Mathematical Model of Composite Fibre-Glass Aramide-Wired Cord Rheological Properties

Mikhail Pavlov[1, a)], Sergey V. Ponomarev[2, b)], Nikolay Maritsky[2, c)] and Sergey A. Ponomarev[2, d)]

[1]*National Research Tomsk Polytechnic University, 30 Lenin Avenue, Tomsk 634034 Russian Federation*
[2]*National Research Tomsk State University, 36 Lenin Avenue, Tomsk 634050 Russian Federation*

[a)] corresponding author:mspavlov@tpu.ru
[b)] psv@niipmm.tsu.ru
[c)] nnmar.niipmm@ mail.ru
[d)]doc.sergeyponomarev@gmail.com

Abstract. This paper describes the rheological properties of composite fibre-glass aramide-wired cords which, in its turn, are applied in large-sized structures for space systems. Based on experimental data a new mathematical model describing creeping and relaxation of composite cords is proposed. This model defines the operation time of the composite cords to be 15 years.

INTRODUCTION

Composite cords are being widely used in designing large spacecraft mechanical systems, such as space antenna reflectors with operation life of 15 or more years. It's aperture size could be 50 m or more. These systems should be deployable as the payload bay in the launch rocket is limited, i.e. not more than several meters. The most advantageous type is the cable - stayed shell structures. They are made of flexible cords and membrane elements, tensioned on rigid framework [1-4].

Reflector antennas with high-frequency bands involve precise parabolic geometrical shape (formed by cords elements) of reflecting surface. Cord elements should have fixed size reliability when on orbit. Described composite fibre-glass aramide-wired cords meet these requirements. The cords under discussion are preferable for space application due to its rather insignificant thermal-expansion coefficient. At the same time these cords exhibit rheological properties such as creeping and relaxation. According to experimental results the strain rate of these cords under constant loads depends on temperature.

For space antenna reflectors it is necessary to retain exact geometrical paraboloid shape. It depends on the orbital temperature. Thus, it is essential to consider a mathematical model of cord elements which would takes into account both the rheological properties and temperature.

Based on experimental data a mathematical model to predict the mechanical behavior of described cords was developed. It includes cord rheological properties and temperature effect.

PROBLEM STATEMENT

To describe the strain in viscoelastic materials, Boltzmann heritable viscoelastic theory based on the principle of superposition was applied. In general, relation associating strain and stress to time are expressed by Volterra integral equation of the second kind. In this case, these equations are invertible, i.e. one equation is a resolvent of another. In this case, the final equations should include the influence of the temperature itself. In one dimensional case the strain in composite cords is described by viscoelastic model as [5, 6, 8]:

Prospects of Fundamental Sciences Development (PFSD-2016)
AIP Conf. Proc. 1772, 060002-1–060002-6; doi: 10.1063/1.4964582
Published by AIP Publishing. 978-0-7354-1430-3/$30.00

$$\sigma(t)=E_0\left[\varepsilon(t)-\int_0^t R(t-\tau)\varepsilon(\tau)F\left[T(\tau)\right]d\tau\right]$$
$$\varepsilon(t)=\frac{1}{E_0}\left[\sigma(t)+\int_0^t K(t-\tau)\sigma(\tau)f\left[T(\tau)\right]d\tau\right] \quad (1)$$

where, E_0 - instantaneous elasticity modulus; $\sigma(t)$ and $\varepsilon(t)$ – function of time, describing the strain and stress state changes; , $R(t)$ and $K(t)$ – relaxation and creeping kernels. In this case, temperature effect is considered by functions $F[T(t)]$ and $f[T(t)]$; t - time.

Thus, the mathematical problem statement describing the viscoelastic properties of above – mentioned cords will be solved if the function type for the creeping and relaxation kernels and their parameters are selected. At the same time, rather simple types of creeping and relaxation kernels should be applied.

RESULTS AND DISCUSSION

During corresponding operation time, the cycles number of temperature changes is significant. Thus, to analyses the rheological properties of cords, it is supposed that the temperature is constant (does not depend of time) and equal, for example, averaged cycle value.

Under constant stress (creeping) and strain (relaxation), relation (1) could be as follows [7]:

$$\frac{\sigma_k-\sigma(t)}{\sigma_k}=F\left[T\right]\int_0^t R(t-\tau)d\tau,$$
$$\frac{\varepsilon(t)-\varepsilon_k}{\varepsilon_k}=f\left[T\right]\int_0^t K(t-\tau)d\tau, \quad (2)$$
$$\sigma_k=\sigma(\varepsilon_k,T=const),$$

where, σ_k – stress in k experiment; ε_k – strain in k experiment.

