IOP Conf. Series: Materials Science and Engineering **168** (2017) 012097 doi:10.1088/1757-899X/168/1/012097

Modeling of radiative - conductive heat transfer in compositing materials

P A Luchnikov[1], V I Nefedov[1], N A Trefilov[1], A N Dementiev[1] and A P Surzhikov[2]

[1]Moscow Technological University (MIREA), 119454, Vernadsky 78, Moscow, Russia

[2]Institute of Non-Destructive Testing, National Research Tomsk Polytechnic University, 30, Lenina Avenue, 634050, Tomsk, Russia.

E-mail: xamdex@gmail.com

Abstract. A layer of composite material is investigated, which is heated one-sidedly with one-dimensional energy transfer accounting for thermal conductivity and radiation. A mathematical model is suggested for non-stationary coefficient thermophysical problem under radiative-conductive heat transfer in a material layer. Temperature dependencies of thermal capacity and thermal conductivity coefficient of composite radio-transparent material have been determined through numerical modeling by solving the coefficient reverse problem of thermal conductivity.

1. Introduction

Composite radio-transparent materials are widely utilized as protection devices for aircraft antennas [1, 2]. In the process of exploitation, these composite materials experience influence of various environment factors, such as discharge plasma [3–5], electrization and optical radiation [6], locally increased temperature and others. The crucial factor determining composite's properties is the increased temperature, which alters such primary dielectric parameters of composite materials as complex relative dielectric permittivity [7, 8]. In order to measure temperature change within the sample one uses high-temperature thermocouples with small conductor cross-section. The thermocouples are embedded into the sample's volume during its formation stage. The coordinates of thermocouple's joint in the sample are defined via X-ray imaging. Thus, several thermocouples with different setting depths are placed in various cross-sections of the sample.

2. The samples and research method

In order to calculate the temperature in spaces between the thermocouples of the sample a specific mathematical apparatus is used applying numerical mathematical methods. One uses solutions for non-standard direct problems of thermal conductivity and inverse coefficient ones under radiative-conductive heat transfer in a flat layer of the absorbing, emitting and scattering medium. One-dimensional problem is commonly considered for the purposes of measurement. In the mathematical setting of the thermophysical problem of determining temperature distribution in the thickness of a flat dielectric sample one considers a flat layer of a thermophysically isotropic material.

When solving thermal conductivity problems, first-, second- or third-type boundary conditions may be used in arbitrary combinations for each of the surfaces. As for the radiation transfer, within each of the spectral ranges the sample boundaries can be either semi-transparent or non-transparent for self-radiation and radiation of external heat sources. The diversity of boundary condition types allows modeling arbitrary heat response conditions and processing the experiment results, which have been conducted on installations varying in design and sample heating conditions [8, 9]. When solving coefficient reverse problems, both separate and integrated determination of temperature dependencies of volumetric heat capacity cq (T) and thermal conductivity coefficient λ (T) are valid.

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

Thermophysical properties of the sample an $d\lambda(T)$ arenon-linear and temeprature-dependant, whileitsoptical-physical properties, such as absorption and scattering factors, refraction index and scattering indicatrix, depend on temperature and wavelength of the heat-associated radiation.

We consider a layer of composite material which is in condition of one-sided heating with energy transferred one-dimensionally via thermal conductivity and radiation. In the case initial temperature distribution within the sample and heat exchange conditions at its boundaries are known. When implementing numerically, the composition of heat exchange parameters set is determined by a concrete type of heat response. Thus, in case of radiative microwave heating or infrared heating, one specifies densitiesof radiation fluxes that are brought to the sample surfaces, their spectral and angular distributions. Under the experimental conditions with convective heating created by gas burners or arc plasma sources, we specify gas environment temperature and temperature dependency of the heat-transfer coefficient, Moreover, in all the cases of thermal response optical properties of the boundaries are considered established.

