

Modeling Large Pneumatic Reflectors Based on Innovative Materials

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Abstract. Large pneumatic structures of deploying space reflector are highly efficient and significantly advantageous due to the following reasons: significant coverage at low material usage and deployment simplicity. At the same time experimental modeling of such structures is time-consuming and costly. So, numerical analysis of such structures is very important in preliminary design. This paper presents the mathematical problem statement of stress-strain state of pneumatic structures, as well as the modal analysis results for two pneumatic reflectors of 50 and 100 meters in size applying such materials as Kevlar and thin polyamide films. The above described problems were solved by well-known nonlinear finite element method.

INTRODUCTION

Although large pneumatic structures are simple in design, they are the result of high-technology progress. Designing such structures has been in the minds of mankind for decades and only today, this idea has become realistic. Updated composite materials furthered the possibility of designing large pneumatic structures which include such properties as strength and durability. This structure type is significantly advantageous to the conventional ones. These structure systems are highly efficient due to the following reasons: significant coverage at low material usage and deployment simplicity. Pneumatic structures have a wide application spectrum from designing structures and its elements for the Earth to space operating functions.

Large pneumatic structures of deploying space reflector have been described in this paper. In designing any pneumatic reflector a few problems arise, i.e. estimating reflector surface accuracy, and stiffness level itself. So, the targets of the following paper are to determine the minimum reflecting surface root mean square (RMS) error relative to paraboloid, and stiffness level estimation by modal analysis. In addition, levels of stress and strain were estimated. Above-described problems were solved by nonlinear finite element method (using ANSYS software) for 50m and 100m structures.

These problems were solved because of the fact that increasing the size of space reflectors is tightly interconnected with the communication domain (for example, mobile communication). Designing large reflector antenna involves the following requirements: 2% RMS error of operating wavelength, distinct frequency interval, portability for transportation to space orbit and simplified deployment in space. All in all, this results in designing a fail-safe system. Designing such structure types as pneumatic space reflectors meets all the above-described requirements.

Although pneumatic reflectors have been widely studied [1 – 4], all existing research data remains non-public information. We consider that our research is a breakthrough in designing large pneumatic reflectors and this information is open for further discussion.

PROBLEM STATEMENT

Stress-strain state simulation problem of cable stayed shell structures is based on nonlinear elasticity theory, including thermal strain.

Deformation-displacement relation is considered as

$$e_{ij} = (1/2)(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j})$$

where, u_i – components of the displacement vector.

To simulate the mechanical behavior in stress state, simplified stress-strain relations were applied. Tensor components of Kirchhoff stress and tensor components of strain are as following:

$$\sigma_{ij} = a_{ijkl}(e_{kl} - e_{kl}^{Temp}),$$

where, $a_{ijkl}=a_{ijkl}(X,T)$ – elasticity tensor components related to different structure elements and their temperatures; $e_{kl}^{Temp} = \mathcal{G}\delta_{kl}\Delta T$ – strain temperature tensor components; \mathcal{G} – linear thermal expansion coefficient; δ_{kl} – Kronecker symbol; $\Delta T=T-T_0$ – temperature difference; T_0 – initial temperature; T – current temperature.

Equilibrium equations are:

$$[\sigma_{ij} \cdot (\delta_{ki} + u_{k,i})]_{,j} + P_i = 0,$$

where, P_i – mass force components.

Displacement boundary conditions are assigned to S_u structure surface, where $\mathbf{u}=0$ is the attachment location to spacecraft. Equilibrium equation for external S_p structure surface in space is:

$$\sigma_{ij}n_j(\delta_{ki} + u_{k,i}) = 0,$$

while on the Earth surface:

$$\sigma_{ij}n_j(\delta_{ki} + u_{k,i}) = p_a,$$

where, p_a – ambient pressure on Earth surface; n_j – components of the normal vector to the boundary surface.

If temperature field in structure elements is stated, then the solid mechanic problem is determined in a closed form as displacement function components correspond to the number of resulting equations.

