Simulation of Coherent Diffraction Radiation Generation by Pico-Second Electron Bunches in an Open Resonator

L G Sukhikh, A P Potylitsyn and D A Verigin

Tomsk Polytechnic University, Lenin avenue, 30, Tomsk, 634050, Russia

E-mail: `sukhikh@tpu.ru`

Abstract. In this report we present new approach for calculation of processes of diffraction radiation generation, storage and decay in an open resonator based on generalized surface current method. The radiation characteristics calculated using the developed approach were compared with those calculated using Gaussian-Laguerre modes method. The comparison shows reasonable coincidence of the results that allows to use developed method for investigation of more complicated resonators.

1. Introduction

Stimulated radiation is responsible for optical and infrared laser operation principle based on atomic properties of various materials. Later the term "laser" was used to specify light generated in undulators in electron accelerators known as Free Electron Lasers (FEL) [1, 2]. In this case electrons moving on a periodical path generate synchrotron radiation. The radiation traveling along the beam in the undulator starts interacting with the electrons stimulating the production of the radiation with the same wavelength and phase, which are the main properties of the laser light.

In the experiments [3, 4] the authors observed Stimulated Transition Radiation (TR) and Stimulated Synchrotron Radiation respectively, which, in contrast to undulator radiation, had a continuous spectrum. Authors of Ref. [3] used a special 4-mirror resonator in order to focus coherent TR generated by the electron bunch on the both surfaces of the target alternatively in the moment of passage of the subsequent electron bunch. In papers [4, 5] the authors proposed a concept of the so-called "broadband FEL" based on stimulated TR in the closed cylindrical resonator where resonator mirrors were used at the same time as TR targets. Afterwards the radiation is recirculated in the resonator. As a result every subsequent bunch generates radiation in the presence of radiation field from all preceding bunches stimulating the radiation production.

In Ref. [6] we proposed to generate intense THz radiation in an semi-confocal resonator to stimulate coherent diffraction radiation (DR) generated both by the flat mirror in forward direction (FDR) and by the concave mirror in backward direction (BDR). Due to DR non-invasive nature [7] there was no direct interaction of the electron beam with the mirrors resulted in fine background conditions and the beam parameters remaining almost the same as the initial ones. It was demonstrated that the radiation intensity stored in such a resonator growed up exponentially as a function of the number of bunches, which has an obvious analogy with

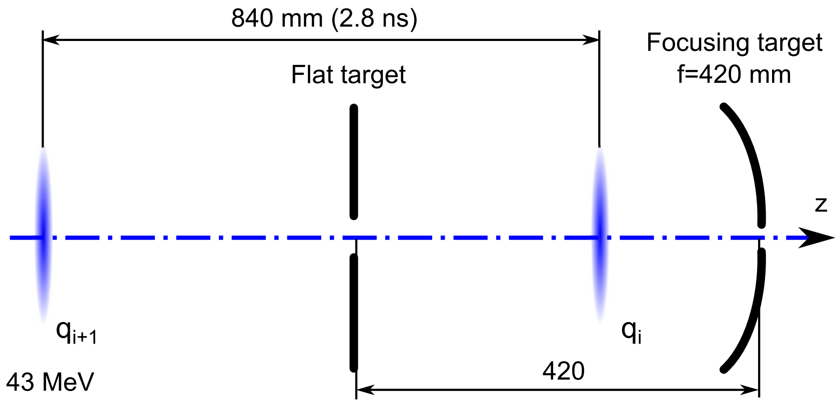

Figure 1. Simulation scheme.

undulator FELs, where the intensity grows up exponentially as a function of the number of undulator periods [6].

The aim of this paper is to show the analytical way for calculation of diffraction radiation characteristics in the open semi-confocal resonator taking into account radiation generation in the presence of external field, its storage and resonator free runs.