The temperature dependencies cq (T) andλ (T) of the material are deternined by solving a reverse problem of thermal conductivity or radiative-conductive heat transfer. When coefficient calculation methods are applied, thermophysical properties of the material are among causal characteristics whereas temperature patterns and radiation fields are consecutive. In the process of the experiment temperatures at separate points of the sample are meausred. In this case measuring temperature at one inner point of the sample allows to restore only one temperature dependency – either cq (T) or λ (T). In order to restore both dependencies one should measure the temperatures in at least two points within the sample thickness and heat flux on one of its surfaces.

3. Mathematical modeling

We shall now take into consideration a mathematical model of the non-stationary coefficient thermophysical problem. The mathematical model of radiative-conductive heat transfer in a layer of absorbing, emitting and scattering medium includes thermal conductivity equation as well as radiation transfer equation transformed using a binomial approximation of the moments method [10, 11]. The equations for inner heat sources and initial condition for the thermal conductivity equation as well as boundary conditions for the thermal conductivity and radiation transfer equations also make up the mathematical model.

$$cq(T)\cdot\frac{\partial T(t,r)}{\partial t}=\frac{1}{r^n}\cdot\frac{\partial}{\partial r}\cdot\left(r^n\cdot\lambda(T)\cdot\frac{\partial T(t,r)}{\partial r}\right)+q_V; \tag{1}$$

$$R_1<r<R_2;\ R_1<r<R_2;\ T(0,r)=f_0(r); \tag{2}$$

$$-K_1\cdot\lambda(T)\cdot\frac{\partial T(t,R_1)}{\partial r}=\alpha_1(T)\cdot T(t,R_1)+\beta_1(\tau)+\gamma_1(T); \tag{3}$$

$$K_2\cdot\lambda(T)\cdot\frac{\partial T(t,R_2)}{\partial r}=\alpha_2(T)\cdot T(t,R_2)+\beta_2(t)+\gamma_2(T); \tag{4}$$

$$R_1<r<R_2$$

$$\frac{1}{r^n}\cdot\frac{\partial}{\partial r}\cdot\left(\frac{r^n}{\varpi_\nu}\cdot\frac{dM_{0_\nu}(r)}{dr}\right)-3\cdot\alpha_\nu\cdot M_{0_\nu}(r)=-3\cdot\alpha_\nu\cdot B_\nu^*(T); \tag{5}$$

$$\begin{aligned}&-\frac{1}{2\cdot\varpi_\nu}\cdot\left(\frac{1}{3}+R_{11_{W1_\nu}}\right)\cdot\frac{dM_{0_\nu}(R_1)}{dr}+\frac{1}{2}\cdot\left(\frac{1}{2}-R_{01_{W1_\nu}}\right)\cdot M_{0_\nu}(R_1)=\\&=q_{0_{W1_\nu}}\cdot\left(2\cdot\eta_{m_{W1}}\cdot Q_{01_{W1_\nu}}+\eta_{\delta_{W1}}\cdot Q_{1_{W1_\nu}}\right)+2\cdot E_{W1_\nu}\cdot n^2\cdot B_{W1_\nu}(T_{W1});\end{aligned} \tag{6}$$

$$\frac{1}{2\cdot\varpi_\nu}\cdot\left(\frac{1}{3}+R_{11_{W2_\nu}}\right)\cdot\frac{dM_{0_\nu}(R_2)}{dr}+\frac{1}{2}\cdot\left(\frac{1}{2}-R_{01_{W2_\nu}}\right)\cdot M_{0_\nu}(R_2)= \\ =q_{0_{W2_\nu}}\cdot\left(2\cdot\eta_{m_{W2}}\cdot Q_{01_{W2_\nu}}+\eta_{\delta_{W2}}\cdot Q_{1_{W2_\nu}}\right)+2\cdot E_{W2_\nu}\cdot n_\nu^2\cdot B_{W2_\nu}(T_{W2}); \tag{7}$$

