UDC 537.874.4

STUDY OF ELECTROMAGNETIC SCATTERING BY STRUCTURES CONSISTING OF SEVERAL UNCROSSING CONDUCTORS

Ju.A. Keller

Tomsk State University
E-mail: kua1102@fpmk.tsu.ru

The algorithm for solution of problems of electromagnetic scattering on the structures composed from a finite number of uncrossing thin conductors is built on the basis of the method of auxiliary sources. The built algorithm is realized in the form of a code for colculation of scattering characteristics of a set of structures differing by relative location of conductors. The influence of relative location of conductors on bistatic scattering sections of considered structures as well as current distribution alongside conductore are researched.

The theory of excitation and scattering of electromagnetic waves by a thin conductor was developed a long time ago [1]. At the initial stage this theory has been generated by needs of the antenna equipment. However, since the middle of 60-s' years of the last century, researchers start to pay the greater attention to the analysis of scattering properties of thin conductors. Interest to similar sabjects has been caused by needs of creation of objects with the given scattering properties. The most commonly used approach to solve both problems of excitation of a thin conductor, and problems of scattering on it is the method of the integral equations which order is equal to number of conductors in the structure. In the presented work to solve problems of scattering on the structures containing thin conductors, the variant of the auxiliary sources method is used. It allows to exclude a stage of building of the integral equations system, thus simplifying process of the solving of a problem and reducing expenses in use of computer resources. The mathematical formulation of the variant and the short description of abilities of the program realized on its basis for calculation of current distributions along conductors and characteristics of a scattered field of various structures are given. Results of the numerical calculations describing mutual influence of conductors on current distribution along them and bistatic scattering section (BSS) are presented.

1. The formulation of the problem and the method of its solving

The geometry of the problem is shown on Fig. 1. Let`s consider the stationary (dependence on time is chosen as $\exp(-i\omega t)$) problem of diffraction of electromagnetic field $\{\vec{E}_0,\vec{H}_0\}$ on the structure consisting of U un-crossed conductors, limited by surfaces S_u (u=1,2,...,U) and located arbitrarily under relation each to other. We shall understand as a thin conductor the ideal conductor of round section which diameter is small in comparison with a wave length and length of the conductor. This structure is placed in the homogeneous unbounded medium D_e with dielectric and mag-

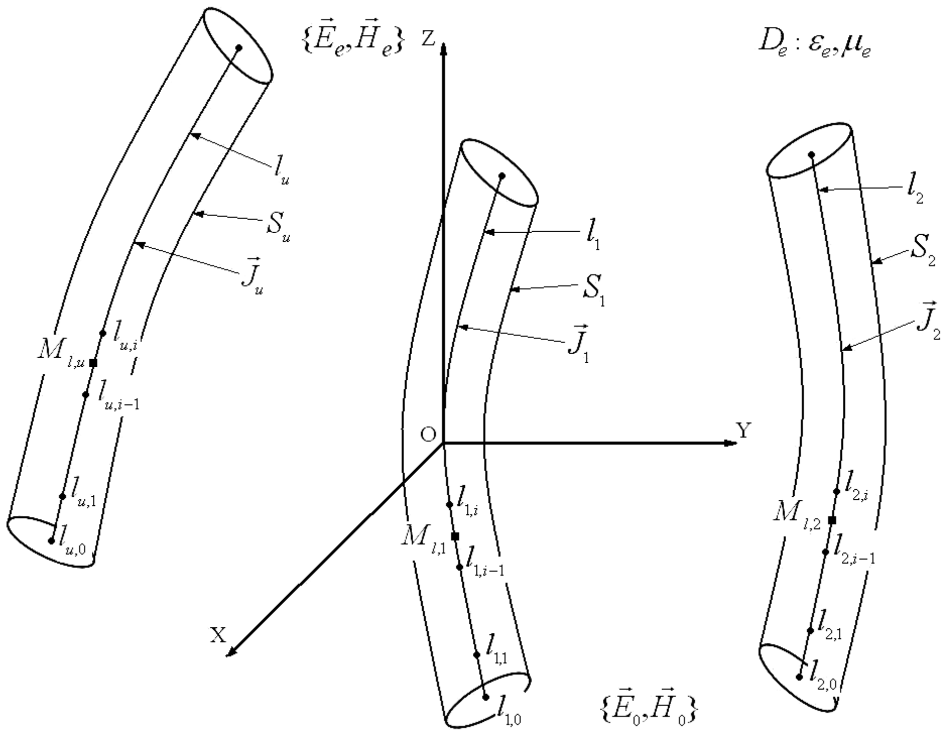

Fig. 1. *Geometry of the problem*

netic permeability ε_e and μ_e accordingly in the Cartesian system of coordinates with the origin, laying inside the conductor with a serial number u=1. It is required to find $\{\vec{E}_e,\vec{H}_e\}$ the scattered field in area D_e.