2. Theoretical model of DR generation in open resonator

Passing through openings in the first and second resonator mirrors point-like electron bunches with a charge q generate FDR and BDR, consequently. The first bunch passes through the opening in the first mirror generating FDR that travels along z-axis together with the electron. The slippage effect could be neglected due to a broadband DR spectrum. The bunch reaches the second mirror and generates BDR in presence of the FDR from the first mirror. Both BDR and reflected FDR travels back and reaches the first mirror at the moment when the second bunch arrives. Thus, both FDR and BDR are generated in the presence of the external electric field that was generated previously.

Let us formulate the problem as follows. Train of the point-like electron bunches with 43 MeV energy travels along z-axis passing through an open resonator that consists of flat and focusing mirrors (see figure 1). The distance between bunches in the train is equal to 840 mm (2.8 ns) while the distance between resonator mirrors is equal to 420 mm. The second mirror is paraboloidal one with radius of curvature equal to $R = 840$ mm (focal distance $f = 420$ mm). In the transverse plane both mirrors are assumed to be circles with a diameter equal to $a = 100$ mm. The openings in the mirrors are assumed to be round ones. The opening diameter in the focusing mirror is equal to $a_1 = 5$ mm, in the flat mirror – $a_0 = 5$ mm. The resonator parameters coincide with ones from the previous experiment [6].

For our calculation we assumed that both mirrors were perfect conductors. According to generalized surface current method [8] the electric field of FDR generated by the first bunch $\mathbf{E_f^{(1)}}$ could be written as integration of the double cross-product:

$$\mathbf{E_f^{(1)}}(\mathbf{r_0}, \omega) = \frac{1}{2\pi} \int_{S_1} dS_1 \left[\mathbf{n_1}, \mathbf{E_e}(\mathbf{r}, \omega)\right] \times \nabla_0 G(\mathbf{r_0}, \mathbf{r}, \omega), \tag{1}$$

where $\mathbf{r_0}$ is the observation point; $\mathbf{r}$ is the radiation point; ω is the radiation angular frequency ($\omega = 2\pi c/\lambda$, c is the speed of light); $\mathbf{n_1} = \{0, 0, 1\}$ is the surface normal to the first mirror; $\mathbf{E_e}(\mathbf{r}, \omega)$ is the electric field of the bunch; $\nabla_0 G(\mathbf{r_0}, \mathbf{r}, \omega)$ is the gradient of free space Green function. The integration is carried out over the surface of the flat mirror S_1, where $dS_1 = \rho d\rho d\phi$.

The electron bunch field $\mathbf{E_e}(\mathbf{r},\omega)$ could be written as follows [8]:

$$\mathbf{E_e} = \frac{q}{\pi c^2}\frac{\omega}{\beta^2\gamma}e^{i\frac{\omega}{\beta c}z}\left\{\frac{\vec{\rho}}{\rho}K_1\left[\frac{\omega\rho}{\beta c\gamma}\right] - \frac{i}{\gamma}\frac{\mathbf{V}}{\beta c}K_0\left[\frac{\omega\rho}{\beta c\gamma}\right]\right\}. \tag{2}$$

Here q is the bunch charge, $\beta = v/c$ is the electron reduced velocity, $i = \sqrt{-1}$, $\vec{\rho} = \{x = \rho\cos\phi, y = \rho\sin\phi\}$, $\mathbf{V} = \{0, 0, \beta c\}$, K_n is the modified Bessel function of the second kind of the n-th order.

The gradient of the free space Green function $\nabla_0 G(\mathbf{r_0},\mathbf{r},\omega)$ could be written as follows:

$$\nabla_0 G(\mathbf{r_0},\mathbf{r},\omega) = -(\mathbf{r}-\mathbf{r_0})\frac{\exp\left[i\frac{\omega}{c}|\mathbf{r}-\mathbf{r_0}|\right]}{|\mathbf{r}-\mathbf{r_0}|^3}\left(i\frac{\omega}{c}|\mathbf{r}-\mathbf{r_0}| - 1\right) \tag{3}$$

