

# Analysis of noisy signal restoration quality with Lanczos filter

A S Belyaev, I A Tutov and S A Maykov

Tomsk Polytechnic University, 30, Lenina ave., Tomsk, 634050, Russia

E-mail: belyaewas@mail.ru

**Abstract.** The differential component of the feedback is used to enhance dynamics of controlled mechanisms in control systems. However, the use of the differential component increases the noise interference, resulting in the need to use different filtering methods. This paper describes the research of a differentiated signal input in a differential Lanczos filter. The influence of the filter order on the integrated square error of the filtered signal is defined. The impact of signal dispersion and a sampling interval on the integrated square error of the filtered signal is identified. Moreover, the most convenient parameter values for the Lanczos filter are determined. The paper also includes the comparison of the exponential moving average filter and the Lanczos filter with selected values of noise dispersion and a sampling interval. The research shows the possibility to apply the Lanczos filter for processing of the differentiated signals.

## 1. Introduction

Control systems are used in many different fields of mechanical engineering such as space, aviation, shipbuilding and other industries. For enhancing dynamics of controlled mechanisms, a differential component is used to implement real-time control systems with high control quality; however, the use of the differential component increases the noise interference, resulting in the need to use filtering methods [1-5]. There are 7 basic types of filtering:

- 1) Methods based on substitution of infinitely small increments of differentiated signal  $ds$  and time  $dt$  with their finite values  $\Delta s$  and  $\Delta t$ ;
- 2) Methods based on the use of different interpolating polynomials.
- 3) Methods based on the use of splines.
- 4) Methods based on the use of various approximating functions.
- 5) Methods based on integral equations.
- 6) Methods based on Fourier and Laplace transforms.
- 7) Methods based on wavelet transform [6]

However, algorithmic implementation of these methods requires high computational power and increases signal processing time, which limits their usage in real-time systems. It forces specialists to find a compromise between the quality of the filter and the amount of processing time. A good combination of implementation simplicity, low computational cost and relatively high accuracy lead to widespread occurrence of the exponential smoothing method.

The paper [7] describes a study of a standard filter based on the method of an exponential moving average. However, this filter does not differentiate the signal; therefore, it is rarely used for such



signals. Differentiation filters are used most often; however, these filters do not typically handle noise suppression well. Combining good noise suppression and differentiation, the Lanczos filter is widely used in filtering radio signals.

## 2. Methods

The choice of the structure and parameters of the differentiating filter will be implemented in the Lanczos filter class [8], the general equation of which is as follows:

$$y[nT] = \frac{3}{TN(N+1)(2N+1)} \sum_{k=0}^{2N} (N-k)x[(n-k)T] \quad (1)$$

where  $N$  – filter order.

This filtration method can be implemented in different application [9, 10]. For the processing of a real analog signal, the signal received by the sensor must be converted by means of an analog-to-digital converter (ADC). Because this filter differentiates the original signal, it should be integrated, and then noise with a certain value of dispersion should be added. At the next stage, the signal is converted into a digital value by means of the ADC with a certain quantization level defined by the resolution of the ADC.

In order to define control quality indicators, it is necessary to compare the transformed signal and the source. The comparison method used is the integral square error (ISE), which is defined as:

$$err = \sum_{i=1}^n (S_i - S_i')^2, \quad (2)$$

where  $S_i$  is the value of the filtered signal at the  $i$ -th iteration,  $S_i'$  is the true signal value at the  $i$ -th iteration. Based on this model, the transformed signal will be affected by the following parameters:

- 1) Noise dispersion -  $\sigma$ ;
- 2) ADC quantization level -  $\Delta$ ;
- 3) Filter order.

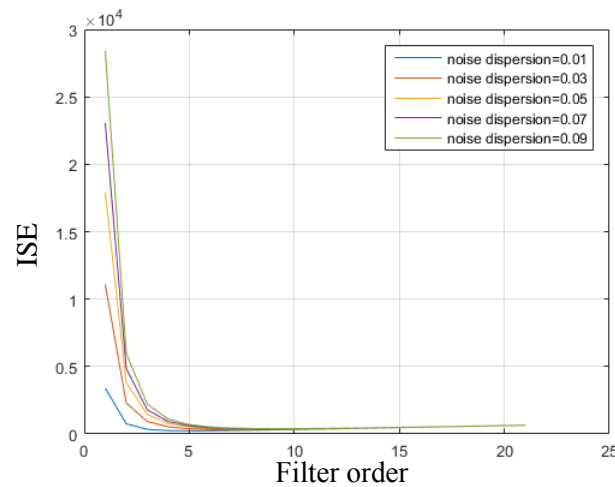
But how changes in these parameters will affect the transformed signal.

## 3. Filter analysis

For the research of these dependencies, a model was implemented in the MATLAB&Simulink environment. As the test signal for the analysis of the ESS, a signal with the following basic levels is used: a negative level (range 0-100), step transition, positive transition (100-300), dynamic transition (300-380), and a zero level.

In order to analyse the relation between the ISE and the filter order, the order was changed in the range from the first to the twentieth. In so doing, the quantization level was set as 0.5, and the noise dispersion values were made equal to the following five values {0.01; 0.03; 0.05; 0.07; 0.09}.

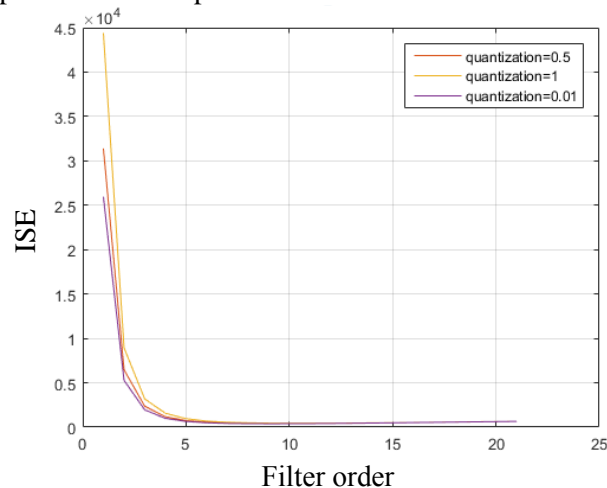
Figure 1 shows that when the filter order is increased, the integral square error first decreases significantly, and then begins to increase; also, an increase in noise dispersion causes an increase in the ISE, i.e. there is a directly-proportional relation between these parameters.



**Figure 1.** Dependence of the integral square error on the filter order at different noise dispersion values.

The diagrams show that an increase in the dispersion value greatly increases the integral error in the region of filter order values of 1-5. In the region of values over 6, the effect of the quantization level on the error is negligibly small. However, under real-life conditions, noise dispersion is not changed; therefore, it is necessary to determine the effect of the quantization level on the integral error.

We will make an evaluation of the effect of the quantization level and the filter order on the integral component. The quantization level for this value was equal to the following three values {0.01; 0.5; 1} while dispersion was equal to 0.5. The result is shown in