Stress-strain relation including temperature is as follows:

$$\sigma(\varepsilon,T)=E(T)(100\cdot\varepsilon)^{a(T)} \quad (3)$$

where, $E(t)$ – elastic modulus as a function of temperature approximated by cubic polynomial; $a(T)$ – model parameter being power function of temperature.

Based on experimental data, to identify the long-term cord mechanical behavior, relaxation and creep kernel functions are introduced:

$$R(t)=\frac{b}{\left[\mathrm{ch}(bt)\right]^2},$$
$$K(t)=\frac{b_1}{\left[\mathrm{ch}(b_1 t)\right]^2}, \quad (4)$$

where, $\mathrm{ch}(t)$ – hyperbolic cosine function; b, and b_1 constant kernel parameters defined by experiment.

To identify state equation resolvents, Volterra integral equation of second kind is considered [9,10]:

$$y(x)=\lambda\int_a^x K(x,t)y(t)dt+f(x), \qquad x\in\left[a,b\right];t\in\left[a,x\right], \quad (5)$$

where, $K(x,t)$ – kernel of integral equation; $f(x)$ – free equation term (continuous function); λ – numerical parameter.

Some particular equations (5) could be solved as differentiation ones. In this case, depending on the parameters differentiation formula is applied for the integral.

Equation (5) has an unique continuous solution at any x. This solution could be determined by the resolvents from

$$y(x)=f(x)+\lambda\int_a^x R(x,t,\lambda)f(t)dt \tag{6}$$

where, $R(x,t,\lambda)$ – resolvent for kernel $K(x,t)$ is defined as recurrence relation

$$\begin{aligned} R(x,t,\lambda) &= \sum_{n=1}^{\infty}\lambda^{n-1}K(x,t); \\ K(x,t)_{n+1} &= \int_t^x K(x,s)K_n(s,t)ds; \quad (n=1,2,...). \end{aligned} \tag{7}$$

Thus, for stated kernel the resolvent is:

$$K(x,t)=\frac{1}{[\cosh(t)]^2}$$

$$K_1(x,t)=K(x,t)=\frac{1}{(\cosh(t))^2}$$

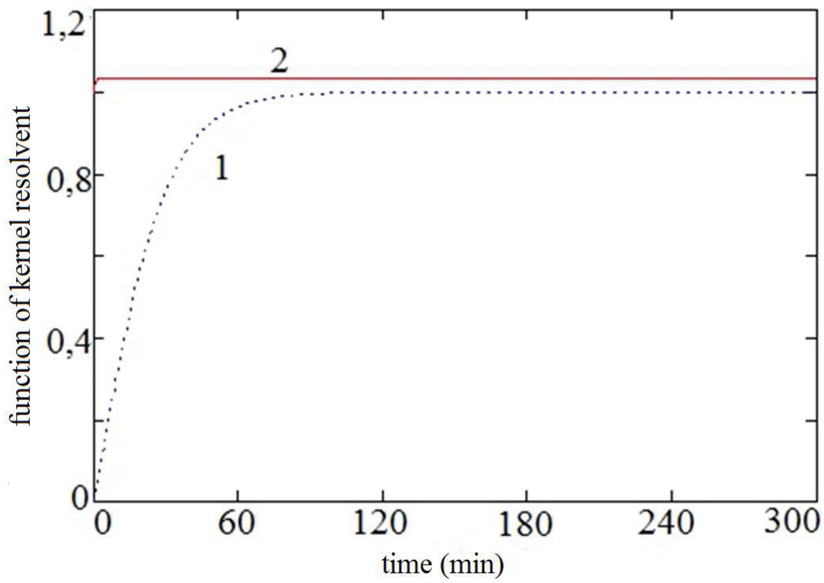
$$K_2(x,t)=\int_t^x K(x,s)K_1(s,t)ds=\int_t^x \frac{1}{(\cosh(s))^2}\frac{1}{(\cosh(t))^2}ds=\frac{1}{(\cosh(t))^2}(\tanh(x)-\tanh(t)) \tag{8}$$

$$K_n(x,t)=\frac{1}{[\cosh(t)]^2}[\tanh(x)-\tanh(t)]^{n-1}$$

$$R(x,t,\lambda)=\frac{1}{[\cosh(t)]^2}\sum_{n=1}^{\infty}\lambda^{n-1}[\tanh(x)-\tanh(t)]^{n-1}$$

If t=0 and parameter $\lambda=b$, then values of resolvents and relaxation kernels are equals. This, in its turn, could be practically used for significant values of current arguments.

