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OPTIMIZATION OF TRANSMISSION LINE CAPACITIES OF CORPORATE NETWORKS USING THE METHOD OF INDIRECT OPTIMIZATION

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Analytic dependences allowing defining average minimum time of package delay and realizing substantiated choice of transmission line capacities at the existing matrix of network node gravitations have been obtained.

Introduction

The existing corporate networks being the systems of collective use are maintained first of all as commercial structures [1]. The problem of efficient use of resources could not be solved only due to attraction of great number of users creating sufficient traffic. Optimization procedures allowing estimating network capacities in supporting the required time-probabilistic characteristics of data exchange should be developed [2, 3]. It is not difficult to determine optimal values of flows in network links it matrix of network capacities is known and Lagrange multipliers are determined. But the contrary is the case if it is necessary to solve the inverse problem (problem of synthesis) when it is impossible to express in explicit form the values of network link capacities through the flows of initial matrix of node gravity. The method of indirect optimization allowing solving the problem of optimization of two variables one of which may act as initial data is used in the article. This method is the most efficient in the case when the investigated function does not contain exptremum but it is convex. Then its conditional minimization may be carried out. Analytic dependences allowing determining average minimal delay time of a package and performing valid selection of link capacities at the existing gravity matrix of network node flows wee obtained. Using the method of conditional optimization in this task solves the problem of selecting a type of function connecting independent variables by a condition in the form of objectively existing interaction between optimized variables. It allows interpreting the results of solving the problem in the form of laws for network and estimating, hereby, its potential capacities.

Problem statement

Let us singled out the main network index – average delay time of the package $\overline{T}_{зад}$ and determine average minimal delay time $\overline{T}^{\min}_{зад}$ for the model of network M/M/1 of a package optimizing flows F on a network layer of master interaction model of open systems. Let us use network capacities V of links and constraint equations of functions of type $\psi(\overline{F})=0$ as initial data, that is:

$$\overline{T}^{\min}_{зад} = \min_F \overline{T}_{зад}(V,F);$$

$$\psi(\overline{F}) = 0,$$

where $\overline{F}$ are the values of flows in backbone lines connected by a certain dependence.

For communication networks with packet switching of constant length L (for example, cells of technology of packet asynchronous transmission) matrix of gravity $\|F_{ij}\|$ is used as initial data, where $F_{ij}=L\cdot\lambda_{ij}$ is the value of flow planned for transmission over link between nodes i and j; λ_{ij} is the intensity of demand (packets) flow at input/output of link between nodes i and j.

Optimal values of network link capacities $\|V_{ij}^{opt}\|$ should be determined. They are considered to be optimal at equality of initial adjacency matrices of planned load $\|F_{ij}\|$ between nodes of gravity matrix (S and T) and calculated adjacency matrices of optimal flows $\|F_{ij}^{opt}\|$ between these nodes ($\|F_{ij}\|=\|F_{ij}^{opt}\|$).

Problem solution

It is necessary to implement functional dependency $V_{ij}=f(F_{ij}^{opt})$. This task is a task of synthesis. Average minimal packet delay time is used for its solving as optimization functional [5–7]:

$$\overline{T}_{зад} = \frac{1}{\gamma}\sum_{i,j}\frac{F_{ij}}{V_{ij}-F_{ij}} \to \min, \qquad (1)$$

where F_{ij} is the value of flow in the link of network unit between nodes i and j; V_{ij} is the value of network link capacity between nodes i and j; γ is the network traffic at initial flow F_j^0 transfer (according to gravity matrix) from node S to node T.

Let us use the objectively existing law for each network switching node – the law of flow conservation as a constraint equation at conditional optimization (1):

$$\sum_{j=1}^{p-1} F_{ij} = aF_j^0, \quad i \neq j, \qquad (2)$$

where $F_{ij}=-F_{ji}$ is the flow in the branch i,j; F_j^0 is the initial flow belonging to the node j; p is the coherence of node j equal to a number of branches adjacent to it

$$a = \begin{cases} 1 & \text{by } j = S, \\ 0 & \text{by } j \neq S, \\ -1 & \text{by } j = T, \end{cases}$$

where S is the source node, T is the node receiver.

