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SIMULATION OF NON-STATIONARY PROCESSES IN CENTRIFUGAL CASCADES

V.A. Vlasov, V.G. Butov, D.N. Goldobin, A.A. Orlov, S.N. Timchenko

Tomsk Polytechnic University
E-mail: richi@inet.tsu.ru

The model of nonstationary hydraulic and dividing processes in rectangular symmetrical counterstream centrifugal cascades is considered. The calculation technique of centrifugal cascade parameters of transition processes has been developed. The results of numerical computation are presented.

Introduction

Enriching uranium isotopes in dividing centrifugal cascade is a complex and expensive process, therefore, the questions of optimizing process flowsheet and conditions of device operation are of great significance for increasing production efficiency. One of the ways of solving mentioned problems is design of appropriate mathematical models of the processes at isotope separation in centrifugal cascades.

Here, the important role is played by modelling non-stationary hydraulic and dividing processes. Non-stationary processes in dividing centrifugal cascades are explained by various reasons. For example, it could be change in frequency of supply current, switching to the new technological mode, switching off a part of equipment at both normal operation and irregularities in circuit operation connected with closing external cascade flows, emergency switching off the part of equipment, power cut and etc.

Simulation of non-stationary processes is also crucial for realisation of corresponding situations in real time scale in training simulator modelling operators' working places of separation centrifugal production.

1.1. Model of counterstream symmetrical cascade

Let us consider rectangular-step centrifugal cascade for separation of two-component isotope mixture. Let supply flow F be fed to some intermediate point of cascade, but from the ends of cascade the flows P (selection) and W (dump) be selected. Such a cascade is schematically presented in Fig. 1.

The magnitudes F, C_f, W, C_w, P, C_p are the external cascade parameters, but $5n$ of the magnitudes N_i, G_i^+, G_i^- ,

$C_i^+, G_i^-, C_i^- (i=\overline{1, n})$ shown in the Scheme are the internal ones. Those parameters are obtained from the external ones in terms of their definitions and known characteristics of steps. The magnitudes P, F and W are the time functions and connected with the balance equation:

$$F = P + W,$$

$$F \cdot C_f = P \cdot C_p + W \cdot C_w.$$

The model of cascade necessary for calculation of non-stationary isotope separation processes includes the equation set of non-stationary hydraulics and non-stationary transport in cascade. Solution of the obtained compatible system allows for determination of time for cascade changing over the stationary conditions, flows, selection and dump concentrations of steps and the cascade in general.

The equation set describing non-stationary hydraulic processes is built in the following way [1]:

Gas content of cascade consisting of n steps is broken into n volumes of gas.

Gas content in each step is defined by gas mass rotating in centrifuge rotors

$$M_0 = \rho V \frac{1 - e^{-A}}{A}, \quad A = \frac{M \Omega^2 r_a^2}{2RT_0}, \quad (1)$$

where M_0 is the gas content of rotor gas centrifuge (GC) with the volume $V = \pi r_a H$, the radius r_a and the height H are defined for ideal gas with molar mass M , equilibrium density ρ , pressure p , at temperature T_0 , rotating around the axis z as a single whole with the angular velocity $\Omega = V_w / r_a$; V_w is the rotor peripheral velocity. Gas content $M_i(t)$ of i -th section is calculated as the product of centrifuge quantity included in it by M_0