$$q_V=\int_{\bar{\nu}_1}\alpha_\nu\cdot\left(M_{0_\nu}(r)-B_\nu^*(T)\right)\cdot d\nu, \tag{8}$$

$$\varpi=\alpha+\beta\,(1-p),$$

where: T is the temperature, K; t is time; r – stand for coordinate; cq – volumetric thermal capacity, J/m^3; λ – thermal conductivity coefficient, W/m·K; α – absorption coefficient, m^{-1}; β – scattering coefficient, m^{-1}; p – scattering indicatrix parameter**;** f_0 – initial temperature distribution; $K_{1,2}$ – the variable indicating the boundary condition type.

With $K_{1,2}=0$ first-type boundary conditions are set. In this case

$$\alpha_{1,2}(T)=-1;\qquad \gamma_{1,2}(T)=0;\qquad \beta_{1,2}(\tau)=f_{1,2}(\tau);$$

$f_{1,2}(\tau)$ is the surface temperature dependency in time.

With $K_{1,2}=1$ second- and third-type boundary conditions are set. And in this case

$$\alpha_{1,2}(T)=-\alpha_{f1,2};\ \beta_{1,2}(\tau)=q_{TW1,W2};$$

$$\gamma_{1,2}(T)=\int_{\bar{\nu}_1+\bar{\nu}_2}\varepsilon_{W1,2_\nu}\cdot q_{\Pi_{W1,W2_\nu}}d\nu-\int_{\bar{\nu}_1+\bar{\nu}_2}\varepsilon_{W1,2_\nu}\cdot B_{W1,2_\nu}d\nu+\alpha_{f1,2}\cdot T_{f1,2};$$

where: $\alpha_{f1,2}$ is the heat transfer coefficient, W/m^2; ε – is the emissivity of the boundary surface; q_Π – stands for the density of the radiation flux incident, W/m^2; q_T – density of the flux absorbed by the surface, W/m^2; T_f – environment temperature, K; B – is the Planck function; n – is refraction index; $B^*=4\cdot n^2B$; R_{mn}, Q_{mn}, Q_m, E are integral optical characteristics of the boundary surfaces; η_m – portion of diffuse radiation in the incedent radiation flux; $\eta_б$ – portion of the directed radiation in the incident radiation flux; q_0 – radiation flux incident to the semi-transparency area of the material, W/m^2; M_0 – intensity moment of the zero-order radiation, W/m^2. The ν index indicated frequency dependence, and $W1$, $W2$ are related to the front and rear surfaces accordingly.

Solving the system (1)–(8) is carried out by the method of finite differences with implicit difference scheme applied. The coefficient problem of radiative-conductive heat transfer is usually formulated in extreme setting [10]. The algorithm for determining the coefficients of equation (1) – λ (*T*), cq (*T*) – is realized via minimizing the functional linking the experimental and estimated temperatures

$$S=\sum_{i=1}^{N}\int_0^{\tau^e}\left(T^e(\tau,r_i)-T(\tau,r_i)\right)^2d\tau=\min, \tag{9}$$

where: N – is the number of temperature sensors; $T^e(t,r_i)$ are experimental temperature values; $T(t,r_i)$ – estimated temperature values, determined from the solution of (1)–(8); t^e – experiment duration.

Solving the radiative-conductive heat transfer is usually carried out applying the principle of iterative regulation [12, 13]. In this case the condition of residual is used to stop the process of minimization:

$$S\approx\delta^2. \tag{10}$$

$$\delta^2=\sum_{i=1}^{N}\int_0^{\tau^e}\Delta T^e(t,r_i)^2dr, \tag{11}$$

where $\Delta T^e(t,r_i)$ is the error of experimental temperature values.