In [4] theorems 1 and 2 determining the main content of the method of indirect optimization are proved. Let us give their content without proving to illustrate the matter of method of indirect optimization.

Let us consider arbitrary function $z=F(\overline{x},\overline{y})$, where $\overline{x}=(x_1,...,x_i,...,x_n)$ and $\overline{y}=(y_1,...,y_i,...,y_n)$ are independent variables specified by projections of vectors $\overline{x}$ and $\overline{y}$ and function z is additive and has extremum. Each member of sum $z=k\sum_{i=1}^{n} f(x_i,y_i)$ is the function of one pair of projection of vectors x_i and y_i that is separation property is fulfilled.

Theorem 1. The result of optimization of differentiated separable function $F(\overline{x},\overline{y})$ containing extremum by both variables in the range of definition domain with accuracy to a certain constant does not depend on selection of independent variable $\overline{x}$ or $\overline{y}$ by which optimization if carried out.

Theorem 2. If separable function containing extremum by both variables in the range of definition domain $F(\overline{x},\overline{y})$ is smooth then the result of optimization does not depend on selection of independent variable by which optimization is carried out.

Let us note that function $F(\overline{x},\overline{y})$ is smooth in the case if all members of sum $f(x_i,y_i)$ are smooth functions. On the basis of theorems 1 and 2 the problems of optimization of two variables $F(\overline{x},\overline{y})$ one of which, for example $\overline{y}$ may act as initial data y_0, may be solved. Carrying out optimization by parameters y_0 function $F(\overline{x},\overline{y}_0)$ may be converted so that coordinates of optimum point coincide with coordinates of the end of vector $\overline{y}_0=(y_{01},...,y_{02},...,y_{0n})$ specified by its projections. Thus, varying purposefully independent variables the coordinates of optimal points of function $F(\overline{x},\overline{y})$ may be coincided with selected coordinates of vector $\overline{y}$ projections of which are known and may be specified as initial data. If function $F(\overline{x},\overline{y})$ has extremum the advantages of the investigated method are not considered in explicit form. However, if function $F(\overline{x},\overline{y})$ does not contain extremum but it is convex then in this case it may be conventionally optimized. For this purpose it is necessary to specify limiting conditions in the form of function connecting independent variables for that variable by which optimization is carried out. In this case the problem of selecting a type of this function in the form of objectively existing interaction between optimized variables or reflecting the character of the current task occurs. For example, if $\overline{x}$ is the optimized variable and variable $\overline{y}$ is specified by its projections which are connected, besides, with functional dependence while there is no such dependence for variable x_i or it is not known then according to the given above theorems 1 and 2 optimization may be carried out by variables $\overline{y}$ with the same result.

Taking into account the matter of the method of indirect optimization stated above the assigned task is solved. Let us use for its solution the method of Lagrange multipliers [8, 9] according to which optimization functional according to the expressions (1) and (2) takes the form:

$$\Phi=\frac{1}{\gamma}\sum_{i=1}^{v}\sum_{j=1}^{v}\frac{F_{ij}}{V_{ij}-F_{ij}}+\sum_{j=1}^{v}P_j\sum_{j=1}^{v-1}F_{ij}+aF_j^0\ , \qquad (3)$$

where P_j are the Lagrange multipliers.

As the law of flow conservation is valid for all network nodes then always for one of nodes the sum of flows is linear combination of flows of all the rest nodes. Therefore, in expression (3) $j=\overline{1,(v-1)}$ occurs. This circumstance may be not taken into account assuming that $j=\overline{1,v}$ for obtaining symmetrical solution. However, this assumption should be taken into account after solving the optimization problem by specifying $P_v=0$. To determine optimal values of flows F_{ij}^{opt} supporting minimum of functional (1) let us calculate private derivatives