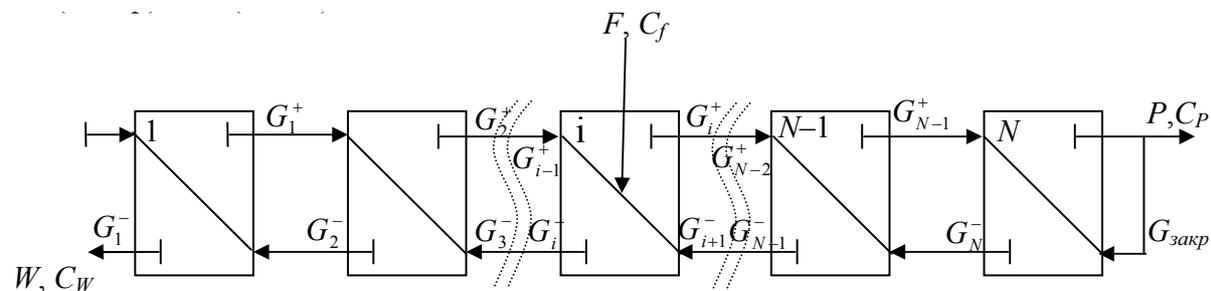


Fig. 1. Counterstream symmetrical cascade: i is the number of step; N is the quantity of steps; W, C_w is the flow and concentration of cascade dump; P, C_p is the flow and concentration of cascade selection; F, C_f is the flow and concentration of external supply fed to i -th step; G_i^+, C_i^+ is the flow and concentration of i -th step selection; G_i^-, C_i^- is the flow and concentration of i -th step dump; G_i, C_i is the flow and concentration of i -th step supply

$$M_i = N_i \cdot M_0, \quad i = 1..n.$$

Rewrite this relationship taking into account the eqp. (2), replacing $\rho = M_p / RT_0$:

$$M_i = Ep, \quad E = \frac{N_i VM}{RT_0} \cdot \frac{1 - e^{-A}}{A}. \quad (2)$$

In general case the flows entering the i -th volume are the selection flow of previous steps G_{i-1}^+ ; the dump flow of the next steps G_{i+1}^- ; external supply flow of the given step F , being the specified time function t . Besides, in case of closing the neighbouring steps, in i -th volume the selection G_{i-2}^+ and dump flows come G_{i+2}^- from the nearest operating step.

From the volume of step the selection G_i^+ and dump flows come G_i^- out.

1.2. Basic equations of non-stationary hydraulics

These are the equations of substance balance in the set off volumes. The change in gas content in the set off volume is determined by the difference of entering and coming out flows.

For i -th step one can write down the following equations of nonstationary hydraulics:

$$\begin{aligned} \frac{dM_i}{dt} = & \delta_F F + \delta_{i-1}^+ G_{i-1}^+ + \delta_{i-2}^+ G_{i-2}^+ + \\ & + \delta_{i+1}^- G_{i+1}^- + \delta_{i+2}^- G_{i+2}^- - G_i^+ - G_i^-, \end{aligned} \quad (3)$$

where: $\delta_F, \delta_{i-1}^+, \delta_{i-2}^+, \delta_{i+1}^-, \delta_{i+2}^-$ are the indicators (0 or 1) of flow in section, t is the time. Therefore, in general case gas content of the gas centrifuge depends on the frequency of its rotation:

$$\frac{dM}{dt} = E \frac{dp}{dt} + p \frac{dE}{dt}.$$

The initial condition for the equation (3) is obtained for $\Omega(0) = \Omega_0, A(0) = A_0$ and reduces the eqp. (2) to the view

$$M_i(0) = \frac{N_i M V}{RT_0} \frac{1 - e^{-A_0}}{A_0} p_i(0), \quad (4)$$

Where $p_i(0)$ is the initial pressure in i -th step.

1.3. Equations of flows

The supply flow of separate GC comes through critical cell. The expression for GC supply flow of the i -th step can be written down in the form:

$$G_i = N_i \cdot K_i \cdot P_i,$$

where: N_i is the number of GC in the i -th step; K_i is the discharge coefficient; P_i is the supply pressure at GC output.

Following from laminar character of gas flow in the sorter of GC sinking fraction for the GC dump flow of the i -th section G_i^- we have [3]:

$$\frac{G_i^- \zeta_i^-}{N_i} = P_w^2 - (P_i^-)^2,$$

where ζ_i^- is the coefficient of hydraulic resistance.

