Planar channelling of relativistic electrons in half-wave silicon crystal and corresponding radiation

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Abstract. New experimental data on planar channeling of 255 MeV electrons in a 0.74 µm Si Half-Wave Crystal (HWC) obtained at SAGA-LS facility are presented. The computer simulation showed that the angular distribution of electrons after penetration through the HWC revealed the number of unknown before peculiarities is connected with specific electron trajectories in HWC. These specific trajectories lead to specific radiation, the properties of which are analyzed.

1. Introduction

The first prediction of charged particles deflection using a bent crystal was made in [1]. Since then, the extensive studies have been performed (see [2] and references therein). Recent works at CERN have been demonstrated the possibility to apply this effect to the beam collimation [3–5]. These early studies have been carried out using positively charged particles. Recently, these techniques has been proven to be useful even for negatively charged particles - relativistic electrons - at MAMI (855 MeV [6]) and at SLAC (3.35 and 6.3 GeV [7]). These experiments showed that there appear a large deflection angle in the channeling orientation and a smaller one in the volume-reflection orientation.

When the charged particles penetrate through the unbent crystal, the similar deflection effect may occur in a crystal of the thickness equal to a half wavelength of the planar channeling oscillation. In this case, the beam is deflected as if it is mirror-reflected by the crystal planes, as predicted in 1995 [8]. The mirroring effect for 2 MeV protons channeled in a Half-Wave (HWC) Si crystal was demonstrated for the first time in [9] in 2012. In 2014, the experimental studies were performed at ultra-relativistic energies using 400 GeV protons at CERN [10].

In 2014, the first experiments on channeling of 255 MeV electrons in a 1µm-thick Si crystal were performed at SAGA LS [11]. Although the results were suggestive of a mirroring effect, the crystal thickness remained slightly greater than the optimum HWC thickness for mirroring. Here we present new theoretical and experimental results on angular distributions of 255 MeV electrons planar

channeled in a thinner 0.74-µm Si crystal. Also we present estimations of spectral-angular properties of electromagnetic radiation appearing in mirroring process.

2. Channelling experiment in HWC Si

The experiments were performed at a linac at SAGA-LS, a Japanese synchrotron radiation facility [12]. The schematic of the setup is shown in figure 1. The electron beam energy was 255 MeV (corresponding to a Lorentz factor γ of 500). The details are described in [13]. The repetition rate of the beam acceleration was 1 Hz, and the average beam current was 7 nA. The horizontal and vertical beam sizes at the target were $\sigma_x \approx 0.3$ mm and $\sigma_y \approx 0.9$ mm, respectively (σ denotes one standard deviation). The horizontal and vertical angular divergences of the incident beam were $\sigma'_x \approx 0.1$ mrad and $\sigma'_y \approx 0.1$ mrad, respectively.
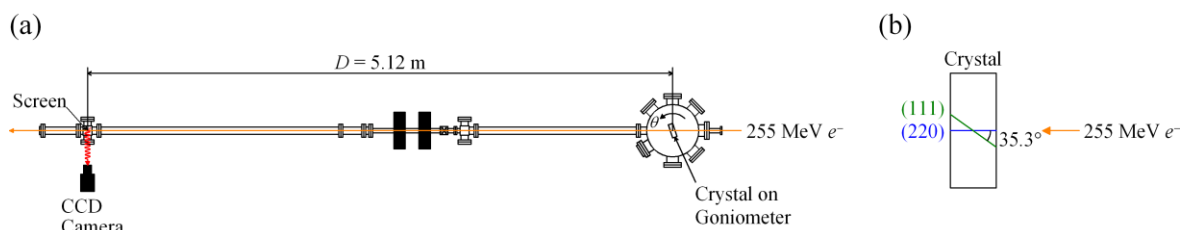

Figure 1. (a) Experimental setup (top view). (b) Orientation of the (220) and (111) planes.

A 0.74-µm-thick Si crystal was employed as a target. The crystal thickness has been selected based on simulations of the mirroring of 255 MeV electrons in an ultrathin crystal [11, 14]. The <001> crystallographic axis was normal to the crystal surface. The Si crystal was installed on a two-axis goniometer in a vacuum chamber. The (220) plane was aligned with the vertical plane. The profiles of the electron beam transmitted through the crystal were measured using a screen monitor, which consists of a 100-µm-thick alumina screen and a CCD camera. The distance between the crystal and the screen was $D = 5.12$ m. The (220) and (111) planes make an angle of 35.3°, and so the crystal was rotated by 35.3° from the normal incidence condition to observe (111) planar channeling (see figure 1(b)). Accordingly, the effective crystal thickness is calculated to be $0.74/\cos(35.3°) = 0.91\,\mu m$ for the (111) channeling experiment.

