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REGULARISING ALGORITHM OF PARAMETER IDENTIFICATION OF ELECTRIC CHARGE EQUIVALENT CIRCUIT. PART II.

Yu.E. Voskoboynikov*, Yu.N. Isayev, V.A. Litasov*, V.A. Kolchanova, E.O. Kuleshova

Tomsk Polytechnic University
* Novosibirsk State Architectural-Building University
E-mail: voscob@mail.ru, isaev_yusup@mail.ru

A new regularizing algorithm of function calculation of indicial admittance in discharge gap equivalent circuit using stable differentiation and integral equation algorithms which allow for errors of initial data is suggested. Application of the least perfect square method at error modelling for the function parameters of indicial admittance is an additional way of «smoothing» modeling error of regularizing solution.

1. Regularising algorithm for calculation of indicial admittance function

In integral equation (1) of the first part of the given paper [1] the intergrand $\dfrac{dU(\tau)}{d\tau}$ is substituted by its estimation $S'_\lambda(t)$ of smoothing cubic spline derivative. It is necessary to find the solution of this equation, i. e. indicial admittance $g(t)$. The solution of such equation is known to be ill-posed problem and to calculate stable solution one must use special regularisation methods [2, 3].

In the work [4] regularising algorithm for impulse function identification in stationary dynamic system (kernel of integral equation) in which input and output signals of identified system are known with random error has been suggested. Use of the discrete Fourier transformation (DFT) and algorithm of the fast Fourier transform (FFT) stipulates high calculation efficiency of regulating algorithm. Without repeating the construction of this algorithm, we are giving the basic calculation relations, *adapting them for the problem of function recovery $g(t)$ and for designations used in this paper.*

The calculation algorithm $g(t)$ can be presented by the following steps [4]:

Step 1. Forming periodic (with N period) sequences:

$$i_p(j)=\begin{cases}\tilde{I}(j\cdot\Delta), & j=0,\dots,N_I-1;\\ 0, & j=N_I,N_I+1,\dots,N-1,\end{cases}$$

$$d_p(j)=\begin{cases}S_\lambda{}'(j\cdot\Delta)\cdot\Delta, & j=0,\dots,N_U-1;\\ 0, & j=N_U,N_U+1,\dots,N-1.\end{cases}$$

Step 2. Calculation of sequence elements

$$D_p(l)=\sum_{j=0}^{N-1}d_p(j)\exp\left(\frac{2\pi i}{N}lj\right),\quad l=0,\dots,N-1,\quad (1)$$

where $i=\sqrt{-1}$.

Step 3. Calculation of DFT sequence coefficients $\{i_p(j)\}$ (direct DFT):

$$I_p(l)=\frac{1}{N}\sum_{j=0}^{N-1}i_p(j)\exp\left(-\frac{2\pi i}{N}lj\right),\quad l=0,\dots,N-1.\quad (2)$$

Step 4. Determination of DFT coefficients (denoted as $\{G_{p\alpha}(l)\}$) of regularizing solution (the calculated relations are presented below).

Step 5. Calculation of periodic solution (inverse DFT from the sequence$\{G_{p\alpha}(l)\}$):

$$g_{p\alpha}(j)=\sum_{l=0}^{N-1}G_{p\alpha}(l)\exp(\frac{2\pi i}{N}l\,j),\quad j=0,\dots,N-1.\quad (3)$$

Step 6. Formation of N_g-dimensional vector g_α according to the rule:

$$g_{\alpha_j}=g_{p\alpha}(j-1),\quad j=1,\dots,N_g,$$

where $N_g=N_I-N_U+1$. If the following condition is met

$$N\geq N_U+N_I-1,$$

then projection g_{α_j} of vector g_α are taken as values of solution regulation $g_\alpha(t)$ in the nodes $t_j=j\cdot\Delta$, $j=0,1,\dots,N_g-1$.

Note that in calculations (1–3) the FFT algorithm is used which reduces the number of operations by the order of 2–3 in comparison with «direct» calculation of corresponding sums. It permits the construction of regularizing solution of some hundreds or even thousands point «length».

