EPJ Web of Conferences **110**, 01045 (2016)
DOI: 10.1051/epjconf/201611001045

MATHEMATICAL MODELLING OF FORCE CONVECTION IN A TWO–PHASE THERMOSYPHON IN CONJUGATE FORMULATION

Atlant Nurpeiis[1] and Alexander Nee[1, a]

[1]*National Research Tomsk Polytechnic University, 634050 Tomsk, Russia*

Abstract. A nonlinear non-stationary problem of the conductive-convective heat transfer is addressed (under forced convection conditions) in the thermosyphon of rectangular cross-section. The thermal energy supply is carried out through the lower horizontal border. The mathematical model is formulated in dimensionless variables of "velocity vorticity vector – current function – temperature". The current and temperature distribution lines are obtained, illustrating the effect of the Reynolds number on the thermodynamic structures formation in the analyzed object.

1 Introduction

The heat pipes [1] are promising and efficient heat exchanging devices. A closed biphasic thermosyphon [1,2] is a vertically located tube made of metal with high heat-conducting capacities and adiabatic lateral surfaces. They are used in the production for cooling nuclear reactors, internal cooling of transformers and electric, and gas heaters [1]. This heat exchanger is distinguished from the heat pipe in that the thermosyphon's condensate is returned back to the evaporation area under the influence of gravity. But such heat exchangers are not widely used yet. This is due to the lack of a general theory of heat and mass transfer processes in such systems, removal of energy, which provides the possibility of their practical development.

There are solutions to the problems of heat transfer in the thermosyphons within the boundary layer model [3,4]. But the approach [3.4] to the steam flow process modelling within the steam channel is insufficiently substantiated (no comparison results calculated according to [3.4] process characteristics method with the experimental data).

The studies [5,6] show the mathematical modelling of heat transfer and steam flow processes with different coolants: distilled water [6], R134a [5] and R404a [5]. The thermosyphon was utilized made of steel with a wall thickness of 0.9 mm, diameter of the steam channel is 22 mm, and height of 500 mm, length of the evaporator is 200 mm, length of the condensation area – 200 mm. The numerical results [5,6] were obtained using ANSYS FLUENT software package. It was found that the proposed mathematical model successfully reproduces the complex phenomena of heat transfer and steam flow of the coolant in the thermosyphon. The study results obtained correlate well with the theoretical and experimental data [7].

[a] Corresponding author: nee_alexander@mail.ru

The results of an experimental study of methanol boiling in a closed biphasic thermosyphon are given in [8]. We have studied the thermosyphon with a height of 1000 mm and diameter of steam channel of 25mm. It is found that the intensity and duration of coolant boiling are affected by the filling ratio, geometrical parameters and mass flow of the methanol.

The purpose of this work is the numerical modelling of the conjugate forced convection in biphasic thermosyphon based on the finite difference method in dimensionless variables of "velocity vorticity vector – current function – temperature".

2 Setting of the problem

The problem was solved of conductive and convective heat transfer in the area shown in Figure 1. The thermosyphon was considered of rectangular cross section made of copper. The energy from the heat source, located near the bottom cover of thermosyphon is fed through the boundary $y=h_1$, $0<x<(l_1+l_2+L+l_3+l_4)$. As a result of intensive evaporation a pressure gradient appears, whereby the steam moves in the direction of y – axis and condenses on the upper lid of the heat exchanger. Gravity drains the condensate layer along the vertical walls to the high temperature area and spreads over the surface of the coolant.

Assumptions are made that allow simplifying the setting of the problem, taking into account the basic physical processes. It was assumed that the thermal physical properties of the housing material, steam, and condensate do not dependent on the temperature and the liquid film thickness on the vertical and horizontal walls do not change over time. Also, it was not taken into account the surface tension forces. The liquid film flow rate was assumed to be constant in time. The steam was taken as Newton's heat-conducting fluid, incompressible and meeting the Boussinesq approximation [9,10].

