Calculation Method for Cable-Beam Shell Structures

Sergey Belov[1, a)], Mikhail Pavlov[1,2, b)], Viktor Ponomarev[1, c)], Sergey Ponomarev[1,d)] and Andrey Zhukov[1,e)]

[1] *Tomsk State University, 36 Lenin Avenue, Tomsk 634050 Russian Federation*
[2] *Tomsk Polytechnic University, 30 Lenin Avenue, Tomsk 634050 Russian Federation*

[a)] corresponding author: belovsv@niipmm.tsu.ru
[b)] mspavlov@tpu.ru
[c)] vsponomarev@niipmm.tsu.ru
[d)] psv@niipmm.tsu.ru
[e)] zh@niipmm.tsu.ru

Abstract. This paper presents a calculation method suitable for cable-beam shell structures. It is based on both nonlinear finite element and force density methods. The main idea is to define the solution sequence for stress – strain state problem of above mentioned structures by nonlinear finite element method. Every successive solution involves the previous one as an initial estimate in convergent domain. To find an initial estimate for the first solution a force density method is used. The proposed method is tested on a new large space umbrella reflector.

INTRODUCTION

Cable-beam shell structures (abbreviated as CBSS) are widely used in modern construction building and architecture. This can be explained by the fact that such structures not only embrace significant space area but also are lightweight. Stadium roofs, tents, cable bridges, and large space antenna reflectors are examples of CBSS. A specific feature of CBSS is its geometrical non-linear behavior, i.e. significant (comparable with its size) displacements of its elements under external loads [1]. Therefore, numerical analysis of such systems, including geometrical nonlinearity, is an important step in its designing. There are research books and papers related to designing CBSS. The most interesting are [2,3] which describe up-dated approaches in form-finding of cable structures and their optimization.

The present paper presents a calculation approach for CBSS strain-stress state problem based on both force density and nonlinear finite element methods (FEM, FDM) [2-7]. The main idea of the proposed method is a construction of the solution sequence (based on nonlinear FEM) where every next solution involves the previous one as initial estimate in certain convergence domain. To obtain the initial estimate for the first solution a FDM is used. Such an approach introduced since an initial estimate randomly selected could result in a divergent solution.

PROBLEM STATEMENT

The essential relations, which describe stress-strain state of CBSS are equilibrium equations, strain tensor and Hooke's law (1) - (3):

$$\frac{\partial}{\partial x_k}\left(\sigma_{kj}\left(\delta_{ij}+\frac{\partial u_i}{\partial x_j}\right)\right)=0, \tag{1}$$

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$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_j}\right), \tag{2}$$

$$\sigma_{ij} = \frac{E_m}{2(1+\nu_m)}(\varepsilon_{ij} + \frac{\nu_m}{1-2\nu_m}\delta_{ij}\varepsilon_{kk}) + \sigma^0_{ij} \tag{3}$$

where, u_i, σ_{ij}, σ^0_{ij}, ε_{ij} – displacement vector components, second Piola-Kirchhoff stress tensor; initial stress tensor and strain tensor, respectively; E_m, ν_m – elasticity modulus and Poisson ratio of m-material, respectively.

Boundary conditions for (1) - (3) are:

$$u_i(\mathbf{x}) = u_i^0(\mathbf{x}), \mathbf{x} \in \partial\Omega_u \tag{4}$$

$$n_k\sigma_{kj}\left(\delta_{ij} + \frac{\partial u_i}{\partial x_j}\right) = p_i^n(\mathbf{x}), \mathbf{x} \in \partial\Omega_\sigma \tag{5}$$

where, $\mathbf{x}=\{x_1, x_2, x_3\}$ – coordinates vector of CBSS points in continuum Ω; $u_i^0(\mathbf{x})$ - displacement on the boundary $\partial\Omega_u$; $p_i^n(\mathbf{x})$ - stress on the boundary $\partial\Omega_\sigma$ which is characterized by normal vector $\mathbf{n}$.