The sought-for temperature dependencies are expanded into series by the number of basis functions:

$$\lambda(T)=\sum_{m=1}^{M_\lambda}\lambda_m\cdot\psi_{\lambda_m}(T); \tag{12}$$

$$cq(T)=\sum_{m=1}^{M_C}C_m\cdot\psi_{C_m}(T), \tag{13}$$

where: λ_m , C_m are expansion factors; ψ_λ , ψ_C are basis functions; M_C, M_λ stand for the number of members in the expansion.

Cubic and linear *b*-splines are often used as basis functions [13]. So the sought-for spline coefficients appear to be the values of thermophysical parameters of the sample material in the spline-approximation vertices.

Expansion of the sought-for dependencies cq (*T*) and λ (*T*) to basis functions allows one to bring problem (1) to the problem of determining the $M_C + M_\lambda$ function minimum of the variables:

$$S = S(\lambda_1, \lambda_2, ..., \lambda_{M_\lambda}, c_1, c_2, ..., c_{M_c}) = \min. \qquad (14)$$

In order to solve this problem we can use, for instance, a gradient minimization method – the Davidon-Fletcher-Powell method [12]. The valuesof $\partial S / \partial \lambda_i$ gradient tvector; $i = \overline{1, M_\lambda}$; $\partial S / \partial C_i$; $i = \overline{1, M_C}$ are determined from the dual problem. The Davidon-Fletcher-Powell method involves determining the minimum of function *S*along the chosen search direction.

Acknowledgments
This work was supported by the Ministry of Education and Science of Russian Federation.

References

[1] Prigoda B A, Kokunjko B C Radomes of aircraft M.: Mashinostroenie 1970 340 p (in Russian)
[2] Walton J D Radome Engineering Handbook N.V.: Marcel Dekker, 1970 520 p
[3] Luchnikov P A, Rogachev A V, Yarmolenko M A, Rogachev A A, Bespal'ko A A Dynamics of combined electron beam and laser dispersion of polymers in vacuum 2016 *IOP Conf. Ser.: Mater. Sci. Eng.* **110** 012081
[4] Luchnikov P A Structure Formation Mechanisms and Electrical Properties of PVD Fluoropolymer Films 2015 *Russian Physics Journal* **57** 9 1239–45
[5] Rogachev A A, Yarmolenko M A, Xiaohong Jiang, Shen Ruiqi, Luchnikov P A, Rogachev A V Molecular structure, optical, electrical and sensing properties of PANI-based coatings with silver nanoparticles deposited from the active gas phase 2015 *Applied Surface Science* **351** 811–18
[6] Aleutdinov A D, Ghygazov S A, Mylnikova T S, Luchnikov P A Thermal shock removal of defective glass-enamel coating from cast-iron products 2015 *IOP Conf. Ser.: Mater. Sci. Eng.* **81** 012069
[7] Luchnikov P A, Sigov A S, Trefilov D N, Trefilov N A Wave dielcometric materials: monograph under the editorship A.S. Sigov. M.: Scientific world, 2014 300 p (in Russian)
[8] Alifanov O M Identification of heat transfer processes in aircraft. M.: Mashinostroenie, 1979. 216 p (in Russian)
[9] Prosuntsov P V, Reznik S V Determination of thermophysical properties of translucent materials 1985 *Engineering-physical journal* **496** 971–76 (in Russian)
[10] Prosuntsov P, Reznik S Mathematical model of the inverse problem of radiative-conductive heat transfer in semitransparent scattering medium Proceedings of the Siberian Department. USSR Academy of Sciences. Series technique Sciences. 1986 **2** 10 3–9 (in Russian)
[11] Surzikov A P, Lysenko E N, Ghyngazov S A, Frangulyan T S 2002 *Russian Physics Journal* **45** 10 989-96
[12] Himmelblau D. Applied nonlinear programming. M.: Mir, 1975. 534 p (in Russian)
[13] Forsyth J, Malcolm M, Moler K Computer methods for mathematical computations. M.: Mir, 1980 279 p (in Russian)