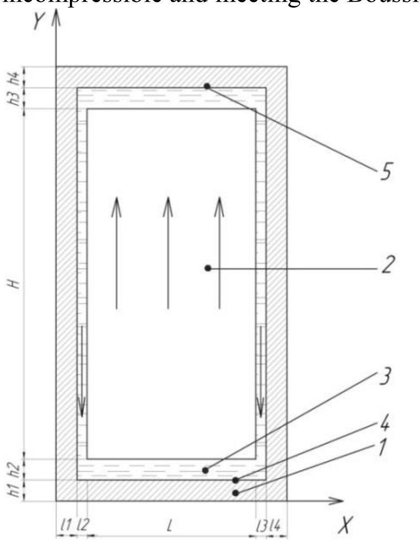


Figure 1. Chart of the closed thermosiphon: 1 - metal corps; 2 - steam channel; 3 - a liquid film; 4 - evaporation surface; 5 - condensation surface. The vertical arrows indicate the direction of steam and condensate movement.

The process of heat transfer under forced convection conditions in given thermosyphon (Figure 1) within the accepted physical model are described by the differential equations system in partial derivatives. The dimensionless vortex transport equation, Poisson and energy for the gas under forced convection conditions in the steam channel and heat conductivity equation for the liquid layer, thermosyphon walls are as follows:

$$\frac{\partial \Omega}{\partial \tau} + \frac{\partial \Psi}{\partial Y}\frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \Omega}{\partial Y} = \frac{1}{\mathrm{Re}} \cdot \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right), \tag{1}$$

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega, \tag{2}$$

$$\frac{\partial\Theta_2}{\partial\tau}+\frac{\partial\Psi}{\partial Y}\frac{\partial\Theta_2}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\Theta_2}{\partial Y}=\frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\cdot\left(\frac{\partial^2\Theta_2}{\partial X^2}+\frac{\partial^2\Theta_2}{\partial Y^2}\right), \tag{3}$$

$$\frac{\partial\Theta_3}{\partial Fo_3}=\frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\cdot\left(\frac{\partial^2\Theta_3}{\partial X^2}+\frac{\partial^2\Theta_3}{\partial Y^2}\right), \tag{4}$$

$$\frac{\partial\Theta_1}{\partial Fo_1}=\frac{\partial^2\Theta_1}{\partial X^2}+\frac{\partial^2\Theta_1}{\partial Y^2}, \tag{5}$$

Initial equations settings (1) – (5):

$$\Psi(X,Y,0)=\Omega(X,Y,0)=0, \tag{6}$$

$$\Theta_1(X,Y,0)=\Theta_2(X,Y,0)=\Theta_3(X,Y,0)=0, \tag{7}$$

Boundary equations settings (1) – (5):

$$X=0,\ 0<Y<\frac{h_1+h_2+H+h_3+h_4}{H}:\ \frac{\partial\Theta_1}{\partial X}=0, \tag{8}$$

$$X=\frac{l_1+l_2+L+l_3+l_4}{H},\ 0<Y<\frac{h_1+h_2+H+h_3+h_4}{H}:\ \frac{\partial\Theta_1}{\partial X}=0, \tag{9}$$

$$Y=0,\ 0<X<\frac{l_1+l_2+L+l_3+l_4}{H}:\ \frac{\partial\Theta_1}{\partial Y}=Ki, \tag{10}$$

$$Y=\frac{h_1+h_2+H+h_3+h_4}{H},\ 0<X<\frac{l_1+l_2+L+l_3+l_4}{H}:\ \frac{\partial\Theta_1}{\partial Y}=Bi\cdot\left(\Theta_1-\Theta_e\right),$$

$$Y=\frac{h_1}{H},\ \frac{l_1}{H}<X<\frac{l_3}{H}:\ \begin{cases}\Theta_1=\Theta_3,\\ \dfrac{\partial\Theta_1}{\partial Y}=\dfrac{\lambda_3}{\lambda_1}\cdot\dfrac{\partial\Theta_3}{\partial Y}-Q_e\cdot W_e,\end{cases} \tag{11}$$

