Delta U_1 \\ \Delta U_i \\ \vdots \\ \Delta U_n \\ \Delta \delta_1 \\ \vdots \\ \Delta P_{ic} \\ \Delta \delta_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ \Delta Q_{ic} \\ 0 \end{bmatrix} \quad (2)$$

In order to calculate the coefficient β_{ic} we suppose that disturbance ΔQ_{ic} at fixed phase of stress vector in equivalent node i ($\delta_i = \text{const}$) which is simultaneously balancing by active power that is reflected by the element $a_{i,n+i} = -1$ in matrix W_β is applied to node i . solving the system (2) relative to ΔU_i at specified ΔQ_{ic} the coefficient

$$\beta_{ic} = \partial Q_{ic} / \partial U_i = D_\beta / A_{\beta(n+i,i)}, \quad (3)$$

is found out where D_β is the determinant, $A_{\beta(n+i,i)}$ is the algebraic complement of matrix W_β .

Calculating the coefficient α_{ic} similarly to the previous one let us consider that external disturbance ΔP_{ic} at condition $\delta_i = \text{const}$ is applied to the node i . The node i is balancing by the reactive power. In order to obtain matrix W_α it is sufficient to consider $a_{n+i,n+i} = -1$ to be a nonzero element in column $n+i$ of matrix W_β . Then:

$$\alpha_{ic} = dP_{ic} / dU_i = D_\alpha / A_{\alpha(i,j)}, \quad (4)$$

where D_α is the determinant, $A_{\alpha(i,j)}$ is the algebraic complement of matrix W_α . In this case $A_{\alpha(i,i)} = -A_{\beta(n+i,i)}$.

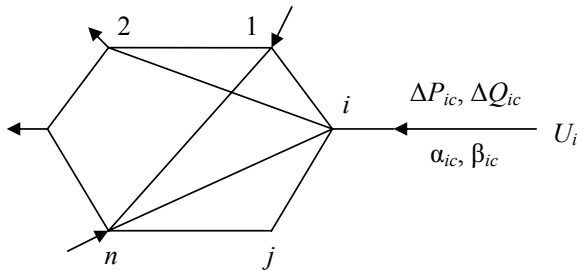


Figure. The equivalent subsystem

Let us consider the substitution of the limited subsystem containing nodes balancing by active power by the equivalent two-pole network with specified static characteristic. For the given subsystem it is appropriate to use the same form of introducing static characteristics as for generator [2]:

$$Q_{ic} = Q(U_i, P_{ic}). \quad (5)$$

We get

$$\Delta Q_{ic} = \frac{\partial Q_{ic}}{\partial U_i} \Delta U_i + \frac{\partial Q_{ic}}{\partial P_{ic}} \Delta P_{ic} = \beta_{ic} \Delta U_i + \eta_i \Delta P_{ic} \quad (6)$$

from (5).

In order to differentiate single-type steepness factors the index of coefficient β determined at condition $P_{ic} = \text{const}$ for subsystems with nodes balancing by active power is parenthesized.

The value of coefficients $\beta_{(ic)}$ and η_i depends on distribution of «unbalance» between generator and load nodes of subsystem. Depending on the solved problem this «unbalance» may be distributed between several nodes of generation and load or fully referred to one node. The linearized system of equations (7) for calculation of $\beta_{(ic)}$ of subsystems (Figure) is obtained provided that disturbance ΔQ_{ic} and $\delta_i = \text{const}$ (stress phase of any node may be taken as a basic one) is applied to the node i .

$$\begin{bmatrix} \frac{\partial P_1}{\partial U_1} - \alpha_{p1} & \frac{\partial P_1}{\partial U_i} & \dots & \frac{\partial P_1}{\partial U_n} & \frac{\partial P_1}{\partial \delta_1} & \dots & -K_{1n} & \dots & \frac{\partial P_1}{\partial \delta_n} \\ \frac{\partial P_i}{\partial U_1} & \frac{\partial P_i}{\partial U_i} & \dots & \frac{\partial P_i}{\partial U_n} & \frac{\partial P_i}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial P_i}{\partial \delta_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial U_1} & \frac{\partial P_n}{\partial U_i} & \dots & \frac{\partial P_n}{\partial U_n} - \alpha_{pn} & \frac{\partial P_n}{\partial \delta_1} & \dots & -1 & \dots & \frac{\partial P_n}{\partial \delta_n} \\ \hline \frac{\partial Q_1}{\partial U_1} - \beta_{q1} & \frac{\partial Q_1}{\partial U_i} & \dots & \frac{\partial Q_1}{\partial U_n} & \frac{\partial Q_1}{\partial \delta_1} & \dots & -\eta_1 K_{1n} & \dots & \frac{\partial Q_1}{\partial \delta_n} \\ \frac{\partial Q_i}{\partial U_1} & \frac{\partial Q_i}{\partial U_i} & \dots & \frac{\partial Q_i}{\partial U_n} & \frac{\partial Q_i}{\partial \delta_1} & \dots & 0 & \dots & \frac{\partial Q_i}{\partial \delta_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial U_1} & \frac{\partial Q_n}{\partial U_i} & \dots & \frac{\partial Q_n}{\partial U_n} - \beta_{qn} & \frac{\partial Q_n}{\partial \delta_1} & \dots & -\eta_n & \dots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix} \times \begin{bmatrix} \Delta U_1 \\ \Delta U_i \\ \vdots \\ \Delta U_n \\ \Delta \delta_1 \\ \vdots \\ \Delta P_{ic} \\ \Delta \delta_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ \Delta Q_{ic} \\ 0 \end{bmatrix} \quad (7)$$

For community all generator and load nodes of subsystems except the equivalent node are taken balancing by active power. A part of general «unbalance» of active power taken by an arbitrary node j is accepted in ratio to increment of the power of node n ($\Delta P_j = K_{jn} \Delta P_n$).