After the FDR is generated by the first bunch it travels in the resonator along z-axis. Both electron bunch and FDR reach the second mirror almost at the same time. In this case BDR is generated in the presence of external field. Thus, electric field of BDR $\mathbf{E_b^{(1)}}(\mathbf{r_0},\omega)$ could be written as follows:

$$\mathbf{E_b^{(1)}}(\mathbf{r_0},\omega) = -\frac{1}{2\pi}\int_{S_2} dS_2\left[\mathbf{n_2}, (\mathbf{E_e}(\mathbf{r},\omega) + \mathbf{E_f^{(1)}}(\mathbf{r},\omega))\right] \times \nabla_0 G(\mathbf{r_0},\mathbf{r},\omega), \tag{4}$$

where $\mathbf{n_2} = -\dfrac{2f\left\{\frac{\rho\cos\phi}{2f}, \frac{\rho\sin\phi}{2f}, 1\right\}}{\sqrt{4f^2+x^2+y^2}}$ is the normal vector to the parabolic mirror surface [9]. The integration is performed over the surface of the parabolic mirror S_2, $dS_2 = \dfrac{\sqrt{4f^2+\rho^2}}{4f}\rho d\rho d\phi$.

After generation of BDR the first bunch leaves the resonator. The BDR travels back to the first mirror and reaches it almost in the same time with second bunch resulting in the generation of the FDR in the presence of the external field. In this case the formula to find field of FDR looks similar to equations (1),(4):

$$\mathbf{E_f^{(2)}}(\mathbf{r_0},\omega) = -\frac{1}{2\pi}\int_{S_1} dS_1\left[\mathbf{n_1}, (\mathbf{E_e}(\mathbf{r},\omega) + \mathbf{E_b^{(1)}}(\mathbf{r},\omega))\right] \times \nabla_0 G(\mathbf{r_0},\mathbf{r},\omega), \tag{5}$$

Thus, the problem consists of the calculation of radiation spatial distributions on the mirror surfaces generated by the electron bunch train. In the case of i-th bunch the FDR and BDR could be calculated in the following way:

$$\begin{aligned}\mathbf{E_f^{(i)}}(\mathbf{r_0},\omega) &= \frac{1}{2\pi}\int_{S_1} dS_1\left[\mathbf{n_1}, (\mathbf{E_e}(\mathbf{r},\omega) + \mathbf{E_b^{(i-1)}}(\mathbf{r},\omega))\right] \times \nabla_0 G(\mathbf{r_0},\mathbf{r},\omega),\\ \mathbf{E_b^{(i)}}(\mathbf{r_0},\omega) &= -\frac{1}{2\pi}\int_{S_2} dS_2\left[\mathbf{n_2}, (\mathbf{E_e}(\mathbf{r},\omega) + \mathbf{E_f^{(i)}}(\mathbf{r},\omega))\right] \times \nabla_0 G(\mathbf{r_0},\mathbf{r},\omega)\end{aligned} \tag{6}$$

The equations (6) allow to calculate radiation characteristics in the resonator for all electron bunches one by one. After the bunch train leaves the resonator the radiation enhancement stops and decay process starts (resonator free runs). The decay of stored radiation could be calculated using the same formalism as described before but assuming that the bunch field is equal to zero.

There exists a significant difficulty that makes it almost impossible to carry out direct calculation of radiation field in the resonator using the proposed approach. Each radiation generation event adds double integration in the final result. For example, in order to calculate FDR from the second bunch (equation (5)) one needs to calculate six-dimensional integral. In order to overcome this problem the following assumption was made. Calculating BDR from the first electron bunch (equation (4)) one needs to know distribution of the FDR from the first bunch on the surface of the focusing mirror. Such a distribution could be calculated easily using