$$\partial\Phi\big/\partial F_{ij}=0. \qquad (4)$$

Differentiation on all set of values $i,j=\overline{1,v}$ assumes that original topological structure of the network is fully connected. Calculation of derivatives (4) results in system of equation of the form:

$$\frac{V_{ij}}{(V_{ij}-F_{ij})^2}=\gamma(P_j-P_i),\quad i,j=\overline{1,v},$$

and $$i\neq j,\quad P_v=0. \qquad (5)$$

A number of these equations is determined by a number of graph branches and for fully connected network equals to $k=v(v-1)/2$ whence the values of flows for each branch is determined by the expression

$$F_{ij}^{opt}=V_{ij}-\sqrt{\frac{V_{ij}}{\gamma(P_j-P_i)}}. \qquad (6)$$

Let us find the Lagrange multipliers P_i and P_j substituting (6) into (2):

$$\sum_{i=1}^{v}\left(V_{ij}-\sqrt{\frac{V_{ij}}{\gamma(P_j-P_i)}}\right)=aF_j\ ,\ \ j=1,\ \overline{v-1} \qquad (7)$$

and joint solution of the system (7). However, owing to irrationality of equation system (7) an attempt of its solution results in the system of nonlinear equations of high order equal to double value of network node amount. Analytical solution of such system by known mathematical methods is not possible. To determine Lagrange multipliers let us carry out their linearization by expansion of left part of the expression (5) in Taylor series in neighborhood of point F_{ij}^0 [10]. As a result of expansion

$$\frac{V_{ij}}{(V_{ij}-F_{ij})^2} \qquad (8)$$

we obtain the system of linear algebraic equations relative to F_{ij}^0:

$$\frac{V_{ij}}{(V_{ij}-F_{ij}^0)^2}+\frac{2V_{ij}}{(V_{ij}-F_{ij}^0)^3}(F_{ij}^{opt}-F_{ij}^0)=\gamma(P_j-P_i), \qquad (9)$$

where

$$F_{ij}^0=\alpha V_{ij},$$

α is the coefficient determining the expansion point F_{ij}^0 on axis V_{ij}. It may be changed in the range of $0\leq\alpha\leq1$.

Function (9) after conversions is reduced to the form:

$$F_{ij}^{opt}=b_{ij}-(P_i-P_j)a_{ij}\,, \qquad (10)$$

where $$a_{ij}=\gamma\,\frac{(V_{ij}-F_{ij}^0)^3}{2V_{ij}}, \qquad (11)$$

$$b_{ij}=\frac{3F_{ij}^0-V_{ij}}{2}. \qquad (12)$$

After substitution of (10–12) in (2) we obtain the equation system which is presented in matrix form:

$$A \cdot X = C, \quad (13)$$

where A, X and C are the matrices of coefficients a_{ij}, Lagrange multipliers P_{ij} and free members b_{ij} respectively.

If matrix is nonsingular then equation (13) has the solution:

$$X = A^{-1} \cdot C, \quad (14)$$

where A^{-1} is the inverse matrix of matrix A. Thus, Lagrange multipliers P_{ij} may be determined by the method of Kramer or multiplying the inverse matrix A^{-1} by matrix C and optimal values of flows F_{ij}^{opt} – by the expression (10). The value of traffic of network γ is determined by a quantity of packets entering the network per unit time (or by a quantity of packets abandoning the network per unit time). That is traffic of network γ is considered to be constant value and does not influence the value F_{ij}^{opt}. Therefore, calculating F_{ij}^{opt} by the expression (10) the value of traffic γ may be taken as a unit or determined from the expression

$$\gamma = \alpha_{np}^{*} \cdot \sum_{ij=1}^{k} V_{ij}, \quad (15)$$

Where α_{np}^{*} is the acceptable coefficient α determining selection of expansion point F_{ij}^{0} at optimal expansion of initial flow F_j^0 between a pair of nodes S and T of gravity matrix; $\sum_{ij=1}^{k} V_{ij}$ is the total capacity of all k-links at optimal expansion of initial flow F_j^0 between pair of nodes S and T.

