Cooling of ultrarelativistic β and μ particles by laser channels

A V Dik[1,3], **S B Dabagov**[1,3,4] **and E N Frolov**[1,2,3]

[1] LPI RAS, Moscow, Russia
[2] NR TPU, Tomsk, Russia
[3] NRNU MEPhI, Moscow, Russia
[4] LNF INFN, Frascati (Roma), Italy

E-mail: `dik.alexey@gmail.com`

Abstract. The descriprion of ultrarelativistic classical particles' movement in interference laser field formed by multichannel "sandwich" structures taking into account the radiative energy losses is present. The muon channeling case is described in detail. The critical angle for muon bound motion in the potential well of laser field is defined. The feasibility of beam cooling for charged particles due to radiation losses is shown.

1. Introduction

The investigation of charge particle dynamic in interference laser field presents great interest. The first article dedicated to this studies was published in the first half of 20^{th} century [1]. Since that many papers describing charge particles interaction with interference electromagnetic field have been published. In particular, new applications of the effect of charge particle interaction with interference electromagnetic field were suggested in [2, 3, 4, 5, 6, 7, 8]. Theoretical investigation of interaction of charge particle with a standing electromagnetic wave were performed in [9, 10, 11]. A new point of view was suggested in the paper [12], in which for describing such interaction mathematical formalism of theory of charge particle channeling in crystals was used [13]. Using the mathematical apparatus of theory of charge particle channeling in crystals the effective potential of channeling was derived that allowed investigation of charge particle interaction in various geometries of interference laser field and some new features, such as the inversion of potential, to be studied in detail. Till now all investigations were made without taking into account the radiation losses of the particle, and only electrons were considered. The charge particle dynamics including the radiation losses, and the probability of muon channeling in laser field are covered in this paper.

2. Muon channeling in laser field

In [14] relativistic electron dynamics in the field of two interfering plane polarized laser waves was considered. Nonzero components of total field formed by the lasers can be written in the form

$$E_x = 2E_0 \sin\alpha \cos(k\cos\alpha x) \sin(\omega t - kz\sin\alpha),$$
$$E_z = 2E_0 \cos\alpha \sin(k\cos\alpha x) \cos(\omega t - kz\sin\alpha), \tag{1}$$
$$H_y = 2E_0 \cos(k\cos\alpha x) \sin(\omega t - kz\sin\alpha),$$

where E_0 is the magnitude of electric field of a single laser wave, $\pi - 2\alpha$ is the angle between the wave vectors $\mid \mathbf{k} \mid= \omega_0/c$ of the lasers waves. The periodic channels can be formed for some specific conditions, and, in generally, they are capable to trap any charged particle. The effective potential describing interaction between the field and the charged particle of the mass m, the charge e, the longitudinal Lorentz-factor $\gamma_{\|}$ and the longitudinal velocity $\beta_{\|}$, in respect to the channel axis, which coincides with the Oz-axis, has the form

$$U_{eff}(x) = U_0(\beta_{\|}, \alpha) \cos(2kx\cos\alpha), \tag{2}$$

where

$$U_0(\beta_{\|}, \alpha) = \frac{e^2 E_0^2(-\cos(2\alpha) - 2\beta_{\|}\sin\alpha + \beta_{\|}^2(1+\cos^2\alpha))}{2\gamma_{\|} m\omega'^2}, \tag{3}$$

$$\omega' = \omega_0(1 - \beta_{\|}\sin\alpha) \tag{4}$$

As seen, the depth of a channel for muon less than for electron, with the same longitudinal Lorentz-factor, because of the muon mass is more than the electron one. Because of it, the only difference between muon and electron interactions with laser channels is quantitative and not qualitative. For example, let consider the critical angle of channeling defined by the following expression

$$\theta_{cr} = \frac{p_{\perp}^{max}}{p_{\|}} \approx \sqrt{\frac{2U_0(\beta_{\|}, \alpha)}{\gamma_{\|} mc^2}} \tag{5}$$

For muon in the field of a standing wave formed by the micrometer laser it can be written in a convenient form for the estimates

$$\theta_{cr}[rad] \approx 1,15 \cdot 10^{-10} \frac{\sqrt{I}}{\gamma_{\|}} \left[\frac{W}{cm^2}\right] \tag{6}$$