If gas pressure p forces on internal pneumatic structures element surfaces is applied, then the boundary conditions are:

$$\sigma_{ij}n_j(\delta_{ki} + u_{k,i}) = p,$$

where, $p(\rho, T)$. Earth gravity is, $p=p_a+\Delta p$ if gas density is identical both on the pneumatic structure and on its surface. While in space it is $p=\Delta p$, where Δp is differential pressure relation between gas in and out of the structure.

In the above-mentioned problem statement, the initial state is the zero stress of the elements in the reflector structure. Due to prescribed boundary conditions pretensioned structure is in equilibrium state, which, in its turn, provides a rather precise designed reflector surface shape.

Analytically, such problem tasks are difficult to solve. It is more preferable to apply the variational approach so as to receive numerical solutions. Detailed variational approach is described in [5].

Root mean square (RMS) error of reflecting surface δ_{rms} is expressed as paraboloid axis error Δz [6]:

$$\delta_{rms,z} = \left[\frac{1}{S_a} \iint_{S_a} (\Delta z)^2 dS \right]^{1/2},$$

where, S_a – reflecting surface area.

This relation is a good approximation for any surface deflection in reference to the paraboloid approximation. In this case, it is preferable to apply the above-mentioned formula not only for radiometric surface deflection approximation, but also for imperfect surface gain loss [7].

In Cartesian coordinates (X,Y,Z) RMS error of structure nodes with coordinates (x_i, y_i, z_i) , $i=1, \dots, N$ is estimated by the integral formula:

$$RMS_z = \sqrt{\frac{1}{N} \sum_{i=1}^N \Delta z_i^2},$$

including $\Delta z_i = z_i - z_i^{ideal}$, where Δz_i – i-node deflection from paraboloid; z_i^{ideal} – theoretical value Z of i-node coordinate on paraboloid; N – number of points. It is presupposed that node distribution on reflecting surface influences the RMS error proportionally.

However, the systematic error is also involved in the RMS error:

$$\overline{\Delta z} = \frac{1}{N} \sum_{i=1}^N \Delta z_i .$$

This could easily exclude reflector displacement on Z axis. It is more suitable to use RMS error without the systematic error being calculated as:

$$RMS_z = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Delta z_i - \overline{\Delta z})^2}.$$

PNEUMATIC REFLECTOR STRUCTURE ELEMENTS

Figure 1 depicts 25 m pneumatic antenna model designed by L'Garde, Inc. [8]. The dome is of film "Mylar" or "Kapton" with a thickness of $0.2 \cdot 10^{-4}$ m; pneumatic torus and stays made of the same material, the thickness of which is $4 \cdot 10^{-4}$ m or even Kevlar for strengthening. The pressure in the dome is 2.8 Pa, while in the torus and stays – 5000 Pa. Offset reflector geometry and required coordinate system are depicted in figure 2 [9]. This is a base model to design finite element model (FEM) of 50 m and 100 m reflectors (Figure 3 and Figure 4).

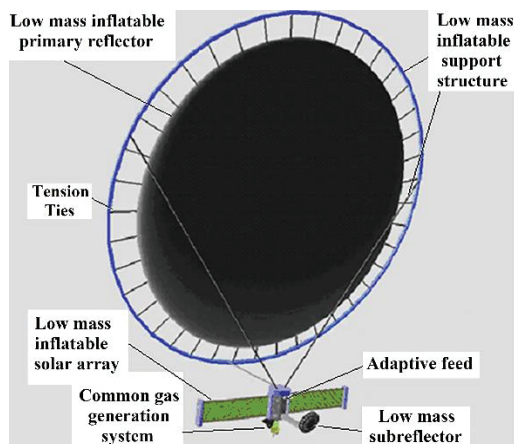


FIGURE 1. Project pneumatic antenna model “ARISE”

