# Adaptive interpretation of gas well deliverability tests

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**Abstract.** The paper considers topical issues of improving accuracy of data obtained from gas well deliverability tests, decreasing the number of test stages and well test time, and reducing gas emissions. The aim of the research is to develop the method of adaptive interpretation of gas well deliverability tests with resulting IPR curve conducted in gas wells with steady-state filtration, which allows obtaining and taking into account additional a priori data on the formation pressure and flow coefficients, setting the number of test stages adequate for efficient well testing and reducing test time. The present research is based on the previous theoretical and practical findings in the spheres of gas well deliverability tests, systems analysis, system identification, function optimization and linear algebra. To test the method, the authors used the field data of deliverability tests run in the Urengoy gas and condensate field, Tyumen Oblast. The authors suggest the method of adaptive interpretation of gas well deliverability tests with resulting IPR curve, which is based on the law for gas filtration with variables dependent on the number of test stage and account of additional a priori data. The suggested method allows defining the estimates of the formation pressure and flow coefficients, optimal in terms of preassigned measures of quality, and setting the adequate number of test stages in the course of well testing. The case study of IPR curve data processing has indicated that adaptive interpretation provides more accurate estimates on the formation pressure and flow coefficients, as well as reduces the number of test stages.

## 1. Introduction

Deliverability tests with resulting inflow performance relationship (IPR) curve run in the gas wells with steady-state filtration are one of the most informative and common methods well tests to characterize the behavior of well and the bottomhole conditions. Currently, the data obtained via deliverability tests are interpreted using the methods described in [1-3], which are based on Forchheimer binomial equation for gas filtration:

$$p_{пл}^2 - p_з^2 = aq + bq^2 \ , \qquad (1)$$

where $p_{пл}^2, p_з^2$ are formation pressure and bottomhole pressure, respectively; $a$ and $b$- flow coefficients dependent on bottomhole zone parameters and bottomhole structure; $q$ - flow rate. The coefficients $a$ and $b$ for IPR curve model (1) should be estimated using least square method, with the formation pressure being known [2-4]. IPR interpretation based on the model (1) and least square method is challengeable as a field method, which is attributed to the following facts: the formation pressure is difficult to determine, estimates should be robust and accurate, the number of test stages (a number of "cycles" characterized by a stabilized flow when the pressure and flow rate are recorded) is reduced.

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To ensure that the estimates are accurate and robust, in the work [5] we suggest to interpret the IPR curve using integrated IPR curve models with account of additional a priori data on the formation pressure and flow coefficients. However, the question is how to provide additional a priori data on the formation pressure and flow coefficients and to determine the adequate number of test stages to secure preassigned estimate accuracy.

To overcome the above-mentioned challenges, the method of adaptive interpretation of deliverability tests with resulting IPR curve with variable parameters is suggested and investigated. The method implies that the parameters depend on the number of test stage and additional a priori data on the formation pressure and flow coefficients obtained according to the empirical power law [6] for gas filtration are taken into account:

$$q = \lambda (p_{пл}^2 - p_з^2)^\gamma \quad (2)$$

where $\lambda$ - productivity index; $\gamma$ - constant factor with theoretical value ranging from 0.5 (turbulent flow) to 1.0 (laminar flow).

It is noteworthy that the empirical law for gas filtration (2) is widely applied in deliverability analysis over the years [7-8].

## 2. Models and Algorithms for adaptive interpretation of IPR curve

The basis to develop algorithms for gas well deliverability test data interpretation is an integrated system of IPR curve models (1) with variable parameters dependent on the number of the test stage and account of additional a priori data on the formation pressure $\overline{p}_{пл,n}^2$ and flow coefficients $\overline{a}_n, \overline{b}_n$ :

$$\begin{cases} y_n^* = p_{пл,n}^2 - a_n q_n - b_n q_n^2 + \xi_n, \\ \overline{p}_{пл,n}^2 = p_{пл,n}^2 + \eta_n, \\ \overline{a}_n = a_n + \nu_n, \overline{b}_n = b_n + \varepsilon_n, n = \overline{1, nk}, \end{cases} \quad (3)$$

where $y_n^* = p_{з,n}^2, q_n$ – values of squared bottomhole pressures and flow rates obtained at test stage number $n$; $nk$ – the number of test stages appropriate to secure preassigned estimates accuracy for the formation pressure and flow coefficients $p_{пл,n}^2, a_n, b_n$ dependent on number of test stage; $\xi_n, \eta_n, \nu_n, \varepsilon_n$, – random variables, i.e. error in measurements, recovery data, and estimates of flow coefficients, as well as deficiencies of gas filtration models (1),(2) etc.