The GC selection flow of the i -th step G_i^+ we define by means of formalized universal hydraulic characteristic of GC in the form:

$$G_i^+ = N_i \left[\begin{aligned} & b1_i + b2_i \frac{G_i}{N_i} + b3_i \left(\frac{G_i}{N_i} \right)^2 + \\ & + b4_i P_i^- + b5_i (P_i^-)^2 + b6_i \frac{G_i}{N_i} P_i^- \end{aligned} \right],$$

where $b1_i - b6_i$ are the coefficients of hydraulic characteristics in general case depending on the GC type, rotor temperature, concentrations of light impurities, rotation frequency Ω , etc.

1.4. Basic equations of non-stationary transfer

The transfer of light isotope from dump to selection in enriching and regeneration parts of cascade are respectively written down in the following form [2, 4, 5]

$$\tau_i = \tau c + G \varepsilon c (1 - c) - \frac{\partial c}{\partial n} \frac{G}{2}, \quad (5)$$

$$\tau'_i = -\tau' c + G \varepsilon c (1 - c) - \frac{\partial c}{\partial n} \frac{G}{2}, \quad (6)$$

where: ε is the coefficient of complete enrichment.

Here, because of the smooth change of enrichment in cascade the number of step can be considered a variable magnitude, then the concentration $c = c(n, t)$, $G = G(n, t)$, $\tau = \tau(n, t)$ and $\tau' = \tau'(n, t)$.

After substitution of τ_i and τ'_i from (5) and (6) into the equation of transfer process in centrifuge cascade

$$\frac{\partial(Mc)}{\partial t} = -\frac{\partial \tau_i}{\partial n}, \quad \frac{\partial(Mc)}{\partial t} = -\frac{\partial \tau'_i}{\partial n}$$

for enriching part we obtain

$$\frac{\partial(Mc)}{\partial t} = \frac{G}{2} \frac{\partial^2 c}{\partial n^2} - \frac{\partial}{\partial n} [G \varepsilon c (1 - c) + \tau c], \quad (7)$$

for regeneration part

$$\frac{\partial(Mc)}{\partial t} = \frac{G}{2} \frac{\partial^2 c}{\partial n^2} - \frac{\partial}{\partial n} [G \varepsilon c (1 - c) - \tau' c]. \quad (8)$$

To solve the equations (7) and (8) one should set the initial condition and the boundary conditions at selection and dump ends of the cascade. It is assumed that concentration distribution at $t=0$ corresponds to some stationary state of the cascade

$$c(n, 0) = c_0(n). \quad (9)$$

The boundary condition at selection end of cascade ($n=N$) at unselection mode of operation is determined in such a way

$$\tau(N, t) = 0; \quad \tau_i = \left[\tau c + G \varepsilon c (1 - c) - \frac{\partial c}{\partial n} \frac{G}{2} \right]_{n=N} = 0.$$

Hence, we obtain

$$\left. \frac{dc}{dn} \right|_{n=N} = 2 \varepsilon c (1 - c). \quad (10)$$

At dump end of the cascade ($n=1$) at dump-free mode of operation the boundary condition is determined in such a way

$$\tau(1, t) = 0; \tau_l = \left[-\tau'c + G\varepsilon c(1-c) - \frac{\partial c}{\partial n} \frac{G}{2} \right]_{n=1} = 0.$$

Hence, we obtain

$$\frac{dc}{dn} \Big|_{n=1} = 2\varepsilon c(1-c). \quad (11)$$

In the case of operation in unselection-dump-free mode with reservoirs having the gas content M_p^I and M_p^II at heavy and light end of the cascade respectively, the boundary conditions have the view:

$$\begin{aligned} \text{at } n=1; \quad \frac{dc}{dn} - 2\varepsilon c &= 2 \frac{M_p^II}{G} \frac{\partial c}{\partial t}, \\ \text{at } n=N; \quad \frac{dc}{dn} - 2\varepsilon c &= -2 \frac{M_p^I}{G} \frac{\partial c}{\partial t}. \end{aligned} \quad (12)$$