Mathematical discription of the problem has the following form:

$$[\nabla,\vec{E}_e]=i\omega\mu_e\vec{H}_e,\quad [\nabla,\vec{H}_e]=i\omega\varepsilon_e\vec{E}_e. \quad (1)$$

In area D_e,

$$[\vec{n}_u,\vec{E}_e]=-[\vec{n}_u,\vec{E}_0] \quad (2)$$

On surfaces S_u, where u=1,2,...,U,

$$[\sqrt{\varepsilon_e}\vec{E}_e,\vec{R}/R]+\sqrt{\mu_e}\vec{H}_e=O(R^{-1}),$$
$$[\sqrt{\mu_e}\vec{H}_e,\vec{R}/R]-\sqrt{\varepsilon_e}\vec{E}_e=O(R^{-1}) \quad (3)$$

at $R\to\infty$. Here $\vec{n}_u$ are individual vectors of normals to conductor surfaces S_u, $R=\sqrt{x^2+y^2+z^2}$.

The solution of the problem formulated above can be obtained as follows. Let`s place the auxiliary continuously distributed current $\vec{J}_u$ (Fig. 1) inside of each thin conductor on its axis. Let`s discrite an unknown scattered field $\{\vec{E}_e,\vec{H}_e\}$ in areas D_e as the sum of fields of the entered auxiliary currents:

$$\vec{E}_e(M)=\frac{i\omega}{k_e^2}\sum_{u=1}^{U}[\nabla,[\nabla,\vec{\Pi}_u]],$$
$$\vec{H}_e(M)=\frac{1}{\mu_e}\sum_{u=1}^{U}[\nabla,\vec{\Pi}_u],$$
$$\vec{\Pi}_u=\int_{l_u}\Psi_e(M,M_{l,u})\vec{J}_u dl, \quad (4)$$

here $k_e=\omega\sqrt{\varepsilon_e\mu_e}$, $\Psi_e(M,M_{l,u})=(4\pi R_{M,M_{l,u}})^{-1}\exp(ik_eR_{M,M_{l,u}})$; $R_{M,M_{l,u}}$ is distance from points $M_{l,u}$ on the axis of conductors up to the observation point M in area D_e; $\vec{J}_u$ are unknown axial auxiliary currents u=1,2,...,U; integration is carried out along axial lines of conductors; l_u is the axial line of a conductor with number u.

The field (4) satisfies to Maxwell equations (1) and to radiation conditions (3) in area D_e. To satisfy to a boundary condition (2), it is necessary to choose the axial auxiliary currents $\vec{J}_u$ (u=1,2,...,U) by appropriate way.

Let's introduce a piece-constant approximation of axial currents. Let's separate a line l_u of each current $\vec{J}_u$ on N_u small sites, within the limits of each of them the current can be considered as constant. Then expression for $\vec{\Pi}_u$ in (4) approximately can be written as

$$\vec{\Pi}_u=\sum_{i=1}^{N_u}J_{u,i}\vec{e}_{u,i}\int_{l_{u,i-1}}^{l_{u,i}}\Psi_e(M,M_{l,u})dl, \quad (5)$$

where $J_{u,i}$ – the current on the i^{th} site of the conductor with number u, $\vec{e}_{u,i}$ is the unit vector, which direction coincides with the direction of the tangent in an average point of the considered site. At such approach finding of unknown distributions of axial currents is reduced to finding $\sum_{u=1}^{U}N_u$ of elements of a current.