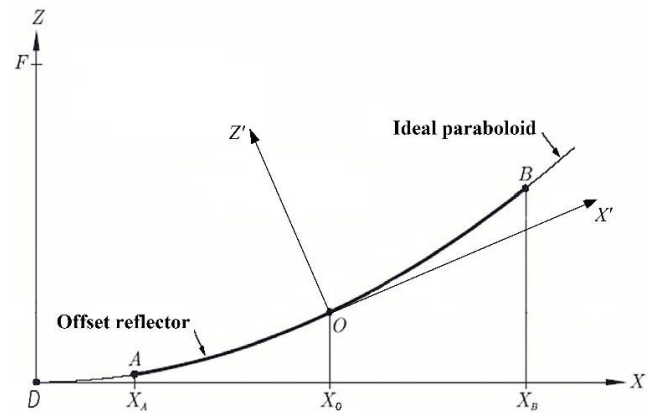


FIGURE 2. Geometry offset reflector

Reflecting surface is an intersection of ideal paraboloid and cylinder of diameter $X_B - X_A$ and its axis parallel to axis Z , where: X_A – distance from intersection border to axis Z reflector is clearance; O – intersection of cylinder axis and ideal paraboloid; F – focal point; and FD – focal distance. Offset paraboloid parameters are:

1. intersection cylinder diameter – 50 m, clearance – 3 m, focal distance – 25 m;
2. intersection cylinder diameter – 100 m, clearance – 3 m, focal distance – 50 m.

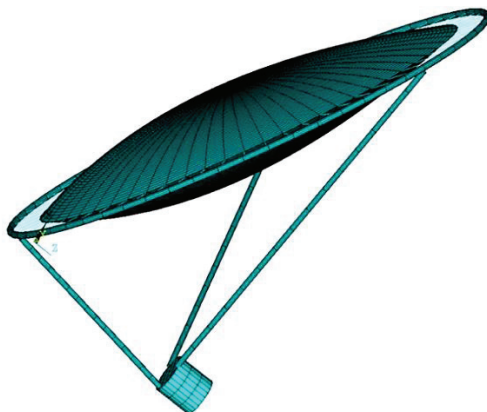


FIGURE 3. FEM reflector (full view)

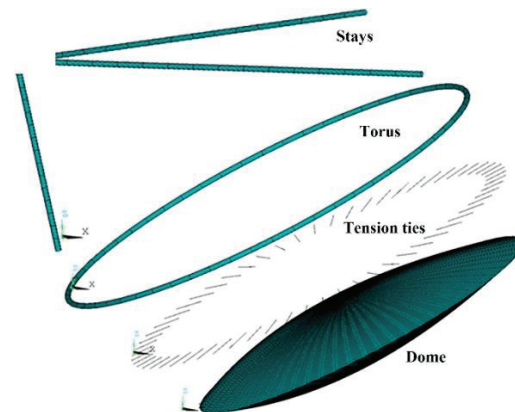


FIGURE 4. FEM reflector (separate elements)

Realistic non-complicated reflector model designed for finite-element calculations in reasonable time was required for simplification. Such simplification is associated with dimension problem variation. For example, reflector dome material thickness of several microns is not considered in this case. As a result, the 2-dimensional problem is applied.

The main features of the finite element formulation for described structures are following:

- Cord elements (tension ties) are one dimensional (LINK10 in ANSYS). These elements behave in tension only providing geometrical non-linear behavior for whole structure.
- Membrane elements (stays, torus and dome) are two dimensional (SHELL181 in ANSYS). The bending stiffness option for these elements is not considered.

More detailed descriptions of above-mentioned elements could be found in ANSYS help system.