In the expression (6) the laser intensity is given in W/cm^2, while the critical angle measured in radians. In this way, for the muon with $\gamma_{\|} = 10^3$ and for the laser intensity $I = 10^{16} \div 10^{20}$ W/cm^2 the critical angle is $\theta_{cr} = 10$ μrad $\div$ 1 mrad. The ratio of muon and electron critical angles is the same as their masses $m^{\mu}/m^{e} \approx 200$. In [12] the estimation of the electrons critical concentration, which can be held by channel field, gives

$$n_0^e \leq \frac{2\gamma_{\|} I}{m\omega_0^2 c d_{ch}^2} \tag{7}$$

As one can conclude from expression (7) the difference in critical angles for muon and electron is defined by the mass, and the range of the laser intensity above the range of the critical concentration for muon is $10^{19} \leq n_0^{\mu} \leq 10^{23}$ cm^{-3}. It should be underlined, the expression (7) valid for non-disperse bunch. For a real bunch the critical concentration is typically less than (7).

3. Radiation losses

Let consider the motion of relativistic charge particle in external laser field. The equation of motion including energy losses is written in the following [15]:

$$mc\frac{du^i}{ds} = \frac{e}{c}F^{ik}u_k + g^i, \tag{8}$$

where F^{ik}, u_k, g^i, s are the electromagnetic field tensor, the 4-vector component of velocity, the 4-vector component of stopping force and the proper length of trajectory, respectively. Hereinafter, lower and upper indexes for 4-vectors correspond to covariance and contravariance components, meanwhile for the space vectors the upper index defines a power, unless it is otherwise specified. For the high energy particle the expression for stopping force can be rewritten in the next form (for convenience we use the 4-vector of momentum instead of the velocity $P^i = mcu^i$, and the 4-vector of momentum is upper-case)

$$g^i = \frac{2e^4}{3m^5c^8}P^i\left(F_{kl}P^l\right)\left(F^{km}P_m\right), \quad P^i = \left(\frac{E}{c}, \mathbf{p}\right), \tag{9}$$

where $E, \mathbf{p}$ are the energy and the space momentum of particle, respectively. Let pass from the proper length to the time, then (8) can be presented in the form

$$\frac{dP^i}{dt} = \frac{e}{c}F^{ik}\frac{P_k}{P^0} + \frac{mc^2}{P^0}g^i \tag{10}$$

Following the technique of our previous papers let present the particle motion as a sum of slow smooth trajectory $\bar{\mathbf{r}}$ and 4-vector of momentum $\bar{P}^i$, and fast oscillation trajectory $\mathbf{r}_\xi \ll \bar{\mathbf{r}}$ and momentum $P^i_\xi \ll \bar{P}^i$. Hereinafter the index ξ denominates the fast oscillation values. In this way for the fast oscillation motion in first approximation we can write

$$\frac{dP^i_\xi}{dt} = \frac{e}{c}F^{ik}\frac{\bar{P}_k}{\bar{P}^0} \tag{11}$$

and for the smooth motion

$$\frac{d\bar{P}^i}{dt} = \frac{e\bar{P}_k}{c\bar{P}^0}\overline{(\mathbf{r}_\xi, \nabla F^{ik})} + \frac{e}{c\bar{P}^0}\overline{F^{ik}\left(P^\xi_k - \frac{\bar{P}_k}{\bar{P}^0}P^0_\xi\right)} + \frac{mc^2}{\bar{P}^0}\bar{g}^i + \frac{mc^2}{\bar{P}^0}\overline{\delta g^i}, \tag{12}$$

where

$$g^i = \frac{2e^4}{3m^5c^8}\bar{P}^iF_{kl}\bar{P}^lF^{km}\bar{P}_m, \tag{13}$$

$$\delta g^i = \frac{2e^4}{3m^5c^8}\Big(P^i_\xi F_{kl}\bar{P}^lF^{km}\bar{P}_m + \bar{P}^i\Big(\bar{P}^l\bar{P}_mF^{km}(\mathbf{r}_\xi, \nabla)F_{kl} + F_{kl}F^{km}\bar{P}_mP^l_\xi + \\ + F_{kl}\bar{P}^l\bar{P}_m(\mathbf{r}_\xi, \nabla)F^{km} + F_{kl}F^{km}\bar{P}^lP^\xi_m\Big)\Big) + \frac{P^0_\xi}{\bar{P}^0}g^i \tag{14}$$