To define values of current elements let`s use boundary conditions (2), satisfying them according to the method of collocations. Let M_j (j=1,2,...,L_u) are points of collocation on the surface S_u; L_u is number of points of a collocation on S_u. Assuming that conductor diameter is small in comparison with conductor length and wave length we shall consider, that the contribution of azimuthal components of currents on thin conductor surfaces into scattered field can be neglected. Then to find unknown elements of currents $J_{u,i}$ (u=1,2,...,U; i=1,2,...,N_u) we shall obtain the following system of the linear algebraic equations with a complex matrix of dimension $(\sum_{u=1}^{U}L_u)\times(\sum_{u=1}^{U}N_u)$:

$$E_{e,u,l}^{j}=-E_{0,u,l}^{j},\quad u=1,2,\ldots,U,\quad j=1,2,\ldots,L_u, \quad (6)$$

where $E_{e,u,l}^{j}$ and $E_{0,u,l}^{j}$ are values of electric components of scattered (4) and exciting fields along an axis of the conductor with number in points of collocation on its surface.

Solution of the system (6) is defined by minimization of functional

$$\Phi=\sum_{u=1}^{U}\sum_{j=1}^{L_u}|E_{e,u,l}^{j}+E_{0,u,l}^{j}|^2. \quad (7)$$

After solution of the problem of minimization (definition of unknown elements of the current $J_{u,i}$, u=1,2,...,U; i=1,2,...,N_u) necessary characteristics of a scattered field are determined from (4). In particular, for a component of the scattered field in far-field zone we have

$$E_{e,\theta}(M)=(\mu_e/\varepsilon_e)^{1/2}H_{e,\varphi}(M)=$$
$$=(\exp(ik_eR)/k_eR)D_\theta(\theta,\varphi)+O(R^{-2}),$$
$$E_{e,\varphi}(M)=-(\mu_e/\varepsilon_e)^{1/2}H_{e,\theta}(M)=$$
$$=(\exp(ik_eR)/k_eR)D_\varphi(\theta,\varphi)+O(R^{-2}), \quad (8)$$

where R is distance from the origin of the coordinate system to the point of observation M, and $D_\theta(\theta,\varphi)$, $D_\varphi(\theta,\varphi)$ are components of the scattering diagram, determined by the expressions

$$D_\theta(\theta,\varphi)=\sum_{u=1}^{U}\sum_{i=1}^{N_u}\tilde{J}_{u,i}(\cos\theta\cos\varphi\cos\alpha_{u,i}+$$
$$+\cos\theta\sin\varphi\cos\beta_{u,i}-\sin\theta\cos\gamma_{u,i})I_{u,i},$$
$$D_\varphi(\theta,\varphi)=\sum_{u=1}^{U}\sum_{i=1}^{N_u}\tilde{J}_{u,i}(-\sin\theta\cos\alpha_{u,i}+$$
$$+\cos\varphi\cos\beta_{u,i})I_{u,i}, \quad (9)$$

where $\tilde{J}_{u,i}=i\omega J_{u,i}$ (u=1,2,...,U;i=1,2,...,N_u) is the solution of system (6), $\cos\alpha_{u,i}$, $\cos\beta_{u,i}$, $\cos\gamma_{u,i}$ are directed cosines of the unit vector $\vec{e}_{u,i}$, θ and φ are the standard angular spherical coordinates of the observation point M, and integral $I_{u,i}$ has the following form:

$$I_{u,i} = \int_{k_e l_{u,i-1}}^{k_e l_{u,i}} \exp\{-i(\sin\theta\cos\varphi k_e x_{l,u} + \sin\theta\sin\varphi k_e y_{l,u} + \cos\theta k_e z_{l,u})\}d(k_e l),$$

where $k_e x_{i,u}$, $k_e y_{i,u}$, $k_e z_{i,u}$ are coordinates on an conductor axis on which integration is carried out.

The proposed method, as well as other variants of a method of auxiliary sources, allows to carry out an aposteriory estimation of accuracy of the obtained solution. As the figure of merit characterising accuracy, we shall choose value of relative norm of boundary conditions discrepancy on surfaces of all conductors S_u in points, intermediate in relation to points of collocation

$$\Delta = (\Phi'/\Phi_0)^{1/2}, \quad \Phi_0 = \sum_{u=1}^{U}\sum_{j'=1}^{L'_u} | E^{j'}_{0,u,l} |^2. \qquad (10)$$

Where Φ' is the value of the functional (7) on the mentioned above set of points; Φ_0 is the value of the falling field norm on same set of points; L'_u is the number of intermediate points on a surface of the conductor with number u.