3. Experimental results and simulations

Simulations of the electron trajectories are carried out using the code "Basic Channeling with Mathematica" BCM–1.0 [15]. This code computes numerical solutions of the classical equations of motion using the Doyle-Turner potential for the crystallographic planes [16]. A series of trajectories for 255 MeV electrons in a Si HWC channeled along the (220) and (111) planes are depicted in figure 2. Because the potential for planar-channeled electrons is not harmonic, the electrons trajectories are not characterized by a single oscillation wavelength λ as they are for planar-channeled positive particles. Instead, for planar-channeled electrons the oscillation period depends on the point and angle of incidence, as shown in figure 2.

Typical electron trajectories channeled in (111) HWC Si are shown in figure 2(b). It can be clearly seen that some electrons change the sign of their perpendicular momentum (in the *X*-direction) for crystal thicknesses of about 0.8-0.9 µm (these trajectories are depicted by red solid lines). Due to this specific motion of channeled electrons in HWC, the mirror effect occurs for incidence angle $\theta = \theta_c / 2$ (figure 2(b) right column).

To simulate the angular spread of electrons the procedure described in [17] is used. For each point of entry, 20 values of the incident angle θ are generated randomly with a normal distribution. The standard deviation of the distribution matches the experimental value of 0.1 mrad. Thus, two thousand

trajectories are calculated for different entry points within one period of the planar channeling potential. The calculated coordinates and angles of exit of these electrons are used to simulate the 2D beam profiles. Horizontal profiles are obtained by projecting the 2D beam profiles onto the horizontal plane.
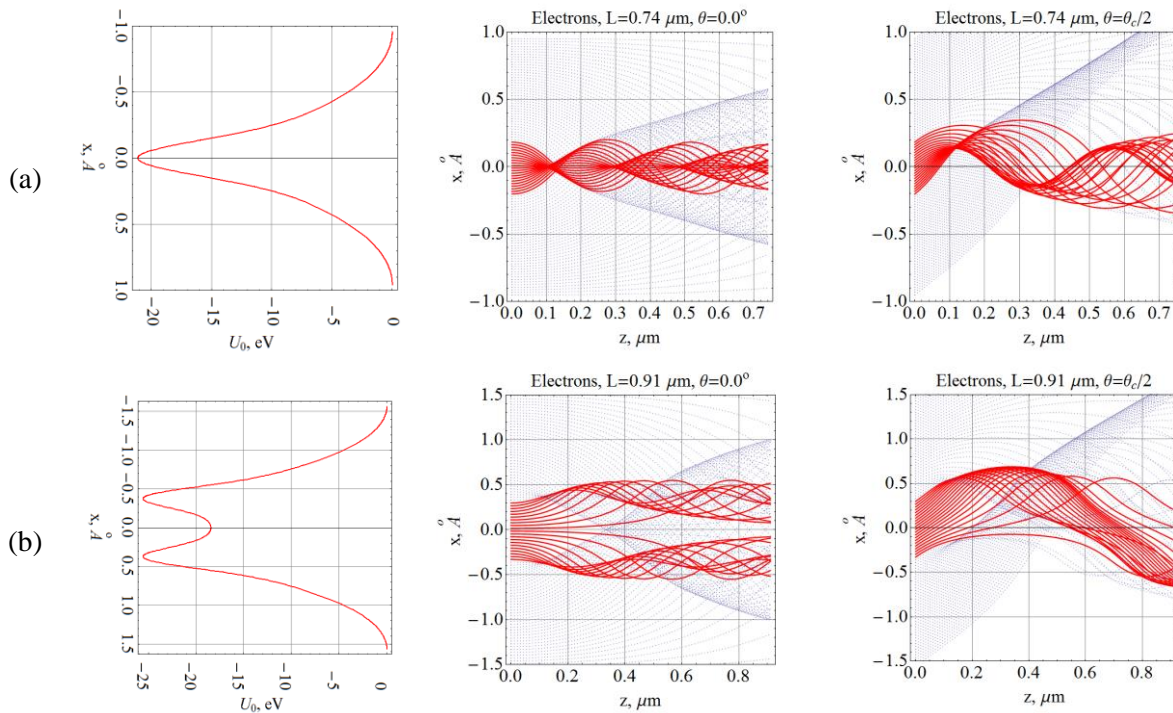