The additional data on the formation pressure $\overline{p}_{пл,n}^2$ and parameters estimates $\overline{\lambda}_n$ and $\overline{\gamma}_n$ of model (2) can be obtained by solving the following optimization problem:

$$\overline{\boldsymbol{\alpha}}_n = \arg\min_{\boldsymbol{\alpha}} \sum_{n=1}^{nk} r(q_n^* - \alpha_1 (\alpha_2 - y_n^*)^{\alpha_3}) \quad (4)$$

where $\arg\min_x f(x)$ is the minimum point $x^*$ of the function $f(x)$ ( $f(x^*) = \min_x f(x)$ ); $\overline{\boldsymbol{\alpha}}_n = (\overline{p}_{пл,n}^2, \overline{\lambda}_n, \overline{\gamma}_n)$ – the vector of estimates; $r(x)$ – the known function. The additional data on flow coefficients $\overline{a}_n, \overline{b}_n$ can be obtained from the system of linear equations

$$z_n = a_n q_n + b_n q_n^2, n = \overline{1, nk} \ , \quad (5)$$

which is the result of grouping models (1),(2) for depression $p_{пл}^2 - p_з^2$ where $z_n = \sqrt[\overline{\gamma}_n]{q_n / \overline{\lambda}_n}$ , $q_n$ – value of flow rate obtained at test stage number $n$; $\overline{\lambda}_n, \overline{\gamma}_n$ – the optimal estimates obtained by solving problem (4).

The optimal values of squared formation pressure $p_{пл,n}^2$ and flow coefficients $a_n, b_n$ of model (3) represented for convenience as a matrix

$$\begin{cases} \mathbf{y}_n = F_n \boldsymbol{\alpha}_n + \boldsymbol{\xi}_n, \\ \bar{\boldsymbol{\alpha}}_n = \boldsymbol{\alpha}_n + \boldsymbol{\eta}_n, n = \overline{1, nk}, \end{cases} \quad (6)$$

are calculated using the method of adaptive identification by solving optimization problems (7),(8)

$$\boldsymbol{\alpha}_n^*(\boldsymbol{\beta}_n, h_n) = \arg\min_{\boldsymbol{\alpha}_n}(J_0(\boldsymbol{\alpha}_n, h_n) + J_a(\boldsymbol{\alpha}_n, \boldsymbol{\beta}_n)), \quad (7)$$

$$\boldsymbol{\beta}_n^*, h_n^* = \arg\min_{\boldsymbol{\beta}_n, h_n} J_0(\boldsymbol{\alpha}_n^*(\boldsymbol{\beta}_n, h_n)), \quad (8)$$

where $\mathbf{y}_n^* = (y_n^* = p_{з,n}^2, n = \overline{1, nk})$ – the vector of initial data on squared bottomhole pressures; $F_n = (\varphi_n^T = (1, q_n, q_n^2), n = \overline{1, nk})$ – the matrix; $\boldsymbol{\alpha}_n = (\alpha_{1,n} = p_{пл,n}^2, \alpha_{2,n} = a_n, \alpha_{3,n} = b_n)$ – the vector of unknown parameters; $\bar{\boldsymbol{\alpha}}_n = (\bar{p}_{пл,}^2, \bar{\alpha}_{2,n} = \bar{a}_n, \bar{\alpha}_{3,n} = \bar{b}_n)$ – the vector of additional a priori data obtained at stage number $n$; $J_0(\boldsymbol{\alpha}_n) = \sum_{n=1}^{nk} \omega_n(h_n) \cdot \psi_0(y_n^* - \varphi_n^T \boldsymbol{\alpha}_n)$, $J_a(\boldsymbol{\alpha}_n, \boldsymbol{\beta}_n) = \sum_{j=1}^{3} \beta_{j,n} \psi_a(kr_{j,n} \cdot \bar{\alpha}_{j,n} - a_{j,n})$ – measures of IPR curve model quality; $\boldsymbol{\beta}_n = (\beta_{j,n}, j = \overline{1,3})$ – vector of control parameters defining the importance (weight) of additional a priori data $\bar{\alpha}_{j,n}, j = \overline{1,3}$; $\psi_0, \psi_a$ – the known functions; $\omega_n(((n-i)/h_n), i = \overline{1, nk-1}, n = \overline{1, nk})$ – weighting functions with decay parameter $h_n$ to secure adaptive identification and interpretation $(\omega(x_1) < \omega(x_2), x_1 < x_2)$; $kr_{j,n}$ – the adjustment parameter for additional data $\bar{\boldsymbol{\alpha}}_n$.