Coefficients η_i in (7) are nonzero only for generator nodes involved in balancing. We have:

$$\beta_{(ic)} = \partial Q_{ic} / \partial U_i = D_{(\beta)} / A_{\beta(n+i,i)}, \quad (8)$$

from (7) where $D_{(\beta)}$ is the determinant, $A_{\beta(n+i,i)}$ is the algebraic complement of matrix $W_{(\beta)}$.

Coefficient η_i shows by how many units the reactive power transferred to the node i of the equivalent subsystem should be changed for U_i to stay constant at changing active power of the node i by a unit stress. Being founded on this determination and retaining the same nodes of active power balancing that at $\beta_{(ic)}$ calculation the linearized system similar to (7) may be obtained. In the main system matrix $W_{(\beta)}$ is replaced by matrix W_η which differs from the first one by the fact that elements of the column i equal zero except $a_{n+i,i} = -1$. Elements of this column are the coefficients at ΔQ_{ic} . The right part of the examined system contains disturbance ΔP_{ic} . At these conditions we have:

$$\eta_i = \partial Q_{ic} / \partial P_{ic} = A_{\eta(i,i)} / D_\eta,$$

where D_η is the determinant, $A_{\eta(i,i)}$ is the algebraic complement of matrix W_η . Let us note that $D_\eta = -A_{\beta(n+i,i)}$.

Dividing expression (6) by ΔU_i we obtain the interaction between the steepness factors of the examined types of static characteristics:

$$\beta_{(ic)} = \beta_{ic} - \eta_i \alpha_{ic}. \quad (9)$$

Conditions of connection (9) may be obtained from physical considerations relying on notions putting in these or those coefficients and conditions of their calculation.

Applying disturbance ΔU_i to the node i of power system (Figure) provided that the whole «unbalance» by active power is received by two-terminal networks and $\Delta P_{ic}=0$, we obtain the reaction ΔQ_{ic} . The ratio of this reaction to disturbance ΔU_i gives the value $\beta_{(ic)}$. The same result may be obtained carrying out two experiments successively and introducing respectively the resultant reaction ΔQ_{ic} in the form of two components:

$$\Delta Q_{(ic)} = \Delta Q'_{ic} + \Delta Q''_{ic}. \quad (10)$$

Really, applying external disturbance ΔU_i and considering that only node i is balancing by active and reactive powers we obtain the reaction in the form of $\Delta Q'_{ic}$ and ΔP_{ic} which are connected with disturbance by known ratios:

$$\Delta Q'_{ic} = \beta_{ic} \Delta U_i; \quad \Delta P_{ic} = \alpha_{ic} \Delta U_i. \quad (11)$$

Condition $\Delta P_{ic}=0$ necessary at $\beta_{(ic)}$ calculation is not observed here. In order to balance ΔP_{ic} reaction let us apply disturbance $-\Delta P_{ic}$ to the node i provided that $U_i=\text{const}$. Considering that «unbalance» of active power in this experiment is received by two-terminal networks we have the second component of the reaction by active power in the node i :

$$\Delta Q''_{ic} = -\eta_i \Delta P_{ic}. \quad (12)$$

Taking into account the (11) and (12) condition (10) is written as:

$$\Delta Q_{(ic)} = \beta_{ic} \Delta U_i - \eta_i \alpha_{ic} \Delta U_i.$$

This equality gives known interaction being divided by ΔU_i (9).

The subsystem having several joint nodes with the main part may be artificially divided into some subsystems limited by a number of abutments. For this purpose it is necessary in the singled out subsystem to open branches in the nodes where «hard» voltage control is fulfilled saving previous values of parameters on the ends of open branches. The procedure of branch cutting in nodes with $U=\text{const}$ is very strict. Branch cutting in the most remote nodes from the equivalent node may be assumed at violation of voltage stability condition. The circumstance that static characteristics and steepness factors are mainly determined by the elements and sections of the system located closer to the equivalent node may serve as the base for this.

At slight changes of normal mode the steepness factors β_{ic} , α_{ic} change insignificantly and may be accepted as constant. At deeper mode change they should be adjusted.

The possibility of presenting the equivalent subsystem in the form of four-terminal network containing the branch of impedance Z , to a free end of which the constant equivalent emf is applied, is considered in [3]. The initial information for such equivalent are the parameters of steady-state mode of the equivalent subsystem and steepness factor β , α .

Conclusion

Substitution of electric system parts by the equivalent two-terminal networks with specified static characteristics and steepness factors allows in some cases reducing considerably the dimension of electric system (a number of required parameters). Such approach is appropriate at studying and modeling the modes of some stations, substations, calculations of ladder network modes limiting by static aperiodic stability (for example, long-distance power transmissions with intermediate systems). It gives an opportunity to take into account in generalized form the reaction of separate system parts and localize directly the studied object.

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