Let us consider the calculation of DFT coefficients $\{G_{p\alpha}(l)\}$ at step 4. Suppose that η_j current registration errors has zero average and variance σ_η^2, but errors ξ_i of derivative calculation has also zero average and variance σ_ξ^2. Then the coefficients $G_{p\alpha}(l)$ are found from the nonlinear equation

$$G_{p\alpha}(l) = \frac{D_p^C(l)}{\left|D_p(l)\right|^2 + \alpha(\theta\left|G_{p\alpha}(l)\right|^2 + 1)\cdot Q_p(l)} \cdot I_p(l), \quad (4)$$
$$l = 0,1,\ldots,N-1,$$

where α is the regularization parameter, $\theta=\sigma_\xi^2/\sigma_\eta^2$ is the variance relation, $D_p^C(l)$ is the magnitude, complex-conjugate to $D_p(l)$. The elements of $Q_p(l)$ sequence are formed according to the rule:

$$Q_p(l) = \begin{cases} Q(l\cdot\Delta_\omega), & l = 0,\ldots,N/2; \\ Q((N-l)\cdot\Delta_\omega), & l = N/2+1,\ldots N-1, \end{cases}$$

where $\Delta_\omega=2\pi/(N\Delta)$ is the discretization step in frequency domain. The function $Q(\omega)$ can be interpreted as frequency characteristic of stabilizing functional [2, 3]: it has to be a monotone increasing function and $Q(\omega)\to\infty$ at $\omega\to\infty$. If the order of regularization is r, then at sufficiently high frequencies ω the asymptotic form is true $Q(\omega)\approx\omega^{2r}$.

To calculate the solution $G_{p\alpha}^*(l)$ of nonlinear equation at fixed parameter let us turn to the scheme of simple iteration

$$G_{p\alpha}^{(n+1)}(l) = \frac{D_p^C(l)}{\left|D_p(l)\right|^2 + \alpha(1+\theta\left|G_{p\alpha}^{(n)}(l)\right|^2)\cdot Q_p(l)} \cdot I_p(l),$$
$$n = 0,1,\ldots \quad (5)$$

«Start point» $G_{p\alpha}^{(0)}(l)$ is given as $G_{p\alpha}^{(0)}(l)=I_p(l)$, $l=0,1,\ldots N-1$. The condition of iteration cease has the view

$$\left[\frac{\sum_{l=0}^{N-1}\left|G_{p\alpha}^{(n+1)}(l) - G_{p\alpha}^{(n)}(l)\right|^2}{\sum_{l=0}^{N-1}\left|G_{p\alpha}^{(n)}(l)\right|^2}\right]^{\frac{1}{2}} \le 0.01. \quad (6)$$

Calculation experiment has shown that not more than 5–8 iterations are necessary to meet the condition (6).

Choice of regularization parameter included in (4), (5) is the main problem of regularization algorithm construction in practice. At aconservative values α in solution g_α the noise terms conditioned by the η_j, ξ_j errors are present. At α pessimistic values the $g(t)$ informative constituents are removed from the g_α solution (solution is «oversmoothed»). Therefore, as α regularizing parameter it is desirable to takevalue α_{opt}, giving the minimum of root-mean-square error determined by the functional:

$$\text{CKO}(\alpha) = M[\sum_{j=1}^{N_g}(g_{\alpha_j} - \bar{g}_j^+)^2],$$

where $\bar{g}_j^+$ is the vector projection $\bar{g}^+$, being the normal pseudo-solution at precise initial data of $u(t)$, $i(t)$, M[•] is the expectation operator in terms of g_α random vectors ensemble g_α haracteristics of $g(t)$ function, which are unknown in practice. Therefore, we restrict our consideration to estimation calculation α_W for α_{opt}.