2. Numerical results

On the basis of the method stated above the code to calculate components of scattered field and to control accuracy of the obtained solution for structures which configuration is presented on Figs. 2–4 is created.

The first structure (Fig. 2) represents the single rectilinear conductor with length L_1 located in the Cartesian system of coordinates OXYZ in such a manner that its axial line is directed along axis OZ, and the middle of an axial line coincides with the origin of coordinate system. The second structure (fig. 3) consists of two parallel conductors with length L_1 and L_2 with the axial lines directed along axis OZ, located on distance from each other; the middle of the axial line of the second conductor is located on axis OX. The third structure (Fig. 4) consists of mutually orthogonal conductors. The axial line of the first conductor of this structure with length L_1 is oriented along axis OZ, its middle coincides with the beginning of the system of coordinates. Other two conductors with length L_2 also L_3 are located on identical distances from the first conductor in such a manner that their axial lines are directed along axis OY.

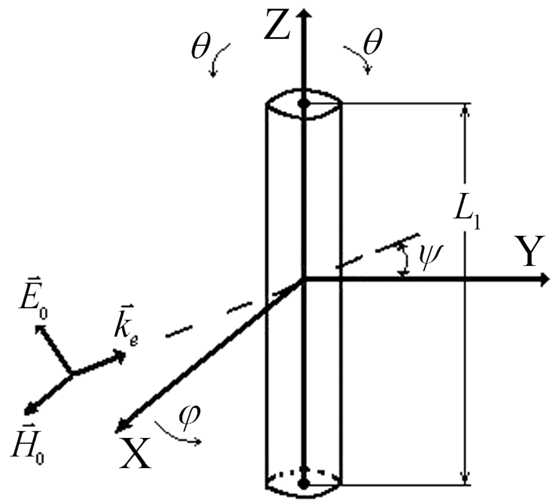

Fig. 2. *Single conductor*

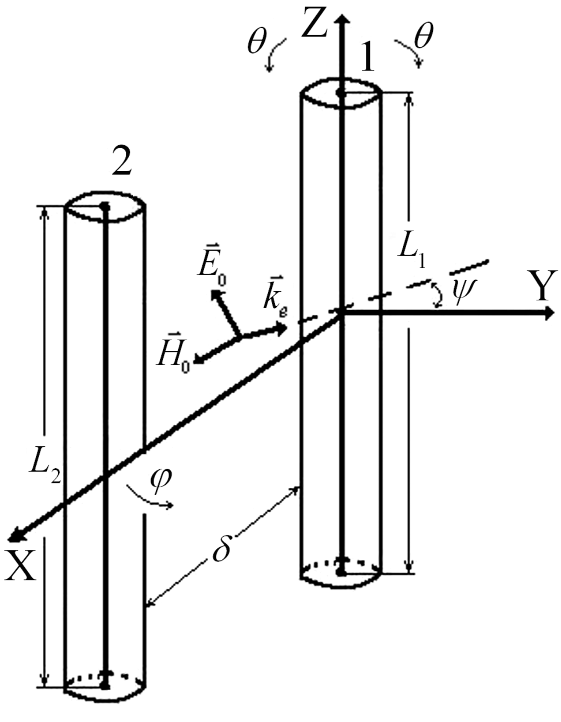

Fig. 3. *Two parallel conductors*

Input values of the program are the structure configuration, a falling field $\{\vec{E}_0, \vec{H}_0\}$, lengths and radiuses of