CALCULATION METHOD

The above-mentioned calculation method for problem (1) - (5) is depicted on Figure 1:

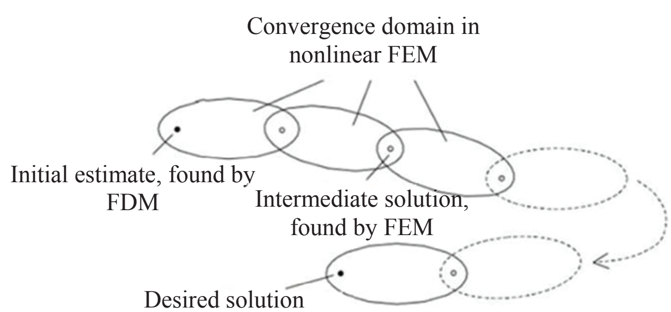

FIGURE 1. Calculation method scheme.

It is known that in FEM the nonlinear equilibrium equation relative to displacement field is essential. It is linearized and solved by the iterative Newton-Raphson method, where the following equation is obtained:

$$\mathbf{Ku} = \mathbf{P} \tag{6}$$

where, $\mathbf{K}$ - stiffness matrix; $\mathbf{u}$ - unknown displacement vector; $\mathbf{P}$ - external loads vector.

Resolution of (6) is:

$$\mathbf{u} = \mathbf{K}^{-1}\mathbf{P} \tag{7}$$

If the displacement field (7) is known we can calculate derivative values such as tension, stress and strain and so on for CBSS. But calculated tension values for cable elements could be insignificant, which, in its turn, could result in small values of stiffness matrix elements. Thus, there is a problem to find $\mathbf{K}^{-1}$ in (7). Also, one should remember that initial estimate should be determined in any iterative method.

FDM allows to find a good initial estimate because it identifies shape of tensioned cable elements in equilibrium with certain constrains. It was developed by Linkwitz and Schek for designing the Munich Olympic Stadium roof in 1971 [4]. Current FDM developments could be found in [8-11]. FDM is based on linearized equilibrium equations for tensioned linear cable elements. These elements are connected into nodes which are subjected to external loads. Some nodes are fixed (known nodes) and others are free (unknown nodes) (Fig. 2).

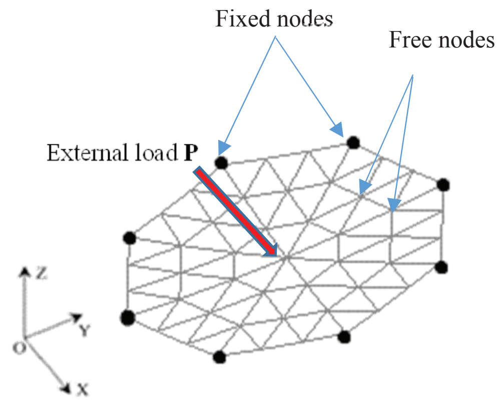

FIGURE 2. Cable elements under external load in FDM.

Let's consider arbitrary node i , which is subjected to external load $\mathbf{P}\left(P_x, P_y, P_z\right)$ (shown in Fig.1 as red arrow). Presuppose that the remaining cable elements nodes are fixed. In this case, equilibrium equations for *i* in Cartesian coordinates O*xyz* are:

$$\sum_{j=1}^{c_i} \frac{T_j}{l_j}\left(x_j - x_i\right) = P_x^i; \sum_{j=1}^{c_i} \frac{T_j}{l_j}\left(y_j - y_i\right) = P_y^i; \sum_{j=1}^{c_i} \frac{T_j}{l_j}\left(z_j - z_i\right) = P_z^i \tag{8}$$

where, x_i, y_i, z_i - unknown coordinates of node *i*; c_i - number of cable elements connected to *i*; x_j, y_j, z_j - coordinates of cable element fixed nodes connected to *i*.; T_j, l_j - tension value and length of *j*- element connected to *i;* P_x, P_y, P_z - coordinates of external load to axes O*x*, O*y*, O*z*.

As one can see, equations (8) are nonlinear to x_i, y_i, z_i . To linearize it a force density parameter $q_j = T_j / l_j$ is introduced. So (8) could be rewritten as (9):

$$\sum_{j=1}^{c_i} q_j\left(x_j - x_i\right) = P_x^i; \sum_{j=1}^{c_i} q_j\left(y_j - y_i\right) = P_y^i; \sum_{j=1}^{c_i} q_j\left(z_j - z_i\right) = P_z^i \tag{9}$$

Varying q_j and resoling (9) certain x_i, y_i, z_i could be found.