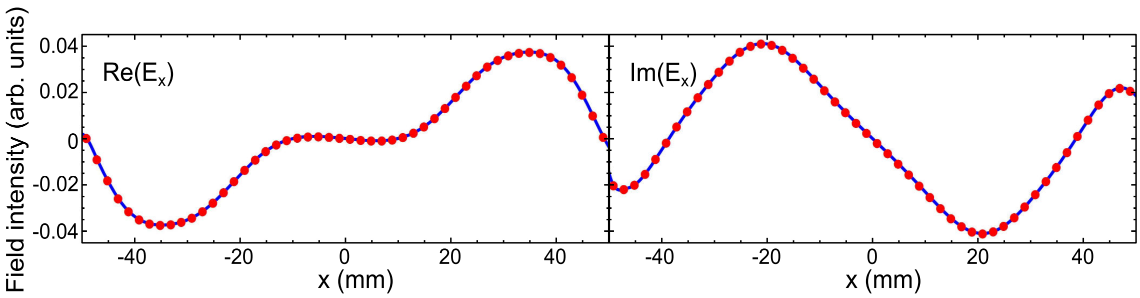

Figure 2. Comparison of calculated field distribution and its interpolation. Left plot – real part of E_x-component, right plot – imaginary part of E_x-component. Red circles – cross-section of calculated E_x-component, blue line – cross-section from interpolation 2D surface, $y = 0$, $\lambda = 3.5\,\text{mm}$.

equation (1) that allows numerical calculation of the spatial distribution of all three components of the electrical field on the mirror surface. Practically these field components are calculated in a finite number of points. Amount of such points should be enough for correct representation of the field. Thus, one can easily calculate the FDR field distribution in the specified number of points situated on the mirror surface and to interpolate obtained distribution assuming its smooth character. In this case the numerical calculation of BDR will include just double integration over the focusing mirror surface that could be easily carried out numerically.

Our calculation was carried out using "Wolfram Mathematica 7.0" software [10]. The FDR field distribution on the surface of focusing mirror was calculated in the range $-50 \leq x, y \leq 50\,\text{mm}$ with the step equal to $2\,\text{mm}$. The calculated field consists of 6 components, namely:

$$\mathbf{E}_{\mathbf{f}}^{(\mathbf{1})}(\mathbf{r_0}, \omega) = \{\Re(E_x) + i\Im(E_x), \Re(E_y) + i\Im(E_y), \Re(E_z) + i\Im(E_z)\} \tag{7}$$

Each field component was interpolated by 2D surface using spline interpolation procedure in "Wolfram Mathematica 7.0". Figure 2 demonstrates comparison of calculated field distribution (x-component of FDR field) and results of interpolation by 2D surfaces in the form of cross-sections along x-axis ($y = 0$) both for real and imaginary parts of the field. The calculation was carried out at $\lambda = 3.5\,\text{mm}$.

One can see in figure 2 that coincidence between calculated field distribution and its interpolation is good and obtained distribution of the field is a smooth function that allows to use proposed method for calculation of further reflections of the field in the resonator avoiding multifold integration.

The spatial distributions of DR on the mirror surfaces has azimuthal symmetry as it was expected from the problem geometry [7]. Both distributions have minimum in the center and one maximum, i.e. one could observe double-lobe distribution expected from the characteristics of DR. The position of maximum differs for flat mirror and concave mirror due to the fact that concave mirror naturally focuses the radiation on the surface of the flat mirror.

3. Gaussian-Laguerre modes in semi-confocal resonator

In a resonator electromagnetic wave is confined by its mirrors. In this case it could be described by superposition of plane waves or transverse electromagnetic modes which occurs in the resonator. Different solutions for modes are used for different types of symmetry. Gaussian-Laguerre modes (GLM) are used for calculation of transverse distributions of radiation stored in confocal resonators with axial symmetry. In order to apply these modes to our problem we "expanded" our semi-confocal resonator and assumed that we have confocal resonator that consists of two focusing mirrors with curvature radii equal to $R = 840\,\text{mm}$. The intermirror