$$Y=\frac{h_1+h_2}{H},\ \frac{l_2}{H}<X<\frac{L}{H}:\ \begin{cases}\Theta_3=\Theta_2,\\ \dfrac{\partial\Theta_3}{\partial Y}=\dfrac{\lambda_2}{\lambda_3}\cdot\dfrac{\partial\Theta_2}{\partial Y},\end{cases} \tag{12}$$

$$Y=\frac{h_1+h_2+H}{H},\ \frac{l_2}{H}<X<\frac{L}{H}:\ \begin{cases}\Theta_2=\Theta_3,\\ \dfrac{\partial\Theta_2}{\partial Y}=\dfrac{\lambda_3}{\lambda_2}\cdot\dfrac{\partial\Theta_3}{\partial Y},\end{cases} \tag{13}$$

$$Y=\frac{h_1+h_2+H+h_3}{H},\ \frac{l_1}{H}<X<\frac{l_3}{H}:\ \begin{cases}\Theta_3=\Theta_2,\\ \dfrac{\partial\Theta_3}{\partial Y}=\dfrac{\lambda_2}{\lambda_3}\cdot\dfrac{\partial\Theta_2}{\partial Y}-Q_e\cdot W_e,\end{cases} \tag{14}$$

$$X = \frac{l_1}{H},\ \frac{h_1}{H} < Y < \frac{h_3}{H}:\ \begin{cases} \Theta_1 = \Theta_3, \\ \dfrac{\partial \Theta_1}{\partial X} = \dfrac{\lambda_3}{\lambda_1} \cdot \dfrac{\partial \Theta_3}{\partial X}, \end{cases} \tag{15}$$

$$X = \frac{l_1 + l_2}{H},\ \frac{h_2}{H} < Y < \frac{H}{H}:\ \begin{cases} \Theta_3 = \Theta_2, \\ \dfrac{\partial \Theta_3}{\partial X} = \dfrac{\lambda_2}{\lambda_3} \cdot \dfrac{\partial \Theta_2}{\partial X}, \end{cases} \tag{16}$$

$$X = \frac{l_1 + l_2 + L}{H},\ \frac{h_2}{H} < Y < \frac{H}{H}:\ \begin{cases} \Theta_2 = \Theta_3, \\ \dfrac{\partial \Theta_2}{\partial X} = \dfrac{\lambda_3}{\lambda_1} \cdot \dfrac{\partial \Theta_3}{\partial X}, \end{cases} \tag{17}$$

$$X = \frac{l_1 + l_2 + L + l_3}{H},\ \frac{h_1}{H} < Y < \frac{h_3}{H}:\ \begin{cases} \Theta_3 = \Theta_1, \\ \dfrac{\partial \Theta_3}{\partial X} = \dfrac{\lambda_1}{\lambda_3} \cdot \dfrac{\partial \Theta_1}{\partial X}, \end{cases} \tag{18}$$

The accepted designations: $Fo = \frac{a \cdot t_0}{H^2}$ – number of Fourier; Re – Reynolds number; $\Pr = \frac{\nu}{a}$ – Prandtl number; $Bi = \frac{\alpha \cdot H}{\lambda}$ – Biot number; a – thermal diffusivity, m^2/s; α – heat transfer coefficient, $\frac{\mathrm{W}}{\mathrm{m^2 \cdot K}}$; g – acceleration of gravity ; L - transverse dimension of the steam channel, m; H - height of the steam channel, m; Q_e – dimensionless analogue of the latent heat by evaporation; Q_c – dimensionless analogue of latent heat during condensation; W_e – dimensionless analogue of the evaporation rate; W_c – dimensionless analogue of condensation rate; T_0 – initial thermal siphon temperature, K; T_h – temperature scale, K; t_0 – time scale, s; u, v – speed of x, y, respectively; U, V – relevant dimensionless speed, u, v; V_0 – the scale of speed in x, y – axis, respectively, m/s; x, y – dimensional coordinates, m; X, Y - dimensionless coordinates corresponding to x, y; ν – kinematic viscosity, m^2/s; λ – coefficient of thermal conductivity, $\mathrm{W/(m \cdot \mathit{K})}$; τ – dimensionless time; Θ – dimensionless temperature; ψ – current function, m^2/s; ψ_0 – current function scale, m^2/s; Ψ – dimensionless analogue, ψ; ω – swirl velocity, 1/s; ω_0 – scale vorticity, 1/s; Ω – dimensionless analogue ω.