Equilibrium equations (9) are written for individual node *i*. Generalized matrix form of equilibrium equations could be found in [3,5].

In form-finding cable structures with specific constrains, for example, certain tension values for elements, nonlinear equations are introduced to describe these constrains:

$$\mathbf{g}^*(\mathbf{q}) = \mathbf{g}\left(\mathbf{x}(\mathbf{q}), \mathbf{y}(\mathbf{q}), \mathbf{z}(\mathbf{q})\right) = \mathbf{0} \tag{10}$$

where, $\mathbf{x}(\mathbf{q}), \mathbf{y}(\mathbf{q}), \mathbf{z}(\mathbf{q})$ - coordinate vectors of unknown nodes; $\mathbf{q}$ - force density vector for all elements. Thus it is necessary to define the force density vector $\mathbf{q}$ which satisfies both equilibrium equations in matrix form and relations (10). The vector $\mathbf{q}$ is determined by iterative Newton method through the following relation :

$$\mathbf{q}_1 = \mathbf{q}_0 + \Delta\mathbf{q} \tag{11}$$

where initial estimate is $\mathbf{q}_0$. To determine $\Delta\mathbf{q}$, nonlinear equations (10) are linearized by means of Taylor series within $\mathbf{q}_0$. As a result, the linear equation system relative to $\Delta\mathbf{q}$ is obtained:

$$\mathbf{G}^T \Delta\mathbf{q} = \mathbf{r}, \tag{12}$$

where, $\mathbf{G}^T = \partial \mathbf{g}^*(\mathbf{q}_0)/\partial \mathbf{q}$ - Jacobian matrix; $\mathbf{r} = -\mathbf{g}^*(\mathbf{q}_0)$. In general, linear equations (12) are not determined. So, it is necessary to find the minimum norm solution in all infinite solutions. This task is solved by Lagrange's method of multipliers which results in:

$$\Delta \mathbf{q} = \mathbf{G}\left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{r} \tag{13}$$

[3,5].

Defined coordinates $\mathbf{x(q)}, \mathbf{y(q)}, \mathbf{z(q)}$ and tensions $\mathbf{T} = \mathbf{Lq}$ in elements are used as an initial estimate in FEM for CBSS where, **L –** diagonal matrix of element's length.

In practice, the proposed method includes additional boundary conditions acting on the displacement field. For example, some nodes are fixed and all cable and shell elements are pre-tensioned. In this case, the displacement field for unfixed nodes is calculated. Further, some of these nodes become free, whereas cable and shell elements pre-tension values are obtained from the previous solution as an initial estimate and etc. This solution process will continue until desired solution with required boundary conditions is achieved.

It should be noted that the algorithm of what nodes should be unfixed is not formalized and depends on concrete CBSS model.

NUMERICAL EXAMPLE OF PROPOSED METHOD

Umbrella Reflector Finite Element Model Description

The proposed method is applied to a large space umbrella reflector (with diameter of 50m). Figure 3 shows its finite element model.

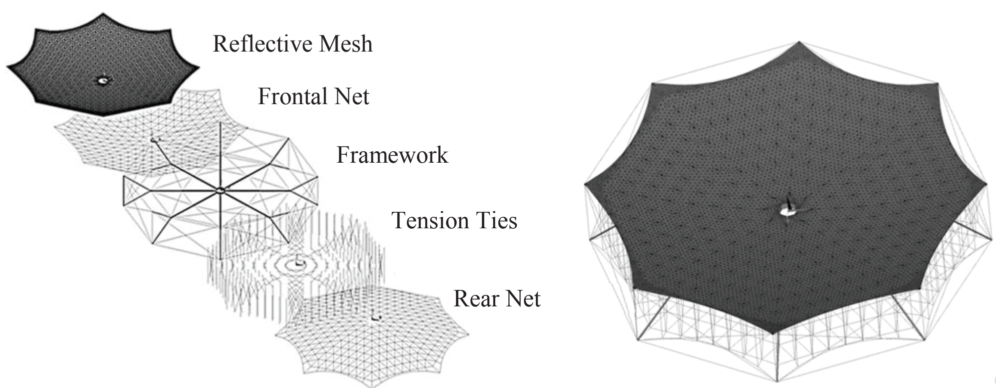

FIGURE 3. Finite element model of space umbrella reflector.