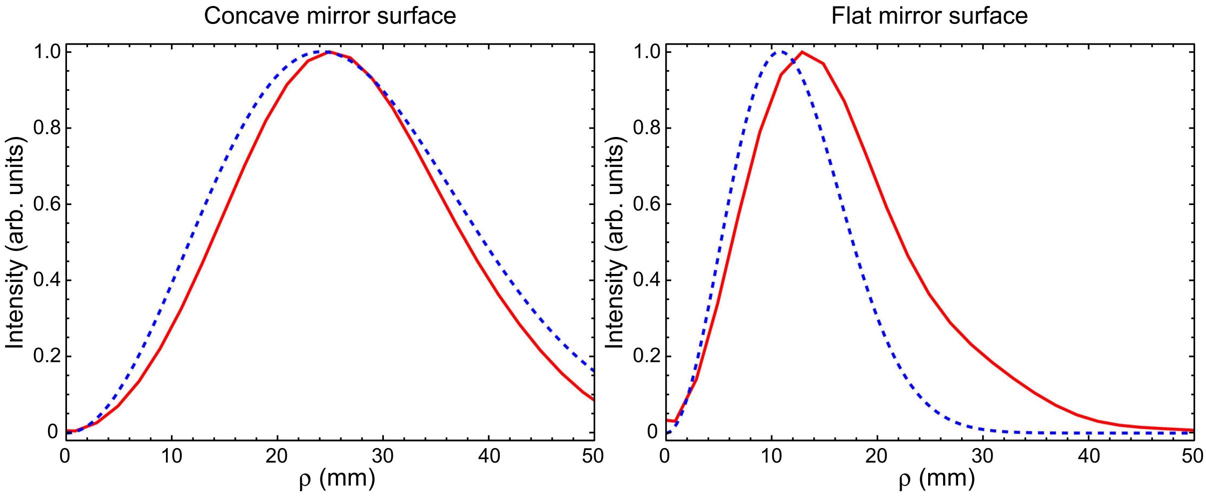

Figure 3. Comparison of radial distribution of GLM (blue dashed curve) and DR in the resonator after 15 electrons (red solid curve). The calculation was carried out at $\lambda = 3.5\,\mathrm{mm}$

distance is also equal to $L = 840\,\mathrm{mm}$. Using Gaussian-Laguerre modes one could estimate the radiation spatial distribution on the surface of focusing mirror and in the center of resonator where the radiation waist is situated.

According to [11] the amplitude of GLM m, n in the radiation waist could be described as following:

$$\Psi^0_{mn}(\rho,\varphi) = \left(\frac{\rho\sqrt{2}}{w_0}\right)^m \exp\left[-\frac{\rho^2}{w_0^2}\right] L_n^m\left(-\frac{2\rho^2}{w_0^2}\right) e^{im\varphi}. \tag{8}$$

Here $w_0 = \sqrt{\frac{L\lambda}{4\pi}}$, L_n^m is the generalized Laguerre polynomial.

Fresnel transformation of GLM allows to obtain amplitude of radiation at any distance z from the waist position inside resonator:

$$\begin{aligned} \Psi_{mn}(\rho,\varphi,z) &= \frac{w_0}{w(z)}\left(\frac{\rho\sqrt{2}}{w(z)}\right)^m \times \\ &\times \exp\left[-\frac{\rho^2}{w^2(z)} + \frac{ik\rho^2}{2R(z)} + im\varphi - i(2n+m+1)\arctan\left(\frac{4z}{L}\right)\right] L_n^m\left(\frac{2\rho^2}{w^2(z)}\right). \end{aligned} \tag{9}$$

Here $w(z) = w_0\sqrt{1+\left(\frac{4z}{L}\right)^2}$, $R(z) = z\left[1+\left(\frac{L}{4z}\right)^2\right]$, $k = 2\pi\lambda^{-1}$.

Due to the fact that the DR distribution has double-lobe structure [7] we took into account only GLM Ψ_{m0}. Figure 3 shows comparison of radial distribution of GLM and DR calculated using equations (6) for 15 point-like electron bunches on the surfaces of both mirrors. GLM on the surface of concave mirror was calculated in the form $|\Psi_{10}|^2$ assuming $z = 420\,\mathrm{mm}$. GLM on the surface of flat mirror was calculated in the form $|\Psi^0_{10}|^2$. During the calculation we keep $\lambda = 3.5\,\mathrm{mm}$.

In figure 3 one can see that radial distribution of DR calculated on the surface of concave mirror coincides rather well with Gaussian-Laguerre mode $|\Psi_{10}|^2$. Coincidence of radial distributions of DR and GLM $|\Psi^0_{10}|^2$ on the surface of flat mirror is not so good. The GLM distribution is narrower than DR distribution, however peak positions are close to each other. Thus, one could say that use of GLM gives reasonable agreement with DR approach and could be used for resonator investigation.