The solution on the time for deliverability test with resulting IPR curve to be completed can be taken via visual analysis of graph (see figures 2–4) or using the criterion for estimates stabilization, where $nk$ is such a test stage $n$ that

$$\left| (\alpha_{j,n}^*(\beta_{j,n}^*, h_n^*) - \alpha_{j,n-1}^*(\beta_{j,n-1}^*, h_{n-1}^*)) / \alpha_{j,n}^*(\beta_{j,n}^*, h_n^*) \right| \le \varepsilon_j, \ j = \overline{1,3}, n = 1,2,3,... \quad (9)$$

is a valid inequality, where $\varepsilon_j$ is preassigned accuracy.

The algorithm given below represents the method of adaptive interpretation of IPR curve with determination of additional a priori data and flow coefficients:

1. Forming vector $\mathbf{y}_n$ and matrix $F_n$ (6).

2. Defining the vector of additional data $\bar{\boldsymbol{\alpha}}_n = (\bar{p}_{пл,n}^2, \bar{\alpha}_{2,n} = \bar{a}_n, \bar{\alpha}_{3,n} = \bar{b}_n)$ by solving problem (4) and system of linear equations (5).

3. Selecting measures of model (6) quality $J_0(\boldsymbol{\alpha}_k, h_k), J_a(\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k)$.

4. Solving problems (7), (8) using the appropriate method of function optimization.

5. Checking condition (9): if the condition is fulfilled, the test is completed; if condition (9) fails to be fulfilled, the next test stage $n+1$ is arranged and one should start new research with step 1 of the algorithm.

## 3. Results of IPR curve interpretation for gas wells.

The results of a case study of deliverability test with resulting IPR curve run in wells 1 and 2 of the Urengoy gas and condensate field are given in figures 1–4 and tables 1, 2.

For example, figure 1 shows the initial data for IPR curves for wells 1 and 2, with eight and seven test stages, respectively.

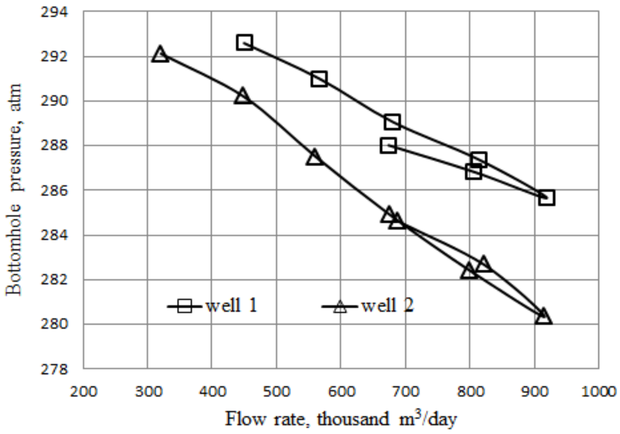


**Figure 1**. Initial data for IPR curves for wells 1 and 2.

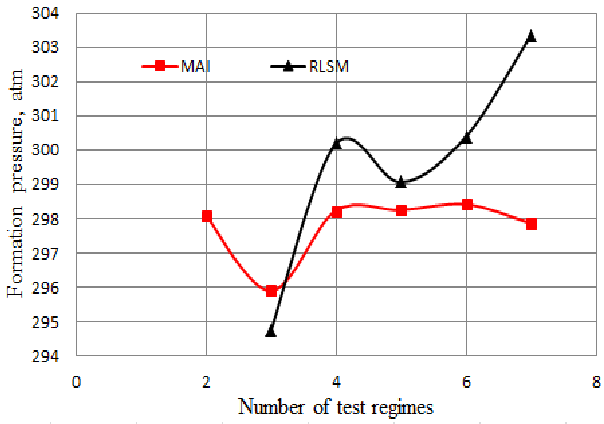


**Figure 2**. Formation pressure estimates for well 1.

Figures 2–4 show the estimates of formation pressure and flow coefficients of well 1, which are obtained using the following techniques:

1. the method of adaptive interpretation (MAI) (7) with quadratic measures of quality $\psi_0(x) = \psi_a(x) = x^2$ by solving the system of linear equations when $\mathbf{kr_n} = (kr_{j,n}, j = \overline{1,3})$ and $\beta_{j,n} = \beta^*_n, j = \overline{1,3}$ [9-10].

$$(F_n^T W_n(h_n^*) F_n + \beta_n^* \mathbf{I})\, \boldsymbol{\alpha}_n^*(\beta_n^*, h_n^*) = (F_n^T W_n(h_n^*) \mathbf{y}_n + \beta_n^* \mathbf{kr_n} \bar{\boldsymbol{\alpha}}_n), \quad (10)$$

where the estimates of control parameter $\beta_n^*$ and decay parameter $h_n^*$ are defined by solving problem (8) using the downhill simplex method [11]; $W(h_n^*) = diag(\exp((n-i)/h_n^*), i = \overline{1, nk-1})$ - diagonal matrix of weighting function values;

2. the regularized least squares method (RLSM) from (10), with $\bar{\boldsymbol{\alpha}}_n = 0$.

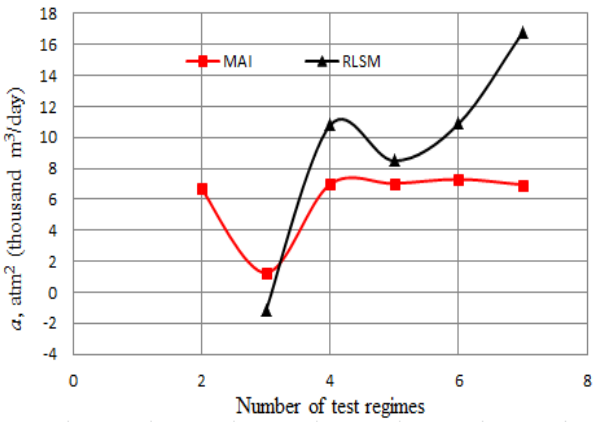


**Figure 3**. Estimates of flow coefficients *a* in well 1.

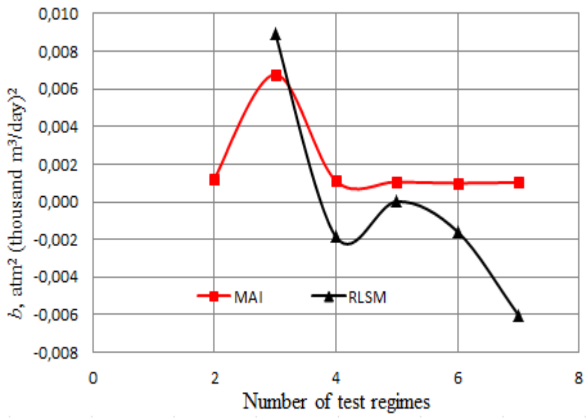


**Figure 4**. Estimates of flow coefficients *b* in well 1.

Table 1 shows the estimates of flow coefficients and formation pressure in well 2.

Table 2 gives the estimates of the formation pressure $\bar{\alpha}_{1,n} = \bar{p}^2_{nл,n}$, and flow coefficients $\bar{\alpha}_{2,n} = \bar{a}_n, \bar{\alpha}_{3,n} = \bar{b}_n$ of wells 1 and 2, which are used as additional data in (3) and obtained by solving optimization problem (4) using Gauss-Newton method with $r(x)=x^2$ [9-10] and the system of linear equations (5).

