

## INTRAMODE WAVE PACKET WITH SPECTRUM IN THE VICINITY OF ZERO FREQUENCY OF THE TM-MODE GROUP VELOCITY OF THIN LEFT-HANDED MATERIAL

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One of the new laws for directed propagation of the optical modes of material left-handed waveguides due to the fact that the direction of the power carried by a waveguide mode in the left-handed material, the opposite direction to portable power with the right-oriented coating medium and substrate [1, 2]. At a certain frequency  $\omega_0$  total power carried by the waveguide mode of the cross section of the waveguide, and, as a consequence, its group velocity vanish [3, 4].

This report presents the results of analysis of the propagation intramode wave packet with frequency range of near zero group velocity of the optical mode *TM*-type thin left-handed material.

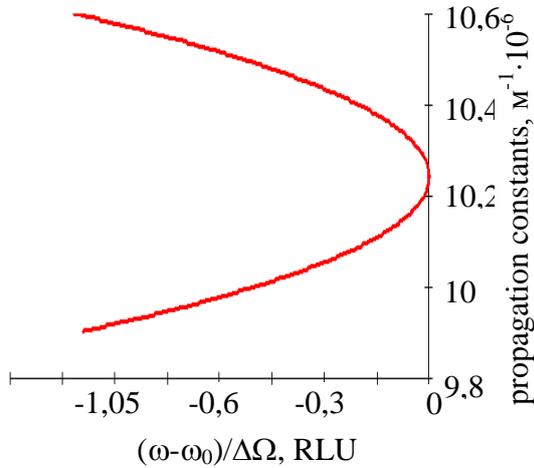


Fig.1 The dispersion dependence of the propagation constant mode of thin left-handed material in the vicinity of the frequency of zero group velocity modes  $\omega_0$  with width of the wave packet spectrum  $\Delta\Omega=10^{11}$  rad/s.

Figure 1 shows the dependence of the dispersion of the propagation constant  $\beta$  with frequency  $\omega$  modes of *TM* for material thickness of  $h=330$  nm with parameters of left-handed metamaterial  $\omega_p=3,46 \cdot 10^{15}$  rad/s (plasma frequency),  $\omega_m=1,63 \cdot 10^{15}$  rad/s (magnetic resonance frequency) and  $F=0,5$  (the filling factor of the metamaterial), air coating medium and the substrate with relative permittivity  $\epsilon_s=2$ . At a frequency  $\omega_0 \approx 1,74 \cdot 10^{15}$  rad/s modes group velocity vanishes ( $v_g=d\omega/d\beta=0$ ). The

dependence of  $\beta(\omega)$  divides into two branches at the point of  $v_g=0$ . The phase and group velocity of modes corresponding to the lower branch of the dispersion relation, have the same direction, and the respective upper branch - opposite. Thus, at a frequency  $\omega$  near the point of  $\omega_0$  ( $\omega < \omega_0$ ), in which the group velocity of the mode *TM* becomes zero, there might be two waveguide modes with propagation constants belonging to the same continuous dispersion branch.

Dependencies  $\beta(\omega)$  near the point of  $\omega_0$  (see. Fig. 1) can be represented as a series in powers of the square root ( $\Delta\omega^{1/2}$ ) from the frequency increment  $\Delta\omega=\omega-\omega_0$ . Dispersion dependencies of the propagation constant  $\beta^+$  (modes with the same directions of the phase and group velocity) and constant  $\beta^-$  (modes with opposite directions of these velocities) in the first approximation largest  $\Delta\omega^{1/2}$  can be described by the following equations:

$$\beta^\pm = \beta_0 \mp \sqrt{\Delta\omega/a} = \beta_0(1 \mp \delta\beta), \quad (1)$$

where  $\beta_0 = \beta(\omega_0)$ ,  $a = -0.5d^2\omega/d\beta^2$ ,  $\delta\beta = (\Delta\omega/a)^{1/2}/\beta_0$ . For the case  $\beta_0 \approx 1,02 \cdot 10^7$  m<sup>-1</sup>,  $a \approx 1,28$  s/m<sup>2</sup>.