Journal of Physics: Conference Series **732** (2016) 012019 doi:10.1088/1742-6596/732/1/012019

4. Quality factor of semi-confocal resonator

During experimental investigation of coherent DR generation in semi-confocal resonator its quality factor (Q-factor) was defined to be equal to $Q = 72.9$ [6]. The Q-factor was defined using the following procedure. After all electron bunches leaved the resonator the radiation stored in it decayed following exponential law. The part of radiation that came out the resonator from the opening in the flat mirror was measured as function of time and was observed as a peak structure with time distance between peaks equal to distance between electron bunches ($\Delta t = 2.8\,\text{ns}$). In order to estimate Q-factor exponential fit of this signal tail was done by the function $A(t) = A_0 e^{-t/t_0}$, where A_0 is free scaling fit parameter and t_0 defines Q-factor in the following form [6]:

$$Q = 2\pi \frac{t_0}{\Delta t}. \tag{10}$$

In our theoretical investigation we tried to estimate the value of Q-factor for the semi-confocal resonator using both DR approach and Gaussian-Laguerre mode approach. During the experiment [6] the flat mirror consisted of glass substrate with reflecting coating and had the opening diameter $a_0 = 5\,\text{mm}$. However, the reflecting layer was deposited not on the whole substrate surface. The area inside the circle with diameter 15 mm was not deposited and thus consisted of pure glass. Using both theoretical approaches we could not take into account this part with no reflecting coating. In order to estimate resonator Q-factor we assumed two cases. In the first case the opening diameter was assumed to be equal to 5 mm that should result in the upper limit of Q-factor. In the second case the opening diameter was assumed to be equal to 15 mm that should result in the lower limit of Q-factor.

In order to estimate Q-factor using DR approach we calculated a train consisted of 15 point-like electron bunches. The DR was calculated using equations (6) at radiation wavelength equal to $\lambda = 3.5\,\text{mm}$ for both opening diameters. After each third bunch starting from third one 25 relaxation free runs were calculated using again equations (6) but assuming that electron field is absent ($\mathbf{E_e}(\mathbf{r}, \omega) = 0$). In order to calculate the part of radiation that comes out of the resonator the DR intensity distribution on the flat mirror surface was integrated over the opening area. This value was assumed to be a “signal” that was measured over time. Figure 4 show an example of radiation yield vs. time that was calculated for $a_0 = 15\,\text{mm}$. Cavity free runs were calculated

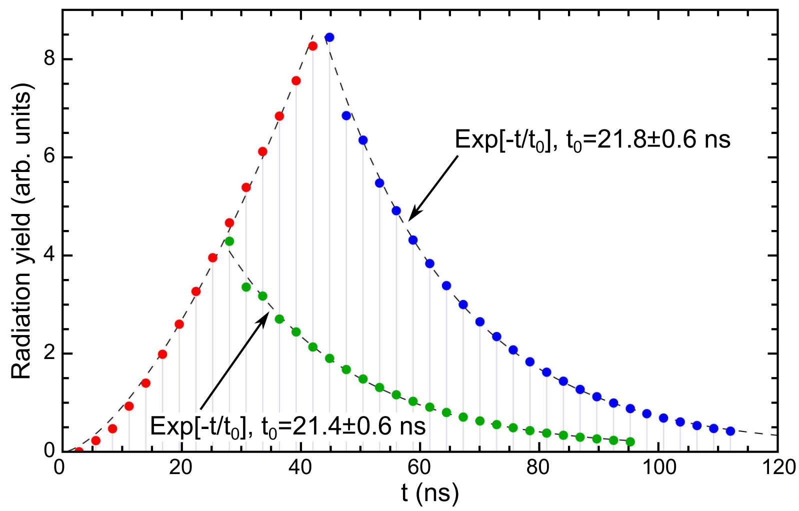

Figure 4. Example of time dependence of radiation yield. Red dots – radiation yield electrons, blue dots – resonator free runs after 15 bunches, green dots – resonator free runs after 9 bunches. The calculation was carried out at $\lambda = 3.5\,\text{mm}$ and $a_0 = 15\,\text{mm}$

for 9 electron bunches in the train and 15 bunches in the train.