**Table 1**. Flow coefficients and formation pressure estimates.

| Number of the test stage ($n$) | Method | Flow coefficient estimate | | Formation pressure estimate |
|---|---|---|---|---|
| | | $a^* = \alpha^*_{2,n}(\beta^*_n, h^*_n)$, atm$^2$/(thousand m$^3$/day) | $b^* = \alpha^*_{3,n}(\beta^*_n, h^*_n)$, atm$^2$/(thousand m$^3$/day)$^2$ | $p^*_{пл} = \alpha^*_{1,n}(\beta^*_n, h^*_n)$, atm |
| 2 | MAI | 2.25 | 0.0084 | 294.82 |
| | RLSM | | | |
| 3 | MAI | 1.47 | 0.0111 | 294.97 |
| | RLSM | -8.91 | 0.0227 | 291.22 |
| 4 | MAI | 1.51 | 0.0104 | 294.79 |
| | RLSM | 2.29 | 0.0096 | 295.10 |
| 5 | MAI | 9.59 | 0.0022 | 297.91 |
| | RLSM | 8.97 | 0.0027 | 297.63 |
| 6 | MAI | 9.27 | 0.0019 | 297.49 |
| | RLSM | 12.10 | -0.0003 | 298.89 |
| 7 | MAI | 9.07 | 0.0019 | 297.36 |
| | RLSM | 11.69 | -0.0001 | 298.65 |
| 8 | MAI | 9.12 | 0.0019 | 297.30 |
| | RLSM | 12.58 | -0.0008 | 299.04 |

**Table 2**. Additional data.

| Number of the test stage ($n$) | Well | Formation pressure $\bar{p}_{пл,n}$, atm | Flow coefficient | | Model parameter (2) | |
|---|---|---|---|---|---|---|
| | | | $\bar{a}$, atm$^2$/(thousand m$^3$/day) | $\bar{b}$, atm$^2$/(thousand m$^3$/day)$^2$ | $\bar{\lambda}$, (thousand m$^3$/day)/atm$^2$ | $\bar{\gamma}$, d e |
| 2 | 1 | 295.30 | 0.06 | 0.00775 | 10.95 | 0.5038 |
| | 2 | 294.20 | 0.30 | 0.01093 | 8.10 | 0.5176 |
| 3 | 1 | 295.30 | 0.001 | 0.00787 | 11.19 | 0.5009 |
| | 2 | 294.40 | -0.12 | 0.01286 | 8.19 | 0.5107 |
| 4 | 1 | 299.00 | 8.48 | -0.00006 | 0.13 | 0.9929 |
| | 2 | 294.30 | 0.30 | 0.01159 | 8.56 | 0.5070 |
| 5 | 1 | 299.00 | 8.38 | 0.00009 | 0.13 | 0.9894 |
| | 2 | 295.10 | 3.37 | 0.00751 | 3.13 | 0.6206 |
| 6 | 1 | 298.40 | 7.27 | 0.00102 | 0.29 | 0.9013 |
| | 2 | 296.30 | 6.86 | 0.00376 | 0.81 | 0.7672 |
| 7 | 1 | 299.50 | 9.88 | -0.00106 | 0.12 | 0.9947 |
| | 2 | 296.50 | 7.34 | 0.00325 | 0.63 | 0.7960 |
| 8 | 1 | | | | | |
| | 2 | 296.90 | 8.31 | 0.00247 | 0.44 | 0.8323 |

It is noteworthy that when the coefficient $\gamma$ of model (2) approaches 1, the flow coefficient $b$ of model (1) approaches 0 (the laminar flow in the well).

As can be seen in figures 2–4 and table 1, the suggested method of adaptive interpretation with account of additional data allows obtaining more accurate estimates of the formation pressure and flow coefficients with less amount of field data compared to the method of least squares. For example, for the adaptive interpretation method three test stages are enough (see figures 2–4 and table 1).

## 4. Conclusion

To overcome the challenges of interpreting deliverability tests with resulting IPR curve of gas wells, the method of adaptive interpretation with account of additional a priori data has been suggested. This method allows:

1. Obtaining additional a priori data on the formation pressure and flow coefficients.
2. Defining optimal, in terms of preassigned measures of quality, estimates of the formation pressure and flow coefficients within the period of test time.
3. Setting the number of test stages adequate for efficient well testing.

The case study of IPR curve interpretation for two wells of the Urengoy gas and condensate field has indicated that adaptive interpretation provides robust and more accurate estimates of the formation pressure and flow coefficients, as well as allows reducing the number of test stages.

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