Figure 2. Simulated trajectories of 255 MeV electrons in a HWC Si and potential energy of electrons in the electric field of the crystal planes. (a) (220) planar potential and simulated trajectories for incidence angle $\theta = 0.0°$, $\theta = \theta_c / 2$. (b) (111) planar potential and simulated trajectories for incidence angle $\theta = 0.0°$, $\theta = \theta_c / 2$ (θ_c is the critical channeling angle). Solid red lines represent trajectories with entry points lying near the bottom of the potential well.
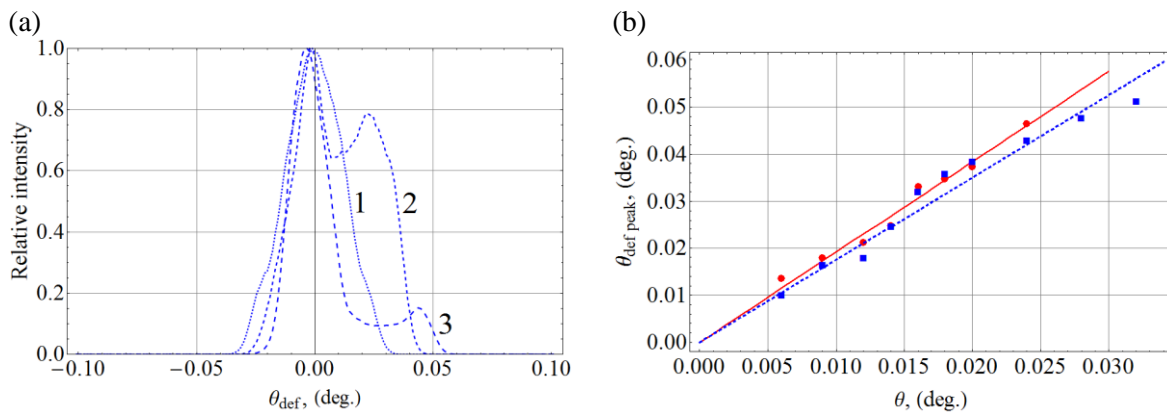

Figure 3. (a) Simulated angular distribution of 255 MeV electrons channeled in (111) HWC after penetrating trough 0.91-µm Si crystal for different angle of incidence θ: (1) $\theta = 0.0°$, (2) $\theta = \theta_c / 2$, (3) $\theta = \theta_c$. (b) The angle corresponding to the peak of angular distribution $\theta_{\text{def peak}}$ versus an angle of incidence θ (● simulation for the parallel beam, ■ simulation for the beam with angular spread).

Figure 3(a) shows simulated angular distribution of 255 MeV (111) channeled electrons after penetrating trough crystal for different angle of incidence. The calculation highlights that the peak of angular distribution lays in region $\theta_{def\ peak} = (1.75 \div 1.91)\theta$ (see in figure 3(b)). But the deflection efficiency decrease if incidence angle $\theta > \theta_c / 2$ (θ_c is the critical channeling angle). Thus the optimal angle of incidence for observation of mirroring effect for electron channeled in HWC is $\theta = \theta_c / 2$, the deflection angles $\theta_{def} = 0.023° = 0.4\ \text{mrad}$ which is comparable to that for planar channeling in bent crystals [6, 7] because the critical angle increases at lower energies. The fraction of deflected particles calculated by Gaussian gives 29% [14]. This result suggests that mirroring in HWC could be an effective method for beam deflection in that energy region, since multiple scattering is suppressed and a complex crystal bender is not required.
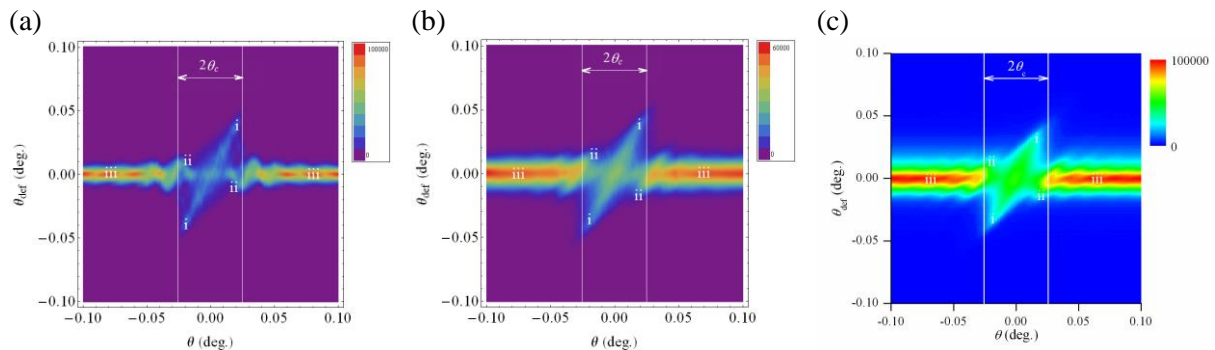
(a)   (b)   (c)