Finite element includes reflecting mesh constructed by shell elements without bending stiffness; frontal/rear nets are connected by tension ties; framework is eight Y-shaped spokes arranged in central hub and connected by cable elements (Fig. 3). Framework spokes members are constructed by shell elements and connected by beam elements, as illustrated in Figure 4.

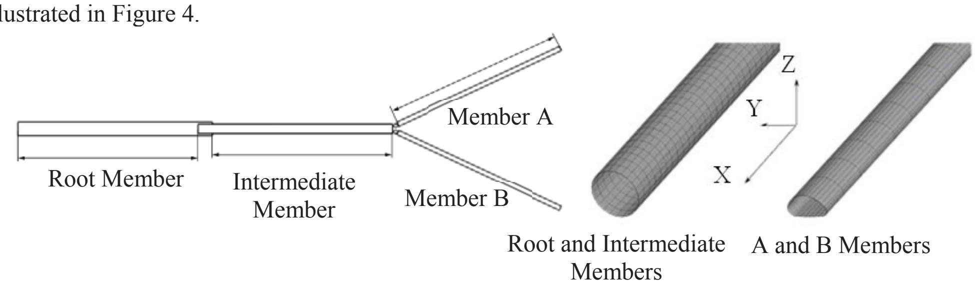

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FIGURE 4. Spoke elements.

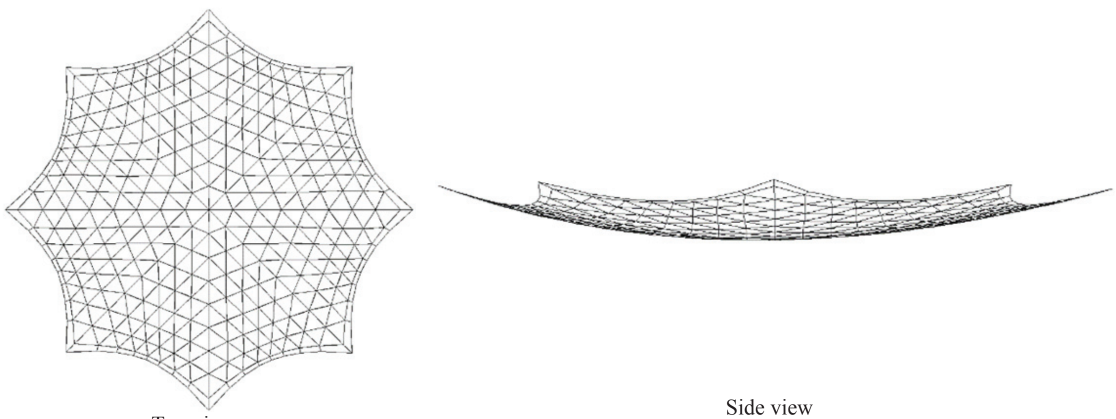
Reflecting surface shape is the intersection of revolution paraboloid and cylinder (so-called offset paraboloid). Cylinder and paraboloid axes are parallel and separated by a clearance value. Offset paraboloid equation could be found in [12].

Frontal/Rear net Form Calculated by FDM

Frontal/rear net form is calculated by FDM with Jacobian matrix for specific constraints on tension values [5].

Calculated net form and corresponding element tension values are illustrated in Figures 6 (a, b) and 7 (a-d), respectively. Accordingly 6 (a), internal frontal net cables should have tensions of 12N. Fixed nodes are considered to be the top net cables.