Material properties of elements are isotropic. Pneumatic dome includes 2 interconnected offset paraboloids made from membrane “Kapton”. One paraboloid surface is covered with radiation coating of 100 nm aluminium oxide (Al_2O_3). Membrane thickness of modeled dome is not more than $15 \cdot 10^{-6}$ m with Young modulus of $2.5 \cdot 10^9$ Pa and Poisson ratio of 0.3. Internal pressure is not more than 0.5 Pa, if intersection cylinder diameter is 50 m; and if intersection cylinder diameter is 100 m, then this pressure is not more than 1 Pa. Pneumatic torus and stays provide structure stiffness which are of the same material, but thicker, i.e. $3 \cdot 10^{-4}$ m with Young modulus of $3.2 \cdot 10^{11}$ Pa and Poisson ratio of 0.3.

Reflector of 50/100 m includes: torus radius – 0.26/0.4 m; stays – 0.4/0.55 m; pressure inside torus and stays 500/800 Pa. Tension ties connecting torus and dome are Kevlar or carbon composite cords strings of $5.024 \cdot 10^{-5}$ m² in cross-section with Young modulus $1.5 \cdot 10^{11}$ Pa and Poisson ratio 0.3. Structure mass reflector of 50/100 m – less than 400/1200 kg, respectively.

RESULTS AND DISCUSSION

Initial geometry of reflecting surface is necessary in designing pneumatic reflector. This is aligned with the fact that existing ideal offset paraboloid shape under pressure (in the reflector dome) is distorted and the reflecting surface, in this case, becomes inapplicable as a functional structure.

Further, it is necessary to determine the initial geometry of reflecting surface by solving stationary problem as following:

- obtain stress-strain state of the reflector structure after gas inflation;
- write all node displacement values Δx , Δy and Δz of reflecting surface;
- adjust initial paraboloid surface to above values with reversed sign.

Practically, a newly paraboloid is designed, involving adjusted displacement values on node coordinates. This requires several iterative steps to determine the optimal reflecting surface shape. The number of iterative steps directly depends on the distortion intensity of the initial offset paraboloid shape. Reflecting surface node displacements relative to ideal paraboloid for reflector of 50 m in Z-direction is illustrated in figure 5. Figure 5(a) shows these displacements before the first iterative shape adjustment, where RMS error equal 58 mm. In this case the permissible RMS error value for such structures is 5 – 7 mm (i.e. 2 – 3 % of operating wavelength). Figure 5(b) shows these displacements after the first iterative shape adjustment where RMS error equals 21 mm.

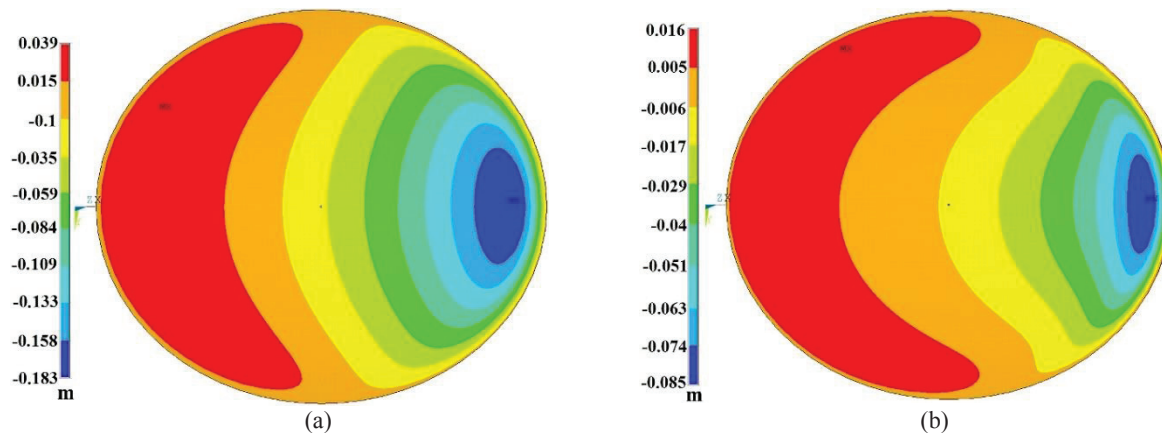


FIGURE 5. Reflecting surface node displacements relative to ideal paraboloid for reflector of 50 m in Z-direction:

- (a) before first iterative shape adjustment (RMS error = 58 mm)
 (b) after first iterative shape adjustment (RMS error = 21 mm)

The second iterative shape adjustment of reflecting surface shows RMS error equal to 11 mm. Reflecting surface node displacement relative to ideal paraboloid in Z-direction after the second iterative shape adjustment is illustrated in Figure 6(a). Figure 6(b) shows the values of reflecting surface node displacement relative to ideal paraboloid in Z-direction after the third iterative shape adjustment.