Figure 4. Beam intensity as a function of the beam deflection angle and of the crystal rotation angle θ relative to the (111) plane for (a) simulations for parallel beam, (b) simulations for beam with angular distribution and, (c) experimentally recorded distribution.

Figure 4(a, b) show calculated beam intensity distributions as functions of the crystal rotation angle θ and of the beam deflection angle θ_{def} for parallel beam (a), beam with angular divergence (b). Experimentally recorded distribution is presented in figure 4(c). The range $-\theta_c \leq \theta \leq \theta_c$ is indicated by the vertical lines. The experimental results are in good agreement with the simulations. Each graph can be divided into three regions. Region (i) corresponds to electrons channeled in the plane, lying along the line $\theta_{def\ peak} = 1.91\theta$ for parallel beam and $\theta_{def\ peak} = 1.75\theta$ for beam with angular spread, i.e., these electrons can be regarded as being mirror-reflected by the plane. Electrons in region (ii) are in the range of angles less than the critical angle. These electrons are slightly deflected into the direction opposite to the channeling direction, which is similar to the volume-reflection process [19]. Such a phenomenon has also been observed for 2 MeV protons [9] and for 400 GeV protons [10]. Region (iii) represents unchanneled (over-barrier) electrons whose deflection angles slightly oscillate with θ affected by the planar potential. A similar oscillation is observed in simulations for 400 GeV protons [10].

4. Simulation of spectral-angular distribution of radiation from electrons under channeling in in HWC Si

If the trajectory is known, the spectral-angular distribution of radiation from electron can be calculated using the well-known formula [18]:

$$\frac{d\varepsilon}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c} \left| \int_0^\tau \frac{[\mathbf{n}[(\mathbf{n} - \boldsymbol{\beta})\dot{\boldsymbol{\beta}}]]}{(1 - \mathbf{n}\boldsymbol{\beta})^2} \cdot e^{i(\omega t - \mathbf{k}\mathbf{r})} dt \right|^2, \qquad (1)$$

where $\mathbf{r}(t)$ is a radius-vector of electron, $\boldsymbol{\beta}(t) = \mathbf{v}(t)/c$, $\mathbf{v}(t)$ is an electron velocity and c is the speed of light, $\mathbf{k} = \mathbf{n} \cdot \omega/c$ the photon wave vector, $\mathbf{n}$ the unit vector determining the photon emission direction. Regarding the channeling radiation (CR), the formula (1) is used under assumption of a large number of oscillations of the channeled particle (the ideal trajectory). In this case it is possible to obtain relatively simple analytical formulas for the spectral-angular distributions of CR (see, for example [19-21]). In the case of channeling in a HWC one deals with approximately half of oscillation and one can expect the appearance of new features in radiation spectrum (let us call it CR-HWC).