At the joints of steps of different capacity the following continuity conditions are to be met

$$\begin{aligned} \tau(+n_k, t) &= \tau(-n_k, t), \quad \tau_l(+n_k, t) = \tau_l(-n_k, t), \\ c(+n_k, t) &= c(-n_k, t). \end{aligned} \quad (13)$$

Signs «+» and «-» denote the meanings of parameters to the right and left of the k -th joint of steps. After substitution of (5) and (6) into (12) we obtain

$$\begin{aligned} \frac{G(-n_k, t)}{2} \cdot \frac{\partial c^-}{\partial n} - \frac{G(+n_k, t)}{2} \cdot \frac{\partial c^+}{\partial n} + \\ + \varepsilon[G(+n_k, t) - G(-n_k, t)] \cdot c^*(1-c^*) = 0. \end{aligned} \quad (14)$$

Here signs «+» and «-» of derivatives have the same meaning, but $c^* = c(+n_k, t) = c(-n_k, t)$.

If at «joint» of steps ($n=n_k$) the supply flow is fed, then instead of the equations (17) the following equations are solved

$$\begin{aligned} \tau(+n_k, t) - \tau(-n_k, t) &= F, \\ \tau_l(+n_k, t) - \tau_l(-n_k, t) &= c_f F, \\ c_f &= c(+n_k, t) = c(-n_k, t). \end{aligned}$$

After substitution of (10) and (11) into (19) we obtain

$$\begin{aligned} \frac{G(-n_k, t)}{2} \cdot \frac{\partial c^-}{\partial n} - \frac{G(+n_k, t)}{2} \cdot \frac{\partial c^+}{\partial n} + \\ + \varepsilon[G(+n_k, t) - G(-n_k, t)] \cdot c_f(1-c_f) = c_f F. \end{aligned} \quad (15)$$

2.1. Numerical calculation of non-stationary processes in separation cascade

Numerical calculation of equation system (3) for determination of gas content and (7), (8) of non-stationary transfer is performed in the following way:

The initial gas content in the steps according to (4) and initial concentration along the cascade in terms of the condition (9) is calculated. Then the minimal step in time $\Delta\tau$ is calculated for the equations (3) and (7),

(8). For the equations (7), (8) the step Δn of the derivative n is also introduced.

For numerical solution of the equations (3) the Euler's modified method of the second order of accuracy [6] in time is used. When solving the equations (7) and (8) the two-level scheme of calculation providing the second order of accuracy in the variables t and n [6] is used.

In the same manner as in the works [7] for calculation with the general second order of accuracy at approximation of conditions at the selection (10) and dump (11) ends of cascade, at the joint of steps of different capacity without (13) and with supply (14) the one-sided difference schemes are used.

$$\begin{aligned} \frac{\partial c^-}{\partial n} &= (-2 \cdot c_{n_k-3}^i + 9 \cdot c_{n_k-2}^i - 18 \cdot c_{n_k-1}^i + 11 \cdot c_{n_k}^i) / 6\Delta n, \\ \frac{\partial c^+}{\partial n} &= (2 \cdot c_{n_k-3}^i - 9 \cdot c_{n_k-2}^i + 18 \cdot c_{n_k-1}^i - 11 \cdot c_{n_k}^i) / 6\Delta n, \end{aligned}$$

which are obtained from superposition of the Taylor series expansion of the function c at the points with the indexes $k-1, k-2, k-3, k+1, k+2, k+3$ respectively.

2.2. Results of calculations

The calculation of non-stationary separation process in symmetrical counter-stream cascade consisting of 6 separating steps has been performed. As an initial condition the initial concentration $C_f=0,00711$ was set. It was considered that all steps of the cascade are filled with the gas mixture at the initial moment of time at the specified magnitude M_n . The external supply flow $F=F(s, t)$ was set as the time function and that of the point supply flow delivery into the cascade. The results of calculation are presented in Fig. 2–4.