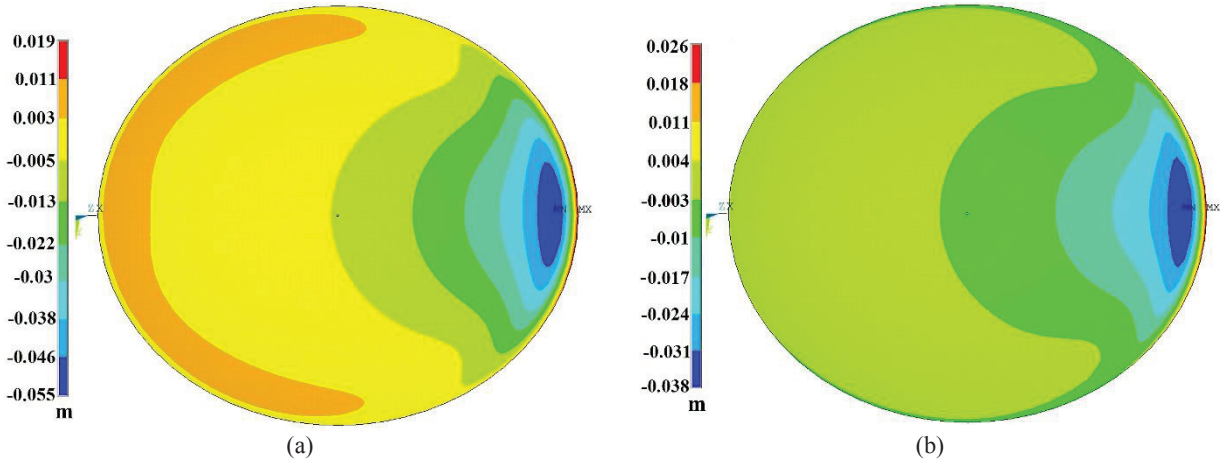


FIGURE 6. Reflecting surface node displacements relative to ideal paraboloid for reflector of 50 m in Z-direction:
 (a) after second iterative shape adjustment (RMS error = 11 mm)
 (b) after third iterative shape adjustment (RMS error = 7 mm)

Similar shape adjustment calculations are used for the reflector of 100 m in Z-direction. Figure 7 illustrates the reflecting surface node displacements relative to ideal paraboloid in Z-direction after the last iterative shape adjustment. In this case two reflectors of 50 and 100 m were compared. Reflector of 50 m involves RMS error equal 4 mm, while reflector of 100 m RMS error is 9 mm.