The g^i is nonzero in Eq. (12) because of momentum squared, therefore after averaging this doesn't equal zero, in generally. In the equation for fast oscillation we neglected influence of the energy losses as follows from the equations above derived. Till now we did not consider the period of averaging, which depends on longitudinal velocity in respect to direction of laser wave propagation [12, 14]. The period of fast oscillations is defined as $T = 2\pi/\omega'$, where ω' is the frequency of fast oscillations (4). Neglecting the losses the longitudinal velocity does not change. In reality that does not work. Nonetheless, we assume longitudinal velocity is constant during the time of one small oscillation. Let consider relativistic particle moving with high speed along the channel axis (Oz-axis), and low velocity along the transverse axis (Ox-axis), i.e. $P^0 \sim p_\parallel \gg p_x$. Then the component of 4-vector can be rewritten as follows

$$P^0 = \gamma mc = mc\sqrt{1+\frac{\mathbf{p}^2}{m^2c^2}} \approx \bar{P}^0 + P^0_\xi, \tag{15}$$

$$P^1 = \bar{p}_x + p^\xi_x, \quad P^2 = 0, \quad P^3 = \bar{p}_\parallel + p^\xi_\parallel, \tag{16}$$

where

$$\bar{P}^0 = mc\sqrt{1+\frac{\bar{p}^2_\parallel}{m^2c^2}} = \bar{\gamma}_\parallel mc,$$

$$P^0_\xi = \frac{\bar{p}_\parallel p^\xi_\parallel}{(\bar{P}^0)^2} = \frac{\bar{p}_\parallel p^\xi_\parallel}{mc\bar{\gamma}_\parallel}$$

Taking into account these approximations losses, we can find the following expressions for the components of space stopping force

$$\begin{aligned} f_x = &-\frac{2e^2}{3m^4c^6}\left[\gamma^2_\parallel m^2 c p_x \omega'^2\left(\frac{(eE_0)^2}{\gamma_\parallel m\omega_0^2}+2U_{eff}\right)+\right. \\ &\left.+\left(\frac{\partial U_{eff}}{\partial x} - 2\beta_\parallel p_x mc\frac{\omega_0}{\omega'}\frac{\sin\alpha-\beta_\parallel}{\cos(2\alpha)}\frac{\partial U_{eff,0}}{\partial x}\right)\frac{\partial U_{eff}}{\partial x}\right], \end{aligned} \tag{17}$$

$$\begin{aligned} f_z = &-\frac{2e^2}{3m^4c^6}\left[\gamma^2_\parallel m^2 c p_\parallel \omega'^2\left(\frac{(eE_0)^2}{\gamma_\parallel m\omega_0^2}+2U_{eff}\right)-\right. \\ &-2\gamma_\parallel mc(1+\beta_\parallel^2)\frac{\omega_0}{\omega'}\frac{\sin\alpha-\beta_\parallel}{\cos(2\alpha)}\frac{\partial U_{eff,0}}{\partial x}\frac{\partial U_{eff}}{\partial x}+ \\ &+\beta_\parallel\frac{(eE_0)^4}{2\omega'^2}\cos^2\alpha\left(1+\frac{2m\omega_0^2U_{eff,0}}{(eE_0)^2\cos(2\alpha)}\right)\left(3\left(\frac{\omega'}{\omega_0}\right)^2-\right. \\ &\left.\left.+\left(\frac{\cos^2\alpha}{\gamma_\parallel^2}-6\beta_\parallel\sin\alpha+3(\sin^2\alpha+\beta_\parallel^2)\right)\frac{2m\omega_0^2U_{eff,0}}{(eE_0)^2\cos(2\alpha)}\right)\right], \end{aligned} \tag{18}$$

where U_{eff} is the effective potential of relativistic particle (2), and $U_{eff,0}$ is the effective potential of nonrelativistic particle ($U_{eff,0} = U_{eff}(\beta_\parallel \to 0)$)

$$U_{eff,0} = -\frac{e^2E_0^2\cos(2\alpha)}{2m\omega_0^2}\cos(2kx\cos\alpha)$$

As seen from the expressions for the components of space stopping force (17) and (18), there is a significant contribution in the losses from both small oscillations under the influence of laser wave and channeling motion. The expression for stopping force includes both the effective potential and the derivative of effective potential. The losses take place when there are no channels. This happens due to the electron scattering in a laser field. The analysis of particle motion in the field of arbitrary geometry becomes difficult. Because of it we consider the standing wave geometry with $\alpha = 0$. In this case the expression for the stopping force is very simplified. Let consider the ultrarelativistic particle $\beta_\parallel \to 1$ in the field of laser channels and assume $p_\parallel \approx \gamma_\parallel mc$, then the equation of motion can be written in the following