The calculated CR-HWC spectrum in a forward direction from a 255 MeV electron planar channeled in (220) Si crystal is shown in figure 5 – only one selected trajectory. It is clearly seen the principal difference of CR-HWC spectrum (electron makes approximately a half oscillation in the transverse direction during its motion through the HWC crystal, figure 5(a) and that in a thicker crystal (electron makes more than a half of oscillation, see in figure 5(b,c)). With increasing the number of oscillations of electron in a crystal peaks in the CR spectrum become narrower and higher.
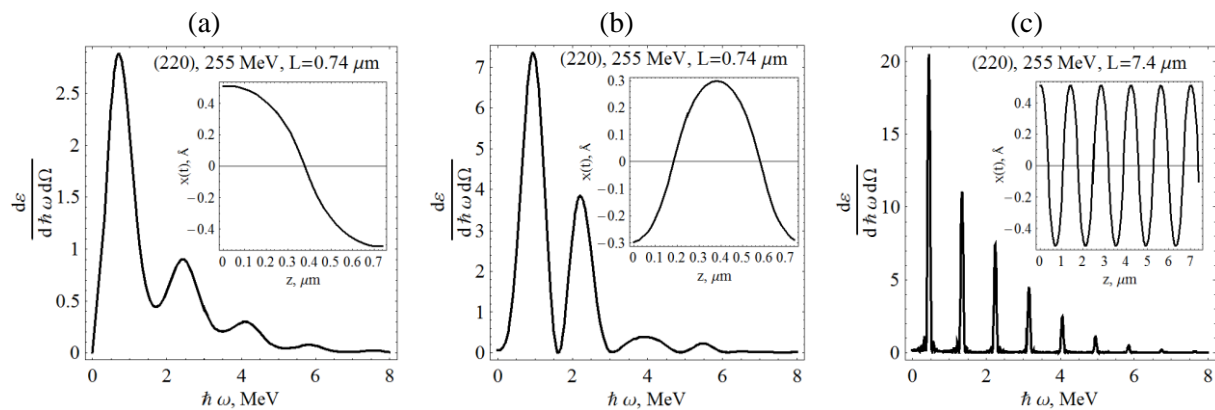

Figure 5. Calculated CR spectra in forward direction from an electron channeled in (220) Si crystal: (a) channeling in HWC – electron makes approximately a half of oscillation , (b) electron makes only one oscillation, (c) channeling in thick crystal.
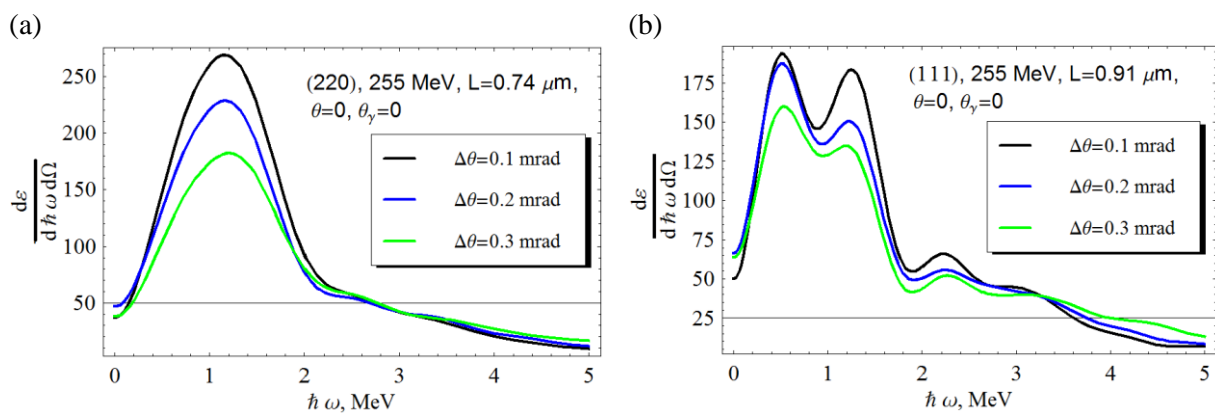

Figure 6. Calculated CR-HWC spectra from 255 MeV electrons in a forward direction ($\theta_\gamma = 0.0°$) averaged over 500 trajectories and for different incident beam angular spread $\Delta\theta$. (a) electrons channeled in (220) HWC Si, (b) electrons channeled in (111) HWC Si. Here, the angle of incidence relative to crystallographic planes equals zero.