The stated equations system with nonlinear boundary conditions is solved by finite difference method as in [11,12], using the locally one–dimensional scheme by A. A. Samarsky [13,14]. The iterative algorithm was used [15,16] developed for the solution of coupled heat transfer in multiply connected domains with intense local heat release.

The algorithm and solution method applied were verified by comparing the results obtained with the experiment [17]. Fig. 2 shows the dependence of the temperature drop in the steam channel of the thermal siphon from the supplied heat flux. The obtained numerical results were compared with experimental data. Dependency analysis showed that the temperature difference increases with the increasing heat flux and differs from the experiment by 1.5°K, which characterizes the reliability of the numerical results obtained.

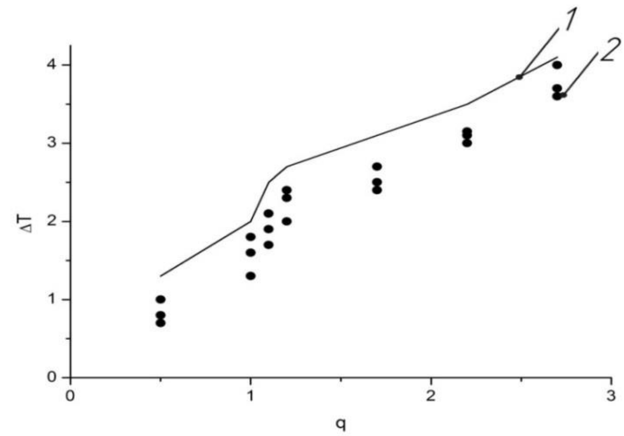
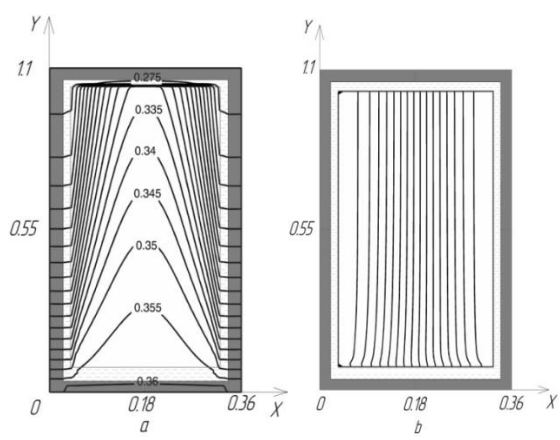


Figure 2. Temperature difference dependence (ΔT,°K) on the heat flux density (q, kW/m²): 1 – numerical results; 2– experiment results [18].

3 Results and discussion

Numerical analysis was performed for the thermosyphon of rectangular cross–section with the following dimensions and dimensionless parameters values: height – 100 mm, width of the steam channel – 21 mm, thickness of the liquid film – 5 mm, wall thickness – 2.5 mm, Re = 185; 277; 370 . The distilled water was used as a coolant. Figure 2 shows typical numerical modelling results illustrating the temperature fields and current functions of the solution area under different Reynolds numbers (heat flux q = 10kW/m² to the lower boundary of the given heat exchanger).