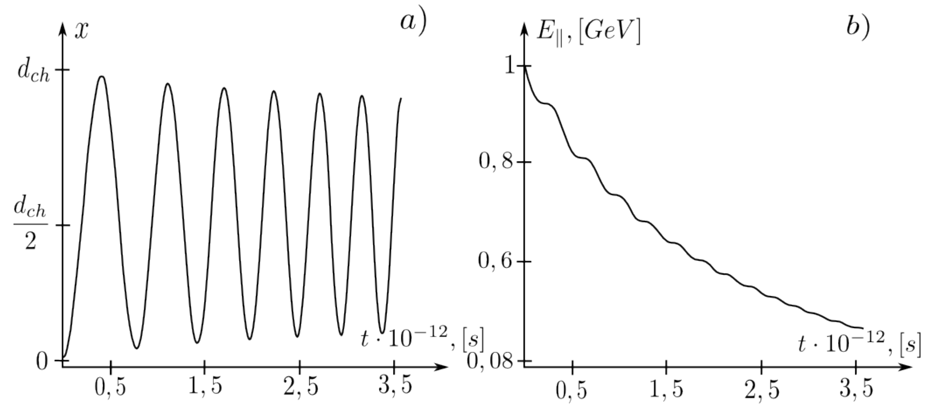

Figure 1. *a) The transverse coordinate x of electron versus time t; b) the longitudinal electron energy $E_{\|}$ versus time t in the field of micrometer wavelength laser with intensity of 10^{19} W/cm^2.*

$$\ddot{x} + \frac{e^6 E_0^4}{3\gamma_z m^5 c^7 \omega_0^2} \sin^2(2kx)\dot{x} + \frac{1}{\gamma_{\|} m}\frac{\partial U_{eff}}{\partial x} = 0, \tag{19}$$

$$\dot{\gamma}_{\|} = -\frac{2e^4 E_0^2 \gamma_{\|}^2}{3m^3 c^5}(1 + \cos(2kx)) \tag{20}$$

In the expression (20) the therms with ratio of effective potential and longitudinal energy are neglected at $U_{eff}/E_{\|} \to 0$. The solutions of the equations (19), (20) are shown in the figure 1. As seen, in such approximation there is no energy loss near the channel axis $x = \pi/(2k)$, while on the hill of potential the losses are maximal.

Hance, the maxima of the energy losses for electron (positron) and muon in the field of laser channels are given by

$$\left.\frac{dE_{\|}^e}{dz}\right|_{max} \approx 2,7 \cdot 10^{-16} I \gamma_{\|}^2, \quad \left[\frac{eV}{cm}\right] \tag{21}$$

$$\left.\frac{dE_{\|}^\mu}{dz}\right|_{max} \approx 7 \cdot 10^{-21} I \gamma_{\|}^2, \quad \left[\frac{eV}{cm}\right] \tag{22}$$

In the expressions above the laser intensity is expressed in W/cm^2. It worth mentioning that the (21) and (22) define energy losses per unit length at peaks of the effective potential, while a particle oscillating in the considered system moves from regions of high potential to regions of minimum potential. Hence, 1 GeV electron in the field of 10^{20} W/cm^2 intensity looses half of its energy at 0,3 mm length, while at the same length in the 10^{22} W/cm^2 intensity field 1 TeV muon looses only ~ 1 % of its energy.

Conclusions

The dynamics of classical charged particle in the field of laser channels including energy losses is considered. It is shown that the critical angle for muon is less than for electron as expected. The analytical expression for the stopping force influence on the charged particle in the channels of two plane polarized laser waves is obtained. A particular case of charged particle dynamics in a standing wave field is considered. The expression for the particle energy losses per length is derived. According to the equation (21) the electron energy losses per length unit are immense, while for muons (22) those are 5 orders less than for electron, but still essential. The use of the channels formed by lasers may become an effective mechanism for cooling and shaping ultrarelativistic charged particle beams.

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