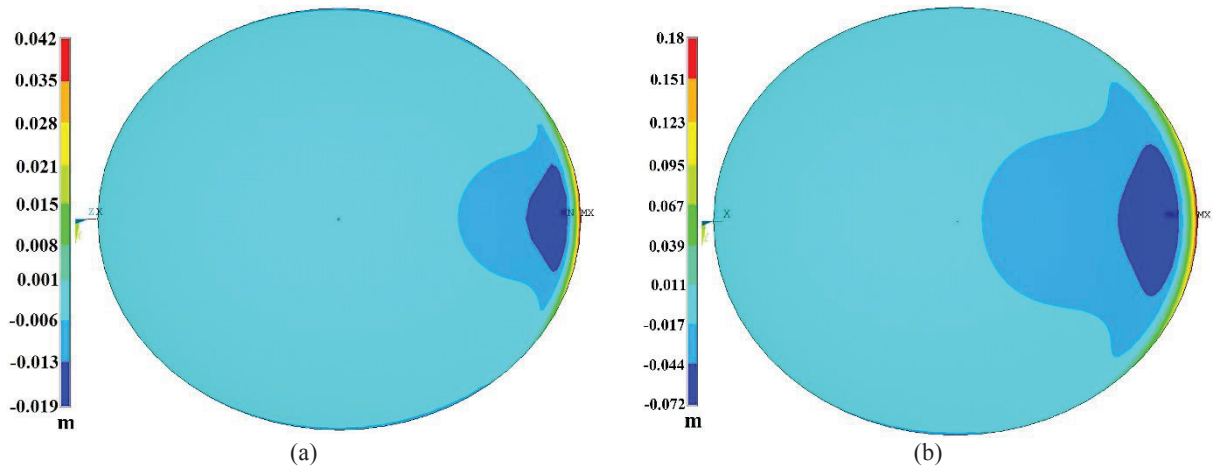


FIGURE 7. Reflecting surface node displacements relative to ideal paraboloid in Z-direction after last iterative shape adjustment:
 (a) reflector of 50 m (RMS error = 4 mm)
 (b) reflector of 100 m (RMS error = 9 mm)

Modal analysis was used for these structures to determine their stiffness. Result data for reflector of 50 /100m was as following: minimum natural frequency 0.41/0.19 Hz, corresponding to maximum effective mass. Mode shapes correspond to above-mentioned natural frequencies are depicted in figure 8.

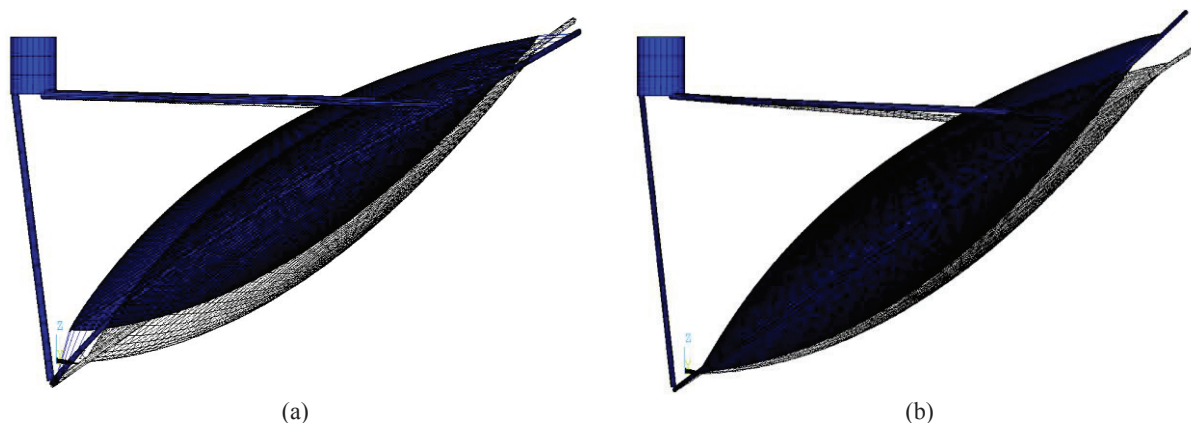


FIGURE 8. Modal analysis:
 (a) reflector of 50 m (mode shape with natural frequency of 0.41 Hz)
 (b) reflector of 100 m (mode shape with natural frequency of 0.19 Hz)
 Dotted lines show initial structure position

Figures 9 and 10 show Von Mises stress and strain level in reflector structures 50m and 100m respectively. It could be seen that strain levels are less than 1%.