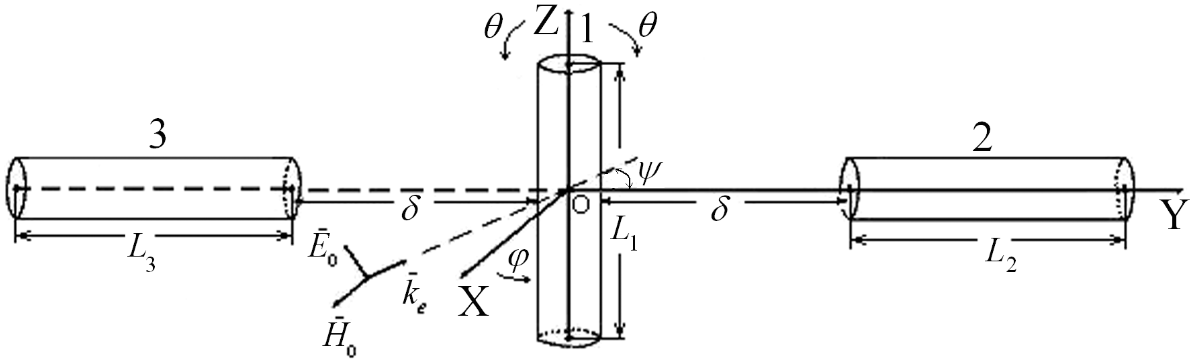

Fig. 4. *Structure from mutually orthogonal conductors*

conductors, distance between conductors, and also number of elements of breaking down of the axial current N_u for each conductor.

Minimization of functional (7) is carried out by the method of the mated gradients; iterative process stops if change of functional after carrying out of the next iteration does not exceed 10^{-5}.

By means of the given program the series of the calculations directed on finding-out of influence of number of elements of breaking down of an axial current on size of boundary conditions discrepancy, in comparison of obtained results with results of other authors, and also on estimation of mutual influence of conductors on current distributions along them and BSS of the structures made of them is carried out. Some results are presented below.

It is supposed, that structures are excited in such a manner that vectors $\vec{E}_0$ and $\vec{k}_e$ lay in planes ZOY and a vector $\vec{k}_e$ forms with axis OY the angle ψ (as shown in Fig. 2–4). It is supposed also, that the radius of conductors is equal 0,02λ in all cases, where λ is length of the exciting wave.

The results of comparison of axial current distribution along a single conductor with length $L=\lambda$ at the angle of a flat wave falling ψ=30° are presented on Fig. 5.

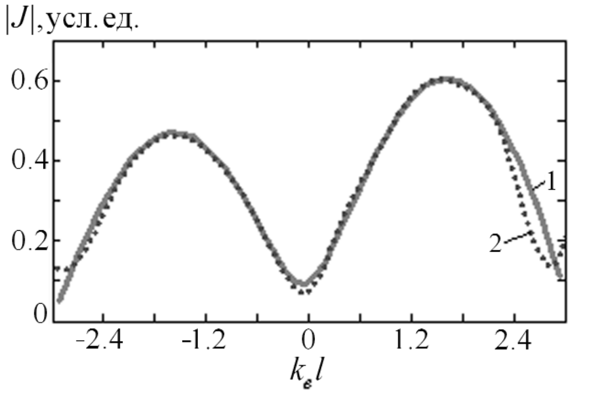

Fig. 5. *Distribution of the axial current along a conductor with length $L=\lambda$*

The coordinate of the conductor axis point is laid on the abscissa axis, and – value of a current in this point in standard units is laid on the ordinate axis. The curve 1 is the results obtained by the given method, the curve 2 is the results from work [5] (they are obtained by the method of integral equations). At obtaining of the curve 1 a current line is broken down into 40 sites. At the given number of breaking discrepancy value is 0,284. As show these curves, good coincidence of compared results takes place. Insignificant observable differences can be explained by errors at graphic recording of information from figure of work [5], and also by errors of calculations both the given method, and a method of the integral equations.

The results presented on fig. 6–9, allow to estimate mutual influence of parallel conductors on current distributions and BSS. Such researches are of interest for the theory and technique of wire (dipole) antennas, and also for an estimation radar-tracking perceptibility of objects containing such wires. The estimation of mutual influence of conductors was carried out by comparison of currents distributions along conductors and BSS of the structures consisting from two (fig. 3) parallel conductors, with characteristics corresponding to a single conductor (Fig. 2).

The Fig. 6 and 7 characterize current distribution along an axis of conductors with length $L=\lambda$ located very close (δ=0,0016λ) and enough far apart (δ=3λ) each of other.