In the case of DR calculation approach the resonator Q-factor was estimated exactly like it was done during the experiment [6]. The values of resonator Q-factor calculated at $a_0 = 5\,\mathrm{mm}$ are shown in figure 5 by blue triangles and at $a_0 = 15\,\mathrm{mm}$ – by red dots. The errors shown in figure 5 are errors caused by fit errors of parameter t_0. It is obvious that Q-factor should not depend on number of electron bunches in the train. In order to demonstrate this the value of Q-factor in figure 5 was plotted versus number of bunches, i.e. resonator free runs were calculated after each third bunch. The resonator Q-factor value $Q = 72.9$ measured experimentally is shown by black solid line.

Cavity Q-factor could be also calculated using GLM as following:

$$Q = 2\pi \frac{1}{c_\Sigma}, \tag{11}$$

where c_Σ is the total losses of energy in the resonator. In our case we took into account geometrical losses (c_G) and diffraction losses (c_D). According to [12]:

$$c_\Sigma = 1 - (1 - c_D^f)(1 - c_D^c), \tag{12}$$

where indexes f, c define flat and concave target, respectively.

According to [13], diffraction losses could be estimated as:

$$c_D = 1 - (1 - c_G)^2 \tag{13}$$

Geometrical losses could be calculated due to the fact that we know spatial distributions of

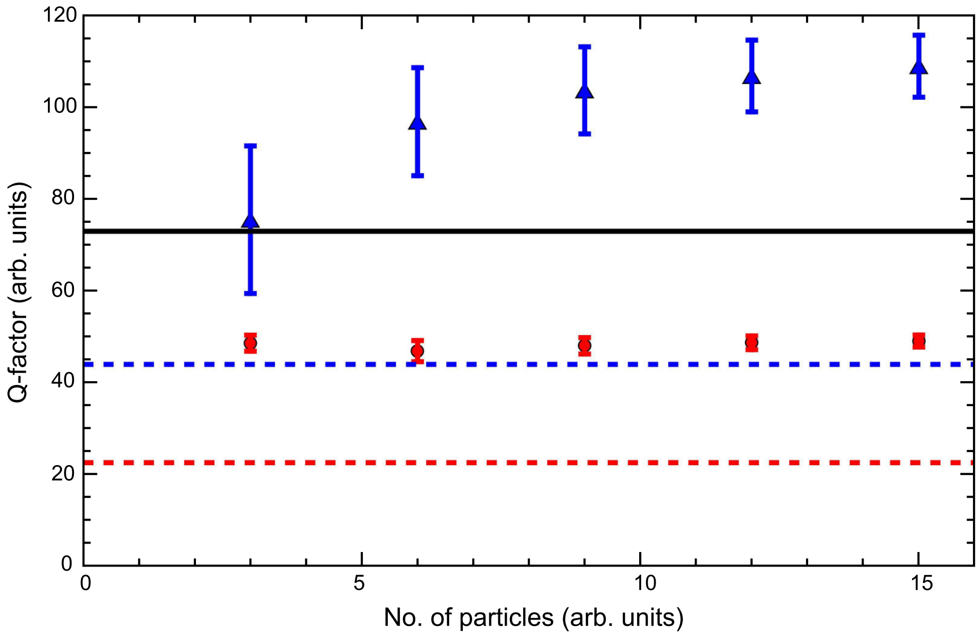

Figure 5. Dependence of the resonator Q-factor on number of electrons. Red dots – DR approach, $a_0 = 15\,\mathrm{mm}$; blue triangles – DR approach, $a_0 = 5\,\mathrm{mm}$; red dashed line – GLM approach, $a_0 = 15\,\mathrm{mm}$, blue dashed line – GLM approach, $a_0 = 5\,\mathrm{mm}$; black solid line – $Q = 72.9$, experimental result [6]