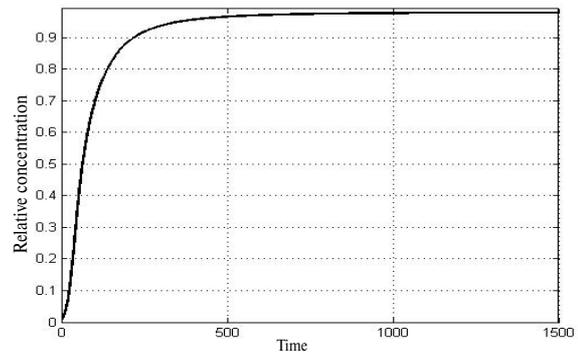


Fig. 2. Change of relative concentration in time at the selection end of separating cascade

In Fig. 2 the time distribution $\tau = \frac{G_N \varepsilon^2 t}{M_N}$ (G_N, M_N is the flow and gas content of the last step at stationary mode of cascade operation) of relative concentration at the selection end of cascade is presented $\frac{C_p}{C_D}$, where C_D is the required concentration of base product. In Fig. 3 the distribution of relative concentration $\frac{C_p}{C_D}$ over the

cascade changing into stationary operation mode is shown. Relative distribution of gas content $\frac{M_i}{M_N}$ over the cascade steps when changing into stationary operation mode is shown in Fig. 4.

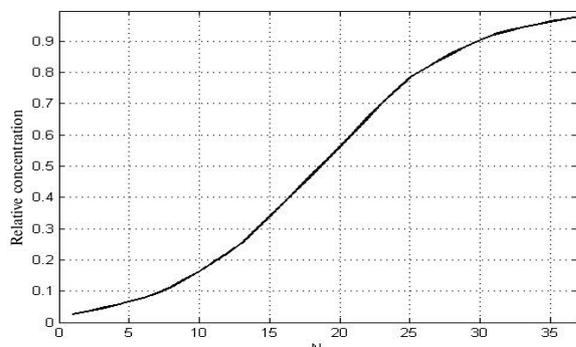


Fig. 3. Distribution of relative concentration over the cascade when switched to stationary mode

Conclusion

The mathematical model for application of non-stationary separation processes in symmetrical counterstream cascade has been designed. The system of equations describing given processes consists of the equations of non-stationary hydraulics which are the equations of substance balance in the isolated volumes. The process of transferring light isotope from dump to selection in the

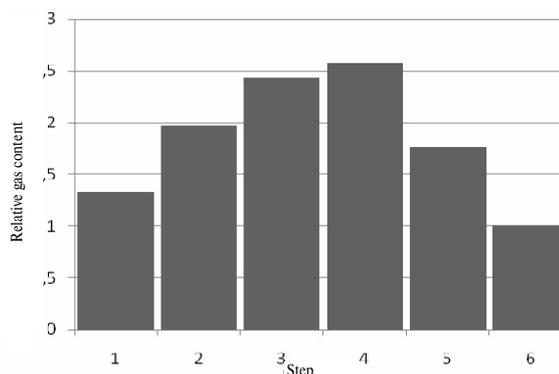


Fig. 4. Distribution of relative gas content over the cascade when switched to stationary mode

enriching and regeneration parts of cascade is described by two equations in partial derivatives of the second order.

At their simultaneous solution for the equation of non-stationary hydraulics the Euler's modified method of the second accuracy order in time is used, in the equations of separation the two-level scheme of calculation providing for the second order of accuracy of variables t and n is used.

The techniques has been developed and the results of simultaneous numerical simulation for non-stationary parameters of hydraulic and separation processes in counterstream symmetrical cascade have been obtained. They allow for simulation of different transfer situations in centrifugal separation cascades.

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