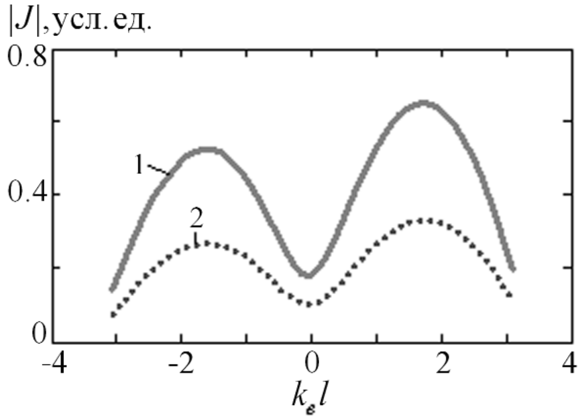

Fig. 6. *Distribution of current along an axis of conductors with length $L=\lambda$ located on distance δ=0,0016λ*

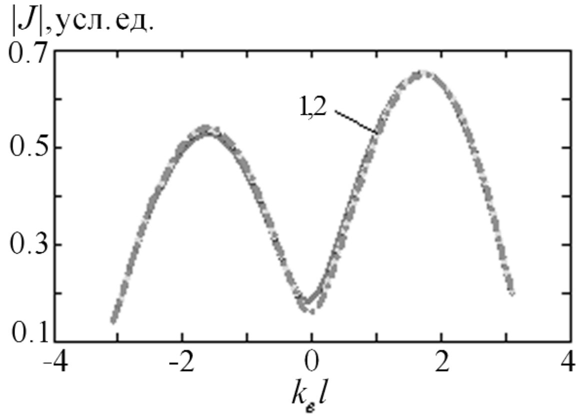

Fig. 7. *Distribution of current along an axis of conductors with length $L=\lambda$ located on distance δ=3λ*

Conductors are excited by the flat wave falling at the angle ψ=30°. The coordinate of the conductor axis point is laid on the abscissa axis, and – value of a current in this point in conventional units is laid on the ordinate axis. Value $k_e l$=0 corresponds to the middle of conductors, values $k_e l$>0 belong to the top part of conductors, value $k_e l$<0 – to the bottom part of conductors. Curves 1 on fig. 6, 7 are current distribution of a along a single conductor; curves 2 are current distribution along each conductor of the system consisting from two conductors (distributions are identical). In the all cases the number of elements of breaking down of axial current was chosen equal to 40.

BSS are presented on figs. 8, 9

$$\sigma(\theta,\varphi) = \lim_{R\to\infty}\{|E_{e,\theta}(\theta,\varphi)|^2 + |E_{e,\varphi}(\theta,\varphi)|^2\}/|\vec{E}_0|^2\} \quad (11)$$

of the same structures in semi-surface φ=0° for the same cases of a relative location of conductors: δ=0,0016λ (Fig. 8) and δ=3λ (Fig. 9).

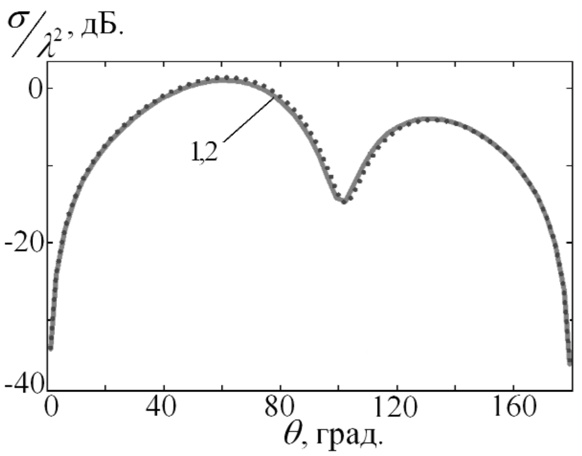

Fig. 8. *BSS of the structures containing of samples with length $L=\lambda$, located on distance $\delta=0{,}0016\lambda$*

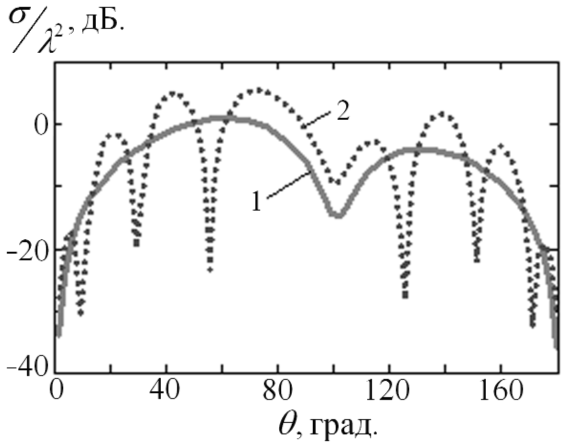

Fig. 9. *BSS of the structures consisting of conductors with length $L=\lambda$, located on distance $\delta=3\lambda$*

Structures are excited by the flat wave falling at the angle $\psi=30°$. On these figures curves 1 is BSS of a single conductor (Fig. 2), curve 2 is BSS of structures from two parallel conductors (Fig. 3).