In a real experiment, one deal with a number of different electron trajectories in a crystal, the shape of every one depends on the point of incidence into a crystal and angle of incidence. Figure 6 shows calculated CR-HWC spectra from 255 MeV electrons in a forward direction ($\theta_\gamma = 0.0°$) averaged over 500 trajectories (in figure 2 only 100 trajectories are presented) and taking account beam angular spread $\Delta\theta$. The black line corresponds to the angular spread of the electron beam at SAGA-LS linac equals to $\Delta\theta = 0.1\,\mathrm{mrad}$. Whereas the angular spread increases three times, the height of the spectrum maximum decreases by 1.6 times for electrons channeled along (220) in HWC Si (see, in figure 6(a)). This is due to the major contribution of electrons involved in the quasi-periodic (over-barrier) motion.

The CR-HWC spectrum from electrons channeled in (111) HWC Si is more complicated. It contains of two peaks in the soft part (see, in figure 6(b)). This is due to the fact that the potential energy of electrons in an electric field of (111) planes has specific shape with two well. As a sequence, the trajectories of the electrons can be characterized by two specific wavelengths (see, in figure 2 (b)). With increase of the incident beam angular spread $\Delta\theta$ the second peak practically disappears. Obviously, the CR-HWC spectra will depend on the angle of incidence of electron beam relative to the channeling planes, i.e. will show the so-called orientation dependence. The calculated CR-HWC spectra (in a forward direction) from 255 MeV electrons channeled in HWC Si, for different incidence angles θ are shown in figure 7. For an incidence angle less than the critical channeling angle θ_c, the spectra contain the peaks in a soft part due to radiation from electrons involved in a bound motion (black and red lines in figure 7.). The heights of these peaks decrease with increase of the angle of incidence from $0.2\theta_c$ up to $0.5\theta_c$ for electrons channeled in (220) HWC, whereas for (111) channeling the heights of the peaks increase. For the angle of incidence $\theta = 2\theta_c$, all the electrons are involved into the quasi-periodic (over-barrier) motion, as a sequence the peaks of CR-HWC spectra are shifted to the region of greater photon energies (blue lines in figure 7(a, b)).
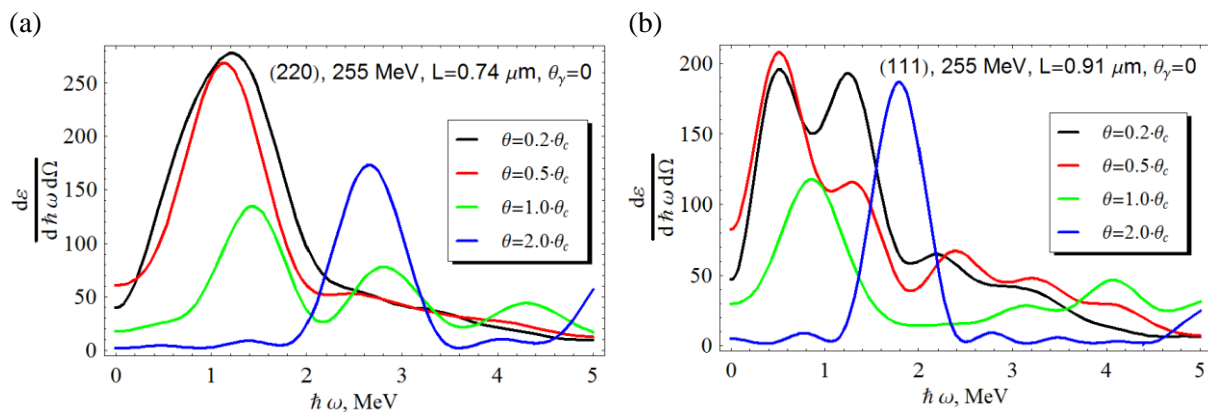
(a)   (b)

Figure 7. Calculated CR-HWC spectra from 255 MeV electrons channeled in HWC Si in a forward direction and for different incidence angle θ: a) (220) HWC Si, b) (111) HWC Si.

Since the efficiency of 255 MeV electron beam bending in a HWC Si crystal was estimated as 29%, it is worth to compare the intensities of gamma-beams emitted at the angles $\theta_\gamma = 0.0°$ and $\theta_\gamma = 0.5\,\theta_{\mathrm{def}}$. The qualitative analysis can be made straightforward by comparison of the typical emission angle of relativistic particle $\theta_\gamma \approx 1/\gamma$ and deflection angle $\theta_{\mathrm{def}} \cong \theta_c$. For parameters chosen (electron beam energy 255 MeV and (220) channeling) one obtains $\theta_\gamma \approx 1/\gamma = 2\,\mathrm{mrad}$ and $\theta_{\mathrm{def}} \cong \theta_c = 0.4\,\mathrm{mrad}$, respectively. Thus, $\theta_\gamma \gg \theta_c$ and we may conclude, no sufficient difference in