Traffic determined by the expression (15) differs from optimal traffic obtained as a result of expansion of initial flow F_j^0 between pair of nodes S and T. It gives rough idea of the value of optimal traffic in the network. To calculate $\overline{T}_{зад}^{\min}$ by the expression (1) it is necessary to calculate real value of the traffic:

$$\gamma = \sum_{ij=1}^{k} F_{ij}^{opt},$$

that is after determining matrix of optimal flows between nodes S and T.

Solution of the problem of flow optimal expansion turns out to be rather complicated even in linear approximation using expressions (10)–(14). It is explained by significant dependence of tolerance range of optimal flows in each branch of network graph ($0 \le F_{ij}^{opt} \le V_{ij}$) on position of initial point F_{ij}^{0} in which (7) is expanded in Taylor series. Accuracy of investigated function expansion was estimated using coefficient a which is determined by the ratio F_{ij}^{0}/V_{ij}. Such presentation simplifies interpretation of the results as it corresponds by implication to the value of relative load on network link (degree of channel load). Acceptable values of coefficient α_{np}^{*} uniform for the network should be selected in the range α_{np} the value of which is determined by two conditions. The first condition is the condition of convergence of Taylor series [3]:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + R_n(x), \quad (16)$$

the remainder term of which equals to

$$R_n(x) = f(x) - T_n(x) = 0, \quad \text{at} \quad n \to \infty, \quad (17)$$

where $T_n(x)$ is the right part of expression (16), n is the degree of the derivative.

For functional (8) expanded in series expressions (16) and (17) take a form:

$$\frac{V_{ij}}{(V_{ij} - F_{ij})^2} = \frac{V_{ij}}{(V_{ij} - F_{ij}^0)^2} + \ldots + \frac{\left[\dfrac{V_{ij}}{(V_{ij} - F_{ij}^0)^2}\right]^{(n)}}{n!}(F_{ij} - F_{ij}^0)^n + R_n(x),$$

$$\frac{V_{ij}}{(V_{ij} - F_{ij})^2} - \frac{V_{ij}}{(V_{ij} - F_{ij}^0)^2} - \ldots - \frac{\left[\dfrac{V_{ij}}{(V_{ij} - F_{ij}^0)^2}\right]^{(n)}}{n!}(F_{ij} - F_{ij}^0)^n = 0.$$

The condition of convergence of series (17) is fulfilled if $(V_{ij} - F_{ij}^0) \ge 1$, that is $V_{ij} - \alpha V_{ij} \le 1$, or $\alpha_{np} \le 1 - (1/V_{ij})$ for a separate connection branch. For the whole network we have

$$\alpha_{np}^{*} \le 1 - \frac{1}{V_{ij}^{\min}}.$$

The second condition is determined by accessible region of possible changes of optimal flows at specified matrix of link capacities $\|V_{ij}\|$. In this case maximum safe load on the applied types of communication lines (cable communication lines, optical fiber communication line, broadcasting) is taken into account that is

$$0 \le F_{ij}^{opt} \le V_{ij}. \quad (18)$$

To determine the interval of acceptable values a (a_{np}^{*}) by the second condition let us express the value of Lagrange multipliers by Kramer rule from the expressions (13) and substitute them in the expressions for determining optimal flows (10). Dependences $\alpha_{ij} = f(F_{ij}^{opt})$ are expressed of the obtained values.

Let us examine the peculiarities of selecting expansion points F_{ij}^{0} of the expression (8) in Taylor series when network unit capacities differs considerably from each other using diagrams typical for two nodes of the network. Diagram of expression (8) for each network unit has the form (Fig. 1).