Results of calculations presented on the Figs. 6–9 allow to draw the following conclusions.

If other conductors are located near a rectilinear conductor distribution of current along conductors essentially differ from distribution of current along a single conductor (fig. 6). If the structure consist of two conductors distribution of current on both conductors are identical. Difference of distribution of current for structures of conductors located near each to other from distribution of current along a single conductor is explained by strong interaction of these conductors. However, despite of essential distinctions of distribution of current of structures of close located conductors from distribution of current along a single conductor, BSS of a single conductor and structures from two conductors differs a little (Fig. 8). It speaks that in relation to a scattered field the structure from close located parallel conductors is equivalent to one conductor. If conductors of considered structures are enough far apart (in this case on distance $\delta=3\lambda$) distribution of current along conductors are close to distribution of current along a single conductor (Fig. 7). It is explained by significant reduction of interaction of conductors of structure. In this case it is possible to consider each conductor as indepen nt scaterer, and a scattered field, hence, is superposition of fields, scattered by each conductor, i.e. has interference structure, that reflects results (Fig. 9).

The results presented on Fig. 10, 11, allow to estimate mutual influence of orthogonal conductors on distribution of current and BSS. As in case of parallel conductors, the estimation of mutual influence was carried out by comparison of distribution of current and BSS of the structure consisting of perpendicular conductors (Fig. 4) with corresponding characteristics for a single conductor (Fig. 2). At carrying out of numerical calculations in all cases the number of elements of breaking down of an axial current was chosen equal 40 for wave length.

Distributions of current along the first conductor of the structure presented on Fig. 4 at falling of a flat wave under $\psi=30°$ and various distances between conductors are shown on Fig. 10. The length of all conductors of structure is equal $\lambda/2$. Value $k_e l=0$ corresponds to the middle of conductors; values $k_e l>0$ belong to the top part of the vertical (first) conductor of structure; values $k_e l<0$ belong to the bottom part of the vertical conductor.

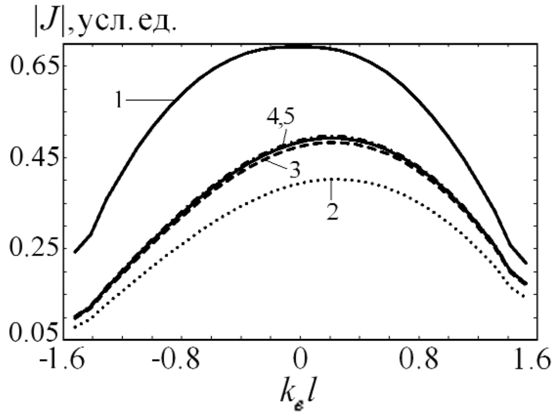

Fig. 10. *Distribution of current along the first conductor of the structure at falling of a flat wave at angle $\psi=30°$*

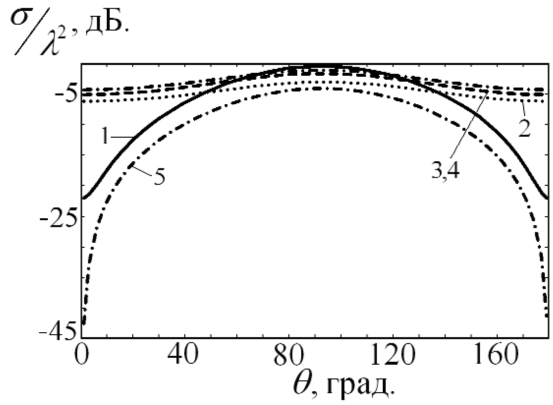

Fig. 11. *BSS of structures from mutually perpendicular conductors at falling of a flat wave at angle $\psi=30°$*

The curve 1 characterizes distribution of current along a vertical (first) conductor of structure at

δ=0,05λ, a curve 2 – at δ=0,2λ, a curve 3 – at δ=0,8λ and a curve 4 – at δ=1,5λ, the curve 5 characterizes distribution of current along a single vertical conductor.