radiation intensity on the mirror surfaces [14]:

$$\begin{aligned} c_G^f &= \frac{\int_0^{a_0/2}\rho d\rho \int_0^{2\pi}|\Psi_{10}^0|^2 d\varphi + \int_{a/2}^{\infty}\rho d\rho \int_0^{2\pi}|\Psi_{10}^0|^2 d\varphi}{\int_0^{\infty}\rho d\rho \int_0^{2\pi}|\Psi_{10}^0|^2 d\varphi}, \\ c_G^c &= \frac{\int_0^{a_1/2}\rho d\rho \int_0^{2\pi}|\Psi_{10}|^2 d\varphi + \int_{a/2}^{\infty}\rho d\rho \int_0^{2\pi}|\Psi_{10}|^2 d\varphi}{\int_0^{\infty}\rho d\rho \int_0^{2\pi}|\Psi_{10}|^2 d\varphi}, \end{aligned} \tag{14}$$

where $a_0 = 5\,\mathrm{mm}$ or $a_0 = 15\,\mathrm{mm}$ (see further), $a_1 = 5\,\mathrm{mm}$, $a = 100\,\mathrm{mm}$.

Combining equations (11)–(13) one could obtain resonator Q-factor value. In figure 5 the calculated values of Q-factor are shown by red dashed line at $a_0 = 15\,\mathrm{mm}$ ($Q = 22.5$) and by blue dashed line at $a_0 = 5\,\mathrm{mm}$ ($Q = 43.9$).

5. Conclusion

In conclusion we would like to note that the well-known theory of Gaussian beams in open resonators and developed computer code allowing to simulate DR characteristics in a semi-confocal resonator give the coinciding results for transversal field components. This fact confirms the validity of the code based on the generalized surface current model [8]. The unified formulation for DR generation in resonator mirrors and the radiation storage can be applied for calculation of DR characteristics at any resonator configuration including the confocal one. The preliminary estimations of Q-factor for such a resonator show a possibility to achieve much higher values of Q-factor than in the semi-confocal resonator considered above.

Acknowledgments

The work was partially supported by the Russian Ministry of Education and Science within the program “Nauka”, Grant No. 3.709.2014/K an by RFBR grant No. 15-52-50028.

References

[1] Dattoli D, Renieri A and Torre A 1993 *Lectures on Free Electron Laser Theory and Related Topics* (Singapore: World Scientific)
[2] Gover A, Dyumin E 2006 *Proceedings of Free Electron Laser conference FEL 2006 (Berlin, Germany)* p 1
[3] chi-Lihn H, Kung P, Settakorn C, Wiedemann H and Bocek D 1996 *Phys.Rev.Lett.* **76** 4163
[4] Shibata Y et al 1997 *Phys.Rev.Lett.* **78** 2740
[5] Sasaki S, Shibata Y, Ishi K, Ohsaka T, Kondo Y, Hinode F, Matsuyama T and Oyamada M 2002 *Nucl. Instr. Meth. A* **483** 209
[6] Aryshev A et al 2014 *Nucl. Instr. Meth. A* **763** 424
[7] Potylitsyn A, Ryazanov M, Strikhanov M and Tishchenko A 2010 *Diffraction Radiation from Relativistic Particles* (Berlin: Springer)
[8] Karlovets D V and Potylitsyn A P 2009 *Phys.Lett.A* **373** 1988
[9] Naumenko G A, Cha V A, Kalinin B N, Popov Yu A, Potylitsyn A P, Saruev G A and Sukhikh L G 2008 *Nucl. Instr. Meth. B* **266** 3733
[10] http://www.wolfram.com/
[11] Allen L, Beijersbergen W, Spreeuw R J C and Woerdman J P 1992 *Phys. Rev. A* **45** 8185
[12] Kubarev V V 2000 *Kvantovaya Elektronika* **30** 826
[13] Kubarev V V, Persov B Z, Vinokurov N A, Davidov A V 2004 *Nucl. Instr. Meth. A* **528** 199
[14] Gurov M G and Dmitriev A K 2009 *Russian Physics Journal* **52** 464