Figure 1 shows the resolvent $R(x,t,\lambda)$ values (8) and function $R(t)$ for kernel $K(t)$ to time from the formulas (4)



1 – $R(t)$; 2 – $R(x,t,\lambda)$

FIGURE 1. Resolvent values $R(x,t,\lambda)$ (8) and function $R(t)$ for kernel $K(t)$ (4)

In the time interval of more than 60 minutes the error between relaxation kernel and creeping kernel resolvent is 3.7 %.

Suggested rheological cord properties in equations (2) describe the mechanical behavior as exhaustion creeping/relaxation to 600 minutes. Experiment data showed steady long-term creeping. Such creeping behavior could be associated with core material creeping of the cord core (quartz glass). The core, itself, supports the main load throughout the operation period. Thus, detailed description of above-mentioned rheological properties and the mechanical behavior of the cord itself during the operation period is based on the modified relation (2) by introducing an additional term retative to the core material creeping as:

$$\sigma(t) = E_0 \left[\varepsilon(t) - \int_0^t R(t-\tau)\varepsilon(\tau) F\left[T(\tau)\right] d\tau - \theta(t-t_0) \int_0^t R`(t-\tau)\varepsilon(\tau) F`\left[T(\tau)\right] d\tau \right],$$
$$\varepsilon(t) = \frac{1}{E_0} \left[\sigma(t) + \int_0^t K(t-\tau)\sigma(\tau) f\left[T(\tau)\right] d\tau + \theta(t-t_0) \int_0^t K`(t-\tau)\sigma(\tau) f`\left[T(\tau)\right] d\tau \right], \quad (9)$$

where, $R'(t)$ and $K'(t)$ – relaxation and creeping kernels of core material; functions $F[T(t)]$, $f[T(t)]$ – temperature influence on creeping/relaxation of core material; $\Theta(t\text{-}t_0)$ – Heaviside function; t_0 – time period during which creeping/relaxation of core material is insignificant (band on experimental data: t_0 = 120 minutes).

By virtue of the fact that predicted operation life is15 years, time t_0 (120 minutes) is insignificant. So, additional term (9) for creeping/relaxation of core material being introduced within-time period of more than 120 minutes would be more significant. Temperature influence on the rheological properties of core materials is insignificant, i.e. the function $F'[T(t)]$ and $f'[T(t)] = 1$. Thus, considering stress for creeping and strain for relaxation to be constant and applying previous assumptions, relations (9) could be as:

$$\frac{\sigma`_k - \sigma(t)}{\sigma`_k} = \int_0^t R`(t-\tau)\,d\tau,$$
$$\frac{\varepsilon(t) - \varepsilon`_k}{\varepsilon`_k} = \int_0^t K`(t-\tau)\,d\tau, \tag{10}$$

where, $\sigma`_k = \sigma(t_0)$ and $\varepsilon`_k = \varepsilon(t_0)$ are calculated by formulas (1)

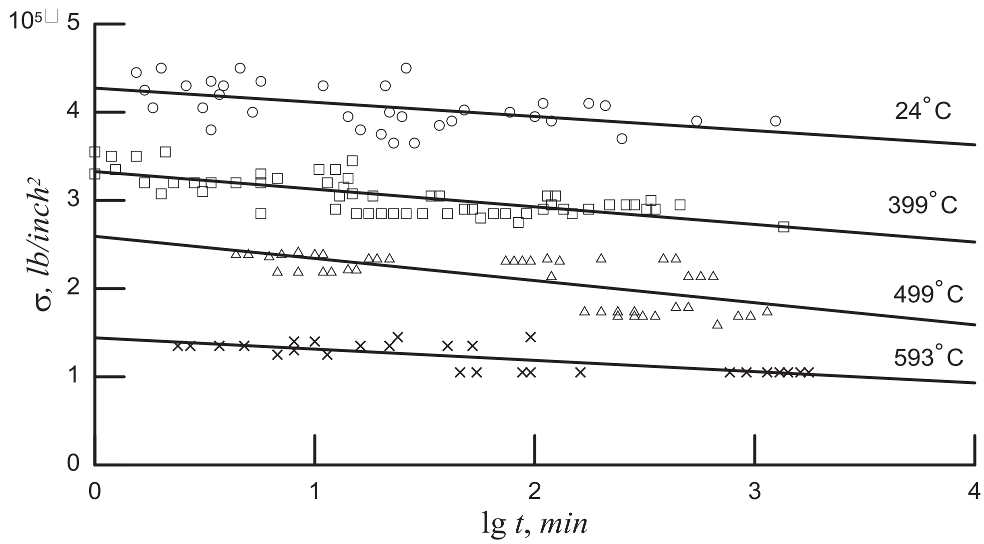
Based on research results [11], long-term strength of core material was determined as following: (Figure 2):

$$\sigma(t) = \sigma_k t^{\alpha}, \tag{11}$$

where, σ_k – stress at initial time α – parameter experimentally determined.