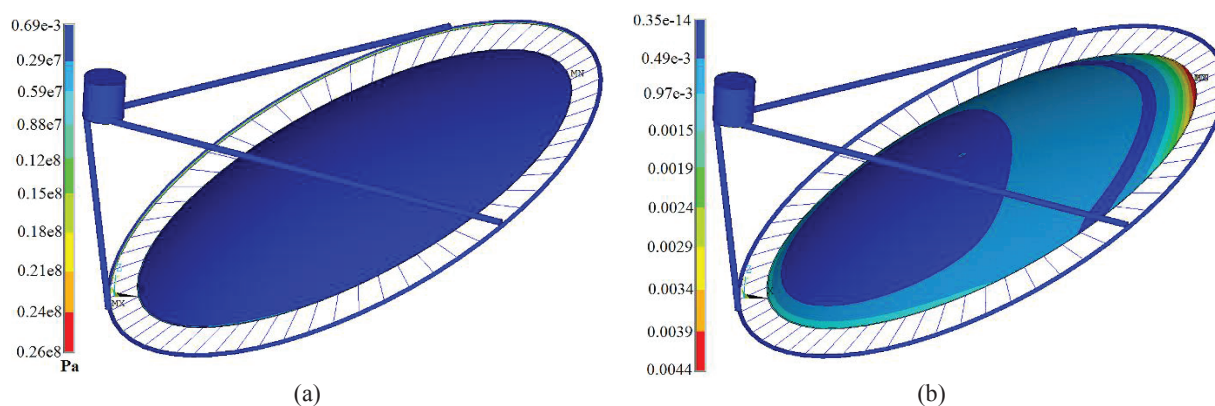


FIGURE 9. Reflector of 50 m
 (a) Mises stress
 (b) Mises strain

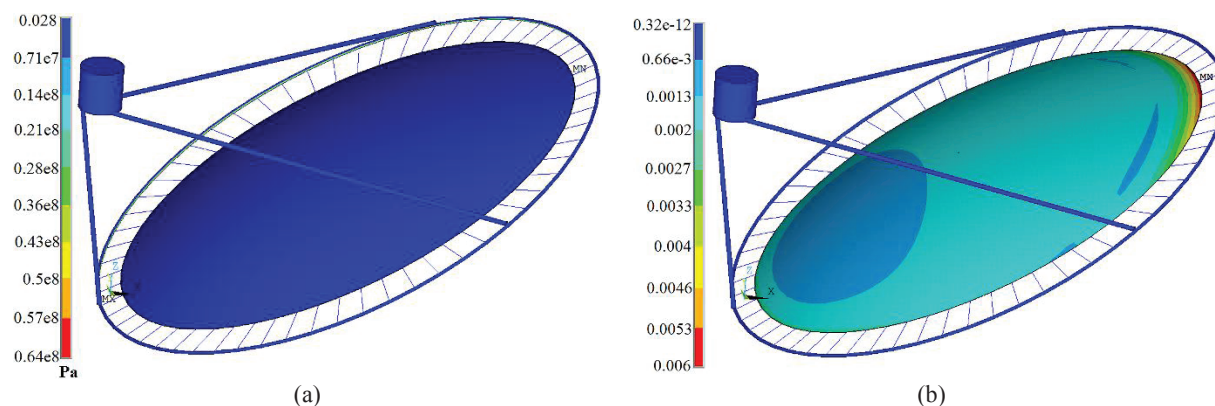


FIGURE 10. Reflector of 100 m
 (a) Mises stress
 (b) Mises strain

CONCLUSION

This research paper presented unique material based on stress-strain state simulation and modal analysis for 50 m/100 m pneumatic reflectors. A minimum RMS surface error for 50m/100m reflector structures was found. According obtained results, RMS error was determined as 4 mm/9 mm, respectively. However, further research is necessary to obtain RMS error (5 – 7 mm) for reflectors of 100 m. In addition, stress and strain levels were estimated. It was shown that strain levels are less than 1% for 50m/100m reflector structures.

For stiffness estimation a modal analysis was done. First natural frequencies with respective mode shapes were obtained.

Obtained numerical results (minimum RMS error, natural frequencies with mode shapes) could be the benchmark in designing operating systems of above-mentioned structures (i.e. large pneumatic reflectors).

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