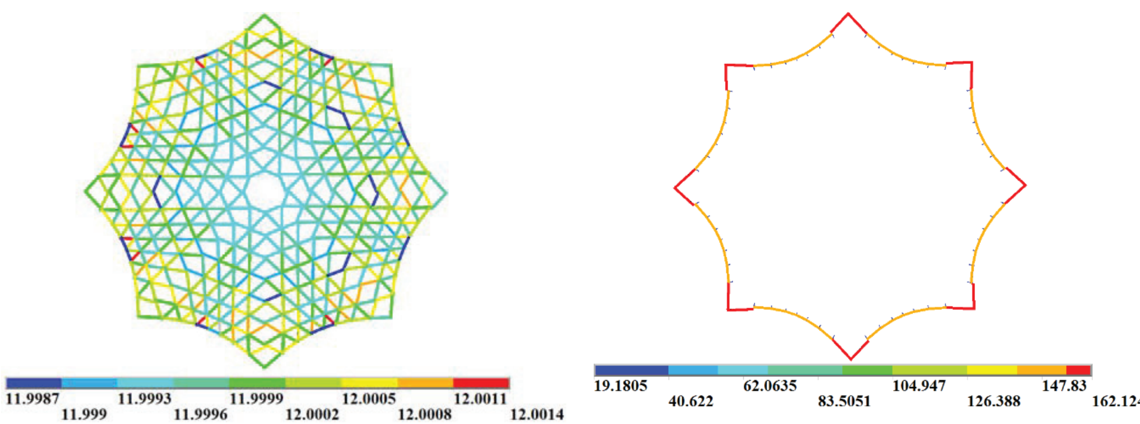
Obtained solution is used as an initial estimate in calculating displacement field for all unfixed reflector nodes.



Top view
(a)

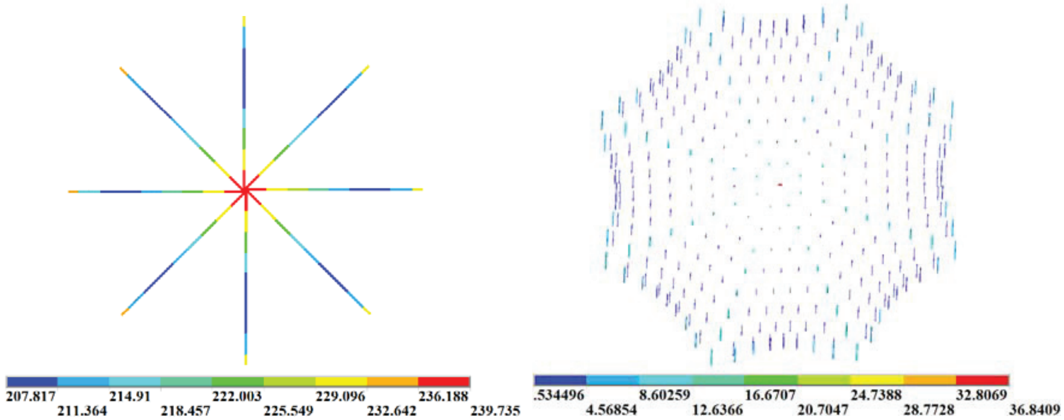
Side view
(b)

FIGURE 5. Frontal/rear net form calculated by FDM.



(a) Inner cable tension, N

(b) Peripheral cable tension, N



(c) Radial cable tension, N (d) Tension tie tension, N

FIGURE 6. Frontal/rear net, tension tie tension calculated by FDM

Solution Sequence for Reflecting Surface Displacement Field

The problem was to calculate the displacement field of reflecting surface nodes under following boundary conditions: central hub nodes are fixed; shell elements of reflecting surface are pre-tensioned to values of 2N/m; pretension value of shortest cable elements which connect A and B and others spokes members are 200N and 100N respectively. Frontal net form and corresponding pre-tension values of its elements are obtained by FDM and used as an initial estimate (Fig. 5 and Fig. 6).

To obtain the first (intermediate) solution, the hub nodes and A and B member nodes were fixed and above-described pretension values were applied. The corresponding result for displacement field of reflecting surface nodes is illustrated in Figure 7.