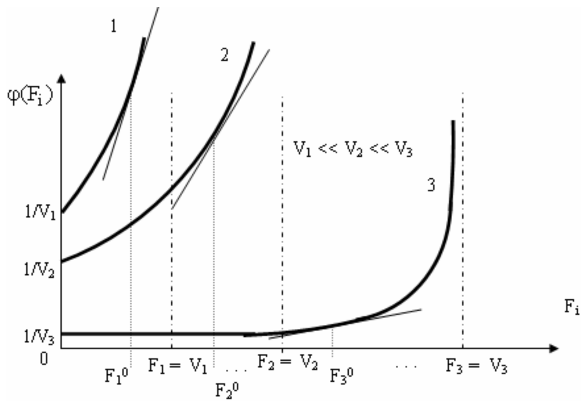

Fig. 1. Selection of expansion point F_{ij}^{0}

In this case selecting a uniform expansion point F_{ij}^0 for the network by selection of α^*_{np} (Fig. 1 $\alpha^*_{np}\approx 0{,}7$), optimal expansion of flows may result in nonfulfilment of condition (18) in some of its units because of selecting the expansion point on different sections of curves 1–3 including underlinear ones (curve 2). Such situations are possible if network unit capacities differ significantly from each other in size (not less than by an order of magnitude) that is not typical for main traffic artery of networks. If condition (18) is not fulfilled after optimal expansion of flows for separate units of network then it is necessary: a) try to find common area of flow acceptable values in the network matching α^*_{np}; b) relocate the planned resource V_{ij} and repeat calculations on optimal flow distribution.

Example. Let us examine determination of optimal flows by the example of graph of the network consisting of two nodes (Fig. 2).

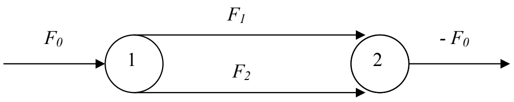

Fig. 2. *Graph of the network of two switching nodes*

Given: F_0=1400 bit/s is the value of input initial flow of source node; F_1 and F_2 are the values of flows in the first and the second adjacent branches of network graph; F_0 is the value of outgoing initial flow of node receiver T; V_1=1000 bit/s is the capacity of the first communication line; V_2=500 bit/s is the capacity of the second communication line; γ=1400 bit/s is the network total traffic.

Determine: the value of optimal flows F_1^{opt} and F_2^{opt} of network communication line.

Functional of optimization for the given network model has the form

$$\bar{T}_{зад} = \frac{1}{\gamma}\left(\frac{F_1}{V_1 - F_1} + \frac{F_2}{V_2 - F_2}\right), \qquad (19)$$

and the constraint equations for two nodes at conditional optimization of functional (19),

$$F_1 + F_2 = F_0.$$

Let us solve the stated problem in three ways: the first one – using geometrical method; the second one – using the developed software; the third way – using the method of undetermined Lagrange multipliers. The obtained optimal values of network parameters and comparison characteristics of calculating errors are given in Tables 1 and 2 respectively.

Table 1. *Values of network optimal parameters*

Ways of solution	Network optimal parameters		
	F_1^{opt}, bit/s	F_2^{opt}, bit/s	$\bar{T}_{зад}^{min}$, s
Geometrical method	942,10	458,90	0,01939
Using the developed software	940,11	459,89	0,02060
Method of undetermined Lagrange multipliers	940,00	457,58	0,01889

Table 2. *Comparison characteristics of calculating errors*

Comparison characteristics of solution ways	Errors of network parameters, %		
	δF_1^{opt}	δF_2^{opt}	$\delta \bar{T}_{зад}^{min}$
The second relative to the first	0,21	0,22	6,24
The third relative to the first	0,22	0,29	2,58
The second relative to the third	0,01	0,50	9,00

In all three cases the results differ insignificantly by their values.

Conclusion

1. For the model of network M/M/1 using the method of indirect optimization the analytic forms allowing determining optimal values of flows in communication lines at specified gravity matrix between network nodes were obtained. The objectively existing law of saving flows in network switching nodes was used as the constraint equation.
2. The obtained values of network optimized indices meet the requirements for accuracy at engineering calculations.
3. The suggested mathematical apparatus of calculating parameters F_{ij}, V_{ij}, $\bar{T}_{зад}^{min}$ may be used at calculation of networks of any topological structure and random coherence.

Thus, the problem of determining the main optimization indices of telecommunication network was solved and analytic expressions allowing implementing their valid choice were obtained.

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