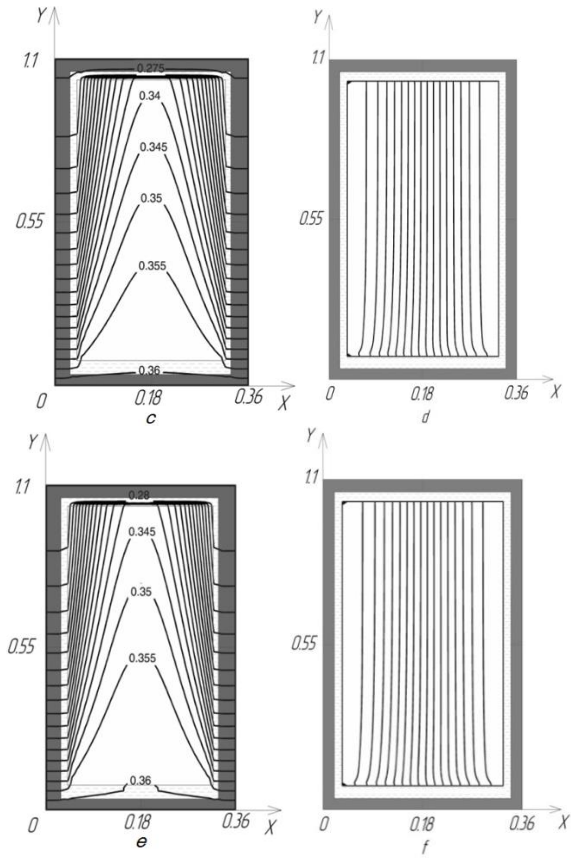


Figure 3. Isotherms (a,c,e) and current lines (b,d,f) at $q=10$ kW/m^2 (a,b)Re=185; (c,d)Re=277; (e,f)Re=370.

By analyzing Fig. 3 we can conclude that within the investigated range of Reynolds numbers in the thermosyphon steam channel the substantially inhomogeneous temperature fields are formed. It is clearly seen that the increase of the Reynolds number results in a corresponding transformation of temperatures isolines along y – axis.

4 Conclusions

A non–stationary conjugated problem of forced convection and conduction in a closed biphasic thermosyphon with the heat-conducting walls in the presence of condensate film with constant thickness was numerically solved. A boundary problem of mathematical physics is stated based on the laws of conservation of mass, momentum, and energy in the dimensionless variables of “velocity vorticity vector – current function – temperature”. The distributions of current function and temperature isolines are showed reflecting the impact of the heat flow and Reynolds number on the heat transfer modes.

Acknowledgments

The reported investigation was supported by grant of State Task “Nauka” №13.1339.2014/К (code of the Federal Target Scientific and Technical Program no. 2.1410.2014).

References

1. Bezrodnyi M. K., Pioro I. L., Kostyuk N. O.,*Transfer processes in the biphasic thermal siphon systems* (Kiev: Fact, 2005) [in russian]
2. Kuznetsov G.V., Al-Ani M.A., Sheremet M.A, J. of Engin Thermosyph., **20**, 2 (2011)
3. Kuznetsov G.V., Sitnikov A.E., High Temp., (2002)
4. Kuznetsov G.V., Sitnikov A.E., Thermophysics and Aeromechanics, **10**, 1 (2003) [in russian]
5. B. Fadhl, L.C. Wrobel, H. Jouhara., Appl. Therm. Eng., **78** (2015)
6. B. Fadhl, L.C. Wrobel, H. Jouhara., Appl. Therm. Eng., **60** (2013)
7. D. Reay, P. Kew, *Heat Pipes: Theory, Design and Applications* (Elsevier Science & Technology, UK, 2009)
8. I. Khazaee , R. Hosseini , S.H. Noie. Appl. Therm. Eng., **30** (2010)
9. Dzhaluriya Y., *Natural Convection: Heat and Mass Transfer* (M.: Mir, 1983) [in russian]
10. Sokovishyn Yu. A., Martynenko O. G., *Introduction to the theory of free-convection heat transfer* (Izd-vo Leningrad University, 1982) [in russian]
11. Kuznetsov G.V., Sheremet M.A., Thermophysics and Aeromechanics, **16**, 1 (2009)
12. Kuznetsov G.V., Sheremet M.A. Russian Microelectronics, **37**, 2 (2008)
13. Samarsky A. A. *The theory of difference schemes* (M.: Nauka, 1977)[in russian]
14. Kuznetsov G.V., Nagornova T.A., Ni A.E. J. of Engin.,Ph. and Thermoph., **88**, 1 (2015)
15. Kuznetsov G.V., Strizhak P.A. J. of Engin Thermosyph., **18**, 2 (2009)
16. Kuznetsov G.V., Strizhak P.A. Int. J. of Heat and Mass Tr., **53**, 5–6 (2010)
17. Dolganov Y.A., Epifanov A.A. European Researcher., **43**, 1–3 (2013)