Introduce the functions [4]:

$$R_W(\gamma) = N\cdot\sum_{l=0}^{N-1}\frac{\dfrac{(1+\theta\left|G_{p\alpha}^*(l)\right|^2)\cdot Q_p(l)}{\gamma\left|D_p(l)\right|^2 + (1+\theta\left|G_{p\alpha}^*(l)\right|^2)\cdot Q_p(l)}\cdot\left|I_p(l)\right|^2}{(C_\sigma\sigma_\xi^2\left|G_{p\alpha}^*(l)\right|^2 + \sigma_\eta^2)}; \quad (7)$$

$$R_W'(\gamma) = \frac{d}{d\gamma}R_W(\gamma) = -N\times$$
$$\times\sum_{l=0}^{N-1}\frac{\dfrac{\left|D_p(l)\right|^2\cdot(1+\theta\left|G_{p\alpha}^*(l)\right|^2)\cdot Q_p(l)}{\left[\gamma\left|D_p(l)\right|^2 + (1+\theta\left|G_{p\alpha}^*(l)\right|^2)\cdot Q_p(l)\right]^2}\cdot\left|I_p(l)\right|^2}{(C_\sigma\sigma_\xi^2\left|G_{p\alpha}^*(l)\right|^2 + \sigma_\eta^2)}, \quad (8)$$

where $C_\sigma = \dfrac{N_U}{N_I}\cdot N^2\cdot\Delta^2$, $\gamma=1/\alpha$. As the value α_W is accepted $1/\gamma_W$, where $\gamma_W=\gamma^{(n+1)}$, but the value $\gamma^{(n+1)}$ meets the condition:

$$\vartheta_{m,\beta/2} \le R_W(\gamma^{(n+1)}) \le \vartheta_{m,1-\beta/2}, \quad (9)$$

where $\vartheta_{m,\beta/2}$, $\vartheta_{m,1-\beta/2}$ are the quantiles of χ^2-distribution with $m=N_I-1$ freedom degrees of significance level $\beta/2$ and $1-\beta/2$ correspondingly (as a rule, β=0,05). The sequence $\{\gamma^n>0\}$ is produced by iteration procedure:

$$\gamma^{(n+1)} = \gamma^{(n)} - \frac{R_W(\gamma^{(n)}) - m}{R_W'(\gamma^{(n)})}, \quad n = 1,2,\ldots;\ \gamma^{(0)} \ll 1.$$

One can show that if the norm square $\|I\|^2$ of $\tilde{I}$ vector meets the condition $\dfrac{\|\tilde{I}\|^2}{\sigma_\eta^2} > \vartheta_{m,\beta/2}$ and $\gamma^{(0)}\ll 1$ (usually $\gamma\approx10^{-10}$), then value $\gamma^{(n+1)}$ meeting (9) is found. Calculation of such value does not require more than 4–5 iterations.

The investigations made [4] have shown high efficiency of estimation α_W. Estimation of regularising solution constructed at $\alpha=\alpha_W$ does not exceed that of optimal solution constructed at $\alpha=\alpha_W$ by more than 15...20 %.

Note that function $R_W(\gamma)$, $R_W'(\gamma)$, includes, variances σ_ξ^2, σ_η^2. Usually in practice the variance values are unknown (it is particularly the case with variance σ_ξ^2 of error in voltage derivative calculation in terms of smoothing cubic spline).

To overcome this difficulty the following estimations of variance are suggested:

- σ_η^2: $\quad \widehat{D}_\eta = \dfrac{N^2}{2N_I L_\eta}\sum_{l=-L_\eta}^{L_\eta}\left|I_p(N/2+l)\right|^2; \quad (10)$

- σ_ξ^2: $\quad \widehat{D}_\xi = \dfrac{1/\Delta^2}{2N_U L_\xi}\sum_{l=-L_\xi}^{L_\xi}\left|D_p(N/2+l)\right|^2. \quad (11)$

These estimations are based on the assumption that in the neighbourhood of $l=N/2$ point (the point, relative to which DFT coefficient modules is symmetric) DFT coefficients are conditioned only by specification errors of initial realizations. Amount of sampling, by which point variance estimates are calculated, equals to $2L_\eta+1$, $2L_\xi+1$ respectively. To specify L_η, L_ξ one can suggest the relation: $L_\eta=L_\xi=(0{,}075...0{,}1)N$. Thus, having accepted $L_\eta=L_\xi=0{,}1N$, we obtain for N=256 the amount of sampling 51 (that permits for reliable estimation of variance).