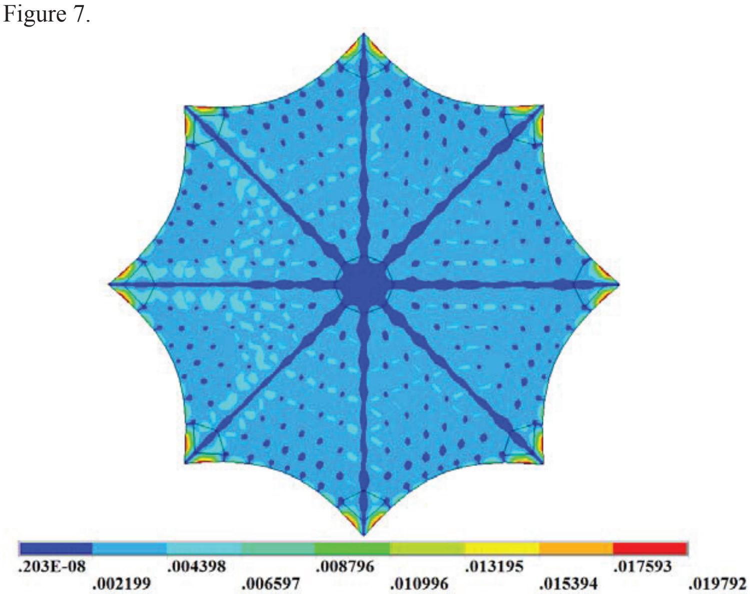


FIGURE 7. Reflecting surface nodes displacement field when A and B spoke member and central hub nodes are fixed (intermediate solution), m

To obtain the second (desired) solution, A and B member nodes were unfixed, whereas initial estimate was the first solution (i.e. displacement field for nodes, tension values of cable elements and so on).

The corresponding result for displacement field of reflecting surface nodes for second solution is illustrated in Figure 8.

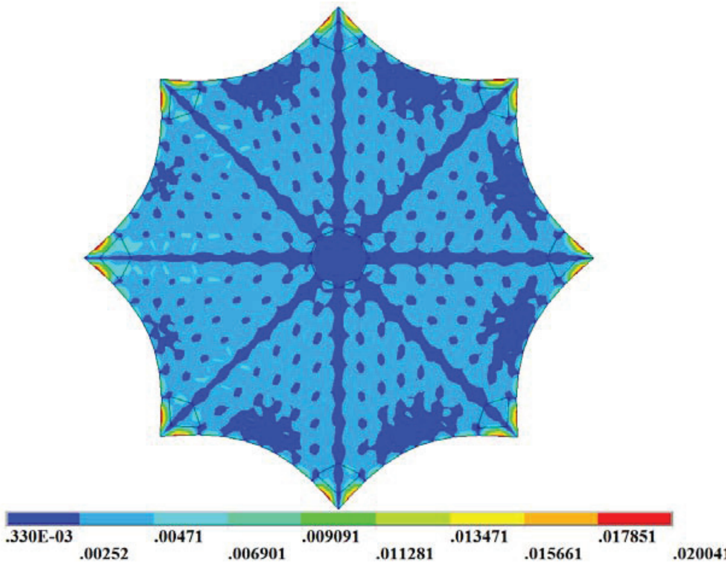


FIGURE 8. Reflecting surface nodes displacement field when central hub nodes are fixed only (desired solution), m

This example illustrates the fact that end-displacement of A and B members is more significant in the displacement field of reflecting surface nodes.

The pretension value of the shortest cable elements connecting A and B spoke members significantly influence on convergence. If this value less than 10N, the iterative Newton-Raphson process is divergent for above-described boundary condition.

Computation Time for Proposed Example

For above-described example, the total number of FEM elements is 88060. The number of line elements in FDM is 928.

Computer characteristics are following: processor Intel (R) Core (TM) i7-3960X CPU@ 3.30GHz; RAM is 32Gb. The number of processors which is used in parallel calculations is 10.

The FDM iterative process is used until $\|\mathbf{g}^*(\mathbf{q})\| < 0.001$.

So, under described number of elements, computer characteristics and FDM calculation accuracy the computation time for proposed example is following:

- The FDM calculation time is 8.37 min
- The first (intermediate) solution calculation time is 4.31 min
- The second (desired) solution calculation time is 5.10 min

So, the total calculation time is 18.18 min.

CONCLUSION

This paper presents a calculation method for geometrically non-linear stress-strain state problem for CBSS. The main idea is to define the solution sequence via FEM where every next solution includes the previous one as an initial estimate. To obtain the initial estimate for first solution, the FDM is used.

In practice, this method involves the gradual unfixing of some CBSS element nodes, whereas obtained pre-tension values for cable and shell elements are obtained from previous solution. However, the drawback is that this algorithm has not been formalized and depends on the specific CBSS model.

This method has been tested in calculating the displacement field of reflecting surface nodes for new large-sized umbrella space reflector.

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