BSS in a semi-plane φ=0° of considered structure for the same angle of falling of a flat wave and distances between conductors are presented on Fig. 11. Designations of curves are similar to Fig. 10.

Conclusions

Basing on the method of auxiliary sources the numerical algorithm is built and the computer program for solution of problems of electromagnetic scattering on the structures made of finite number of uncrossed thin conductors is realized. Influence of a relative position of conductors on bistatic scattering sections of the considered structures, as well as on current distributions along conductors is investigated.

At inclined falling of a wave to an axis of the central conductor of structure the falling wave excites both the central conductor, and lateral conductors located near perpendicularly to it. In this case at small distances between conductors (δ<0,2λ) distributions of current on the central conductor depend on distance; however at distances δ≥0,8λ distributions of current on the central conductor of structure little differ from distribution of current along the same single conductor. It is shown, that bistatic scattering sections of considered structure differ from those for a single conductor.

Literature

1. Calculation methods in electrodynamics // Edition of R. Mitra Transl. from Engl. – M.: World, 1977. – 138 p.
2. Richmond J.H. Scattering by a Dielectric Cylinder of Arbitrary Cross Section Shape // IEEE Transactions on Antennas and Propagation. – 1965. – V. AP-13. – № 3. – P. 334–341.
3. Mei K.K. On the Integral Equations of Thin Wire Antennas // IEEE Transactions on Antennas and Propagation. – 1965. – V. AP-13. – № 3. – P. 374–378.
4. Lifanov I. K., nenashev A. S. New approach to the theory of Thin Wire Antennas // Electromagnetic waves and electronic systems. – 2003. – V. 8. – № 5. – P. 25–41.
5. Harrington R.F. Time-Harmonic Electromagnetic Fields. Ch. 3. – N.Y.: McGraw-Hill, 1961. – P. 358–359.

UDC 535.36

INFLUENCE OF SPACE LIMITED OF THE DISPERSION MEDIUM ON IMAGE QUALITY CHARACTERISTICS

B.V. Goryachev, S.B. Mogilnitsky

Tomsk polytechnic university
E-mail: msb@tpu.ru

Influence of space limited of the dispersion medium on radiation distribution and the quality characteristic of image obtained through dispersing medium of the finite sizes is investigated. The way of calculation of boundary function and contrast function of a light strip is determined. It is shown, that space limited of the dispersion medium and illumination conditions render significant influence on image quality characteristics.

Questions of calculation of radiation distribution are actual for problems of transfer of an image in the dispersion medium [1, 2]. However, the basic values describing the quality of image, obtained through dispersion medium, are determined only for medium, unlimited in a transverse direction (in relation to a direction of radiation distribution).

The radiation distribution on the output from space limited dispersion medium having the form of a parallelepiped is considered in the given work under various conditions of illumination of one of volume sides. Calculations are carried out using the method of repeated reflections [3] on which basis the way for definition of boundary function and contrast function of a light strip is obtained.

Let's consider volume of a dispersion medium in the form of a rectangular parallelepiped with the optical sizes τ_x, τ_y, τ_z, where x, y, z are axes of the Cartesian coordinate system, coinciding with parallelepiped edges. At normal illumination of one of volume sides by a parallel flow of monochromatic radiation the energy components of radiating balance of the given volume are defined by the method given in [3]. Depending on a direction of falling of radiation three variants of radiating balance are realized. At illumination on axis x – components of radiating balance are the following: I_x^+ is intensity of radiation passed through volume, I_x^- is intensity of radiation reflected in volume, $2(I_x^y+I_x^z)$ is intensity of radiation which has left through lateral sides; on axis the same components of balance are equal I_y^+, I_y^-, $2(I_y^x+I_y^z)$; on an axis components of radiating balance are – I_z^+, I_z^-, $2(I_z^x+I_z^y)$ accordingly.

At illumination of volume with a dispersion medium with radiation of intensity I_0=1 the normalizing condit-