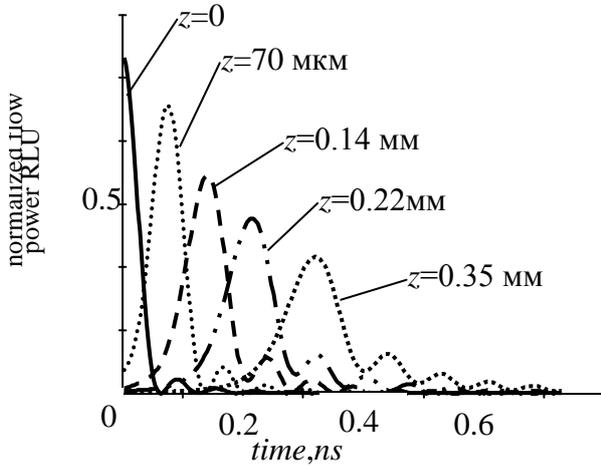
In general, the frequency interval  $\Delta\Omega = \omega_0 - \omega_{\min} \ll \omega_0$  correspond to two different couples intramode packages waveguide modes which have opposite direction of phase velocity. Wherein modes of one package of each pair (with constant propagation  $\beta^+$ ) have the same direction of the phase and group velocities. Modes of another package of each pair (with permanent  $\beta^-$ ) have opposite directions of these velocities. Below is believed that each of the packets has a uniform spectral density within the frequency band  $\Delta\Omega$ .

In this case, the power flow mode and the group velocity at the frequency  $\omega_0$  are equal to zero. Therefore, the dispersion distortion of power of each wave packet, manifested in their spatial-temporal transformation, in this case associated with both the phase dispersion and the form dispersion as well. Normalized (dimensionless) power carried by one of these intramode wave packets can be obtained as:

$$P(z, t) = \text{Re} \left\{ \int_0^{\Delta\Omega} \exp \left[ -i \left( \Delta\omega t + \sqrt{\frac{\Delta\omega}{|a|}} z \right) \right] d\Delta\omega \times \right. \\ \left. \times \int_0^{\Delta\Omega} \sqrt{\frac{\Delta\omega}{|a|}} \exp \left[ i \left( \Delta\omega t + \sqrt{\frac{\Delta\omega}{|a|}} z \right) \right] d\Delta\omega \right\} \quad (2)$$

The time dependence of the power  $p(t)=P(z,t)/\max[P(0,t)]$ , carried by the wave packet width  $\Delta\Omega=100\text{GHz}$  in different sections  $z = \text{const}$ , normalized to the maximum flow at  $z = 0$ , is shown in Figure 2.

Figure 2. Normalized to the maximum time



dependence of the power carried by the wave packet  $TM$  mode in various sections  $z = \text{const}$  left-handed material.

The multi-character of this relationship due to a rectangular range of the original wave packet is not changing during its propagation. Effect of dispersion leads to distortions manifest themselves in alignment depending lobe maxima  $s(t)$  in the process of spreading the package in time with increasing  $z$  while maintaining the package of energy in the absence of absorption. Noticeable distortion of the wave packet dispersion occur even at small distances  $z$ , the order of tenths of a millimeter.

With strong dispersion distortions typical idea of the speed of propagation of a narrow band of the wave packet, as the group velocity is meaningless [5]. In this case, as the package propagation velocity may be considered the speed of the main peak power. The analysis shows that both the instantaneous and average velocity increases from this maximum value, equal to:  $v_0=(136 \cdot (a\Delta\Omega)^{1/2})/105 \approx 5,27 \cdot 10^5 \text{m/s}$  in the section  $z \rightarrow 0$ , to a value equal  $v_\infty=(2 \cdot (a/\Delta\Omega)^{1/2}) \approx 8,14 \cdot 10^5 \text{m/s}$  in the section  $z \rightarrow \infty$ .

Note that in an unbounded (bulk) left-handed environment, the group velocity of light waves does not vanish in any of frequencies under the considered range. At the same time at a frequency  $\omega_0 \approx 1,74 \cdot 10^{15} \text{rad/s}$ , where the group velocity of the guided mode  $TM$  thin left-handed material vanishes, the group velocity of approximately equal  $4 \cdot 10^7 \text{m/s}$ . This is 50 times more than the propagation power velocity, considered tolerable intramode wave packet having order  $7 \cdot 10^5 \text{m/s}$  with a width of its spectrum equal  $\Delta\Omega=100 \text{GHz}$ . Decrease in  $\Delta\Omega$  reduces the speed of power carried by the package in a left-handed material. The propagation velocity of the power

carried by the wave packet with a width range of 1 GHz in this material will be a 500-fold less than the group velocity of the bulk material.

In the limit  $\Delta\Omega \rightarrow 0$  considered the propagation velocity of the wave packet in the material and strive to zero, since in this limit the wave packet in the material degenerates into a guided mode having zero group velocity at the frequency  $\omega_0$ .

Thus, the propagation velocity of a wave packet in intramode left-handed thin material at its spectrum width of 100 GHz can be reduced relative to the group velocity of light in a left-handed bulk material by two orders.

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