2. Parameterisation of conductivity function and parameter estimation

According to the view of $g_\alpha(t)$ regularising solution constructed in the previous paragraph one decides on the view of parametric dependence $g(t,P)$ for $g_\alpha(t)$ approximation, it uniquely defines the structure of discharge equivalent circuit. The argument P means the vector consisting of parameters of parametric dependence.

Sufficiently universal approximation of indicial admittance is the function of the view

$$g(t,P)=A_1e^{\mu_1 t}+A_2e^{\mu_2 t}, \tag{12}$$

where P vector includes four parameters:

$$P=\begin{vmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{vmatrix}=\begin{vmatrix} A_1 \\ \mu_2 \\ A_2 \\ \mu_2 \end{vmatrix},$$

Among them are the roots μ_1, μ_2 of circuit characteristic equation. The admittance function (12) corresponds to electric circuit of the second order, the circuit of which is presented in [5].

To estimate the parameters p_q, q=1,2,...,Q of $g(t,P)$ function let use the least-square method of, i. e. estimations of $\hat{p}_q$ are from the condition of functional minimum:

$$J(P)=\sum_{j=1}^{N_g}(g_\alpha(t_j)-g(t_j,P))^2$$

and are the solution of system consisting of Q (in general case nonlinear) equations:

$$\begin{cases} \dfrac{\partial J(P)}{\partial p_1}=0; \\ \quad\vdots \\ \dfrac{\partial J(P)}{\partial p_Q}=0. \end{cases} \tag{13}$$

Parameters p_q, q=1,2,...,Q are uniquely connected with (i. e. determined by) the parameters $\theta_1,...,\theta_S$ of electrical equivalent circuit (resistance, capacity, inductance magnitudes) and this connection is expressed in algebraic relations of the following view:

$$\begin{cases} p_1=\varphi_1(\theta_1,\theta_2,...,\theta_S); \\ p_2=\varphi_2(\theta_1,\theta_2,...,\theta_S); \\ \quad\vdots \\ p_Q=\varphi_Q(\theta_1,\theta_2,...,\theta_S). \end{cases} \tag{14}$$

Construction of those algebraic relations is made by the analysis method of transient processes in the DC circuits studied in details in the course of theoretical bases of electrical technology [6] and therefore these questions are not considered here.

Having calculated the estimations $\hat{p}_q$, q=1,2,...,Q we solve the non-linear set of equations (14) (probably by using the method of least squares) and find the estimation $\hat{\theta}_s$, s=1,2,...,S. At this point parameter identification of gas discharge equivalent circuit is completed.

3. Result of calculation experiment

To support the efficiency of the identification algorithm suggested and to determine its accuracy capability a large calculation experiment has been carried out, it consisting of the following.

As a «true» function of indicial admittance $g(t)$ the function (12) was taken with complex conjugate roots of characteristic level: $\mu_1=\mu_2^*=-416{,}666-i{\cdot}571{,}304$. Then the admittance function $g(t)$ was preset by the expression:

$$g(t)=0{,}71\cdot e^{-416{,}666\cdot t}\cdot\cos(571{,}304\cdot t+1{,}059)$$

or in general form by the formula

$$g(t)=A\cdot e^{\mu\cdot t}\cdot\cos(\omega\cdot t+\varphi).$$

For such admittance function the identification problem consists in estimation of parameters A, μ, ω, φ.

Interval of $g(t)$ function assignment was equal to [0, 0,015 sec]. The form of specified voltage is presented in Figure a [1] and [0, 0,055 sec] is assignment interval. The current values $I(t)$ were defined by the interval (1) [1] and Interval of $I(t)$ function assignment was equal to [0, 0,070 sec]. The discretization step Δ was specified $\Delta=2{,}5\cdot10^{-4}$ *sec,* it determined the following lengths of discrete sequences: $_U$=220, N_g=60, N_I=280.