intensity spectra at $\theta_\gamma = 0 \div 1/\gamma$ and $\theta_\gamma = 0.5\,\theta_{\mathrm{def}}$ should appear. Indeed, the simulated CR-HWC spectra at $\theta_\gamma = 0.5\,\theta_{\mathrm{def}}$ presented in figure 8 together with ones at $\theta_\gamma = 0.0°$ show only small difference both in maxima positions and maximum values of CR-HWC intensity spectra at $\theta_\gamma = 0.0°$ and at $\theta_\gamma = 0.5\,\theta_{\mathrm{def}}$.
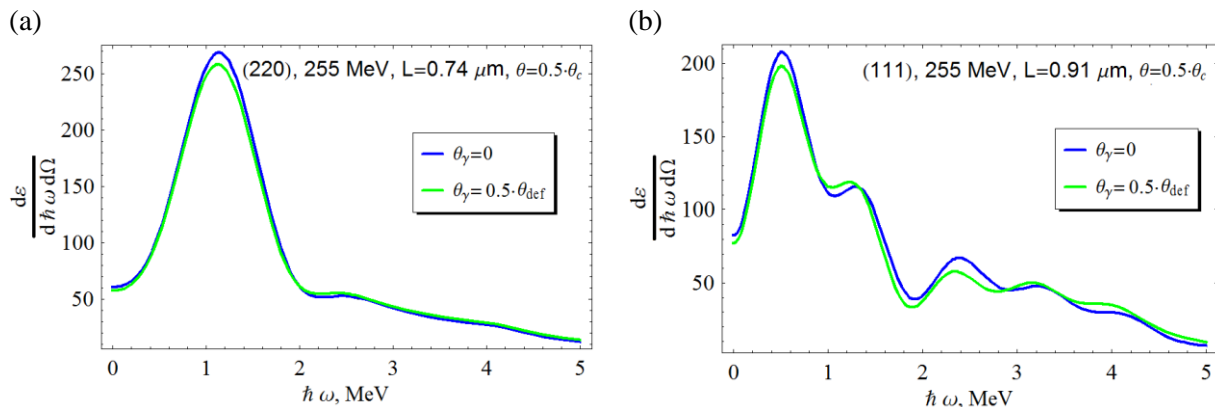

Figure 8. Calculated CR-HWC spectra from 255 MeV electrons channeled in HWC Si (incident angle $\theta = 0.5\theta_c$) at emission angles $\theta_\gamma = 0.0°$ and $\theta_\gamma = 0.5\,\theta_{\mathrm{def}}$: a) (220) HWC Si ; b) (111) HWC Si.

5. Conclusions

The results can be summarized:

New experimental data on angular distributions of planar channeled 255 MeV electrons in a 0.74 μm Si Half-Wave Crystal (HWC) obtained at SAGA - LS facility are presented. The data are explained well by computer simulations.

The computer simulations showed that the angular distributions of electrons after penetration through a HWC revealed the number of unknown before peculiarities and are connected with specific electron trajectories in HWC as well as with initial electron beam divergence and angle of incidence with respect to the channelling planes.

Third, it is closely connected with radiation spectrum from a particle moving in an arc [24-26] or in a short magnet. The experimental studies of so-called CER - coherent edge radiation (or "chicane radiation") have been carried e.g. in [27] to demonstrate the application of CER as a coherent THz radiation source and as a nondestructive beam diagnostic tool. The similarities and differences of CR-HWC considered in our paper and CER [27] are as follows: a) the bent non-periodic trajectories of electrons in both cases, but very different radii of curvature: 1.2 m in the case of CER [27] instead of only about 900 micrometers [11] in the case of CR-HWC in our case); b) the *coherent* radiation from a 61 MeV *compressed electron bunch* [27], but the radiation from a single 255 MeV electrons (our paper); c) the huge difference in spectral regions: THz or $10^{-5} – 10^{-4}$ eV in the case of CER [27], but MeV in the case of CR- HWC from 255 MeV electrons. In view of above, further theoretical and experimental studies of CR-HWC from relativistic electrons are necessary, aiming especially at polarization properties, angular distributions and search for possible applications.

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