In view of the diagram linearity of material strain, it was concluded that in the case of core material under creeping, creeping kernel $K'(t)$ is:

$$K'(t) = Ae^{-\beta t} t^{\alpha-1}\text{, at } A = \alpha \text{ and } \beta = 0. \tag{12}$$



σ – strength at failure, t – time before failure

FIGURE 2. Long-term strength of quartz

According with (12), kernel resolvent (i.e. relaxation kernel) is:

$$R'(t) = \frac{1}{t}\sum_n \frac{[\alpha\Gamma(\alpha)]^n t^{\alpha n}}{\Gamma(\alpha n)}, \tag{13}$$

where, $\Gamma(\alpha)$ – gamma-function.

Thus, relation defining existing rheological behavior of spacecraft antenna cord elements is:

$$\sigma(t) = \mathrm{E}_0\left[\varepsilon(t) - \int_0^t R(t-\tau)\varepsilon(\tau)F[T(\tau)]\,d\tau\right]\left[1 - \theta(t-t_0)\int_0^t R`(t-\tau)\,d\tau\right],$$
$$\varepsilon(t) = \frac{1}{\mathrm{E}_0}\left[\sigma(t) + \int_0^t K(t-\tau)\sigma(\tau)f[T(\tau)]\,d\tau\right]\left[1 + \theta(t-t_0)\int_0^t K`(t-\tau)\,d\tau\right], \tag{14}$$

where E_0 – immediate elastic modulus; $\varepsilon(t)$ and $\sigma(t)$ – functions describing changing strain and stress states depending on time; $R(t)$ and $K(t)$ – relaxation and creeping kernels; $R'(t)$ and $K'(t)$ – relaxation and creeping kernels of core material (quartz glass identical for all proposed cored element types); functions $F[T(t)]$ and $f[T(t)]$ – temperature influence function on mechanical behavior; $\Theta(t-t_0)$ – Heaviside function, t_0 – time period during which creeping/relaxation of core material is insignificant.

CONCLUSION

Suggested mathematical model (14) described the experimental results of cord element strain for the time interval of up to $4 \cdot 10^4$ minutes (more than 550 hours) and provides possible prediction of the cord mechanical behavior throughout the whole spacecraft operating period including the temperature effect.

The experiment analysis results on creeping (at constant stress level) and relaxation (at constant strain level) for cords under different temperatures showed the applicability of proposed mathematical formula to describe rheological properties of cords for deployable spacecraft reflectors.

This model was designed for the unfavorable scenarios, where long-term creeping (relaxation) is steady, i.e. constant rate. However, in reality there is a probability of exhaustion creeping (relaxation) effects at low rate. For more precise prediction behavior of rheological properties of composite cord, further research experiments on long-term creeping (relaxation) are necessary.

ACKNOWLEDGMENTS

This work has been financially supported by Ministry of Education and Science of the Russian Federation. Unique identifier RFMEFI57814X0073

REFERENCES

1. T. Murphey, "Booms and Trusses," *Recent Advances in Glossamer Spacecraft*, edited by C. H. M. Jenkins, Progress in Astronautics and Aeronautics, Vol 212, (AIAA, Reston, VA, 2006), pp. 1 – 43.
2. Unfurlable Mesh Reflector Antennas see https://www.harris.com/solution/unfurlable-mesh-reflector-antennas
3. Astro Mesh see http://www.northropgrumman.com/businessventures/astroaerospace/products/pages/astromesh.aspx
4. A. Meguro, K. Shintate, M. Usui, A. Tsujihata, Acta Astronautica, **65**(9), (2009).
5. M. A. Koltunov. (in Russian – *Polzuchest i relaxacia*), (Vishaya shkola, Moskva, 1976).
6. E. I. Storovoitov and F.B. Nagiyev *Foundation of the theory of elasticity, plasticity, and viscoelasticity* (Apple Academic Press, 1st ed., 2012).
7. Y. N. Rabotnov (in Russian – *Elementi nasledstvennoi teorii mechaniki tverdih tel*) (Nauka, Moskow, 1977)
8. W. E. Morton and J. W. S. Hearle *Physical properties of textile fibres*. (Textile Institute, Manchester, England 2nd ed. London Heinemann [for] the Textile Institute, 1975).
9. G. Robert *The Elements of Real Analysis* (John Wiley & Sons, Inc., New York • London • Sydney 1967).
10. E. Zakon *Mathematical Analysis I, II* (University of Windsor, The Trillia Group West Lafayette, IN, 1975).
11. L. Browtman *Composite materials. Volume 5 Fracture and Fatigue* (Academic Press, New York, London, 1974).