The values $U(t_j)$ were distorted by normally distributed random numbers ζ_j with zero average and variance σ_ζ^2 defined by relative noise level δ_U:

$$\sigma_\zeta^2=\left(\frac{\delta_U\cdot\max|U(t_j)|}{2}\right)^2.$$

Similarly, the values $I(t_j)$ were distorted by normally distributed random numbers η_j zero average and variance σ_η^2, defined by relative noise level δ_I.

Then, in terms of noised, in this way, values $\tilde{U}(t_j)$, $\tilde{I}(t_j)$ the regularizing solution $g_\alpha(t_j)$ was constructed according to algorithms in 3, 4 points of the given paper. Smoothing parameter λ was determined from the solution of non-linear equation (4) [1] at $\Delta_{np}=5\cdot10^{-3}$ sec. Regularizing parameter was calculated from the condition (9), in this case it was suggested that variances σ_ξ^2, σ_η^2 were unknown, and instead of σ_ξ^2, σ_η^2 (7), (8) their estimations were used in (10), (11).

According to the constructed solution $g_\alpha(t_j)$ from the set of equations (13) the estimations were calculated $\hat{A}$, $\hat{\mu}$, $\hat{\omega}$, $\hat{\varphi}$. Accuracy of these estimations was determined by the vector of relative errors:

$$\delta_p = \begin{vmatrix} \left|\widehat{A} - A\right| / \left|A\right| \\ \left|\widehat{\mu} - \mu\right| / \left|\mu\right| \\ \left|\widehat{\omega} - \omega\right| / \left|\omega\right| \\ \left|\widehat{\varphi} - \varphi\right| / \left|\varphi\right| \end{vmatrix}.$$

Since vector projections δ_p are random, then by sampling of random vectors by volume the vector of average $\delta_p^{(n)}$ relative estimation errors $\overline{\delta_p}$ has been calculated, the projections of which are equal to average values of corresponding vector projections $\delta_p^{(n)}$. Vector $\delta_p^{(n)}$ contains relative estimation errors $\widehat{A}^{(n)}$, $\widehat{\mu}^{(n)}$, $\widehat{\omega}^{(n)}$, $\widehat{\varphi}^{(n)}$, that are constructed in terms of initial data $\tilde{U}^{(n)}(t_j)$, $\tilde{I}^{(n)}(t_j)$, obtained by distortions of «accurate» values $U(t_j)$, $I(t_j)$ by n-realisations of errors in voltage ${}_j^{(n)}$ and current-specification $\eta_j^{(n)}$.

In the table the values of vector $\overline{\delta_p}$ projections are presented for different relative levels of δ_I=0,01; 0,10; δ_U=0,01; 0,10.

It is seen that *relative errors of identification depend weakly on error level of initial data.* It can be explained by the two reasons:

- Application of stable algorithms for differentiating and solving integral equations, including errors of initial data of the solved problem efficiently.
- Application of least perfect square method in construction of estimations for indicial admittance function parameters, which is additional way of «smoothing» error in construction of regularising solution $g_\alpha(t_j)$.

Table. *Relative errors of parameter identification of indicial admittance function*

		δ_U	
		0,01	0,10
δ_I	0,01	0,035 0,031 0,005 0,007	0,119 0,096 0,019 0,012
	0,10	0,107 0,088 0,023 0,011	0,136 0,112 0,045 0,023

Comparing vector separate projections $\overline{\delta_p}$, one can note that A,μ parameters are estimated with a little more errors in comparison with ω, φ. It can be explained by the error of regularising solution $g(t_j)$ in amplitude.

Analysis of the table and results of other calculation experiments allow us to conclude: the suggested identification algorithm *identifies the parameters of indicial admittance with acceptable accuracy for construction of electric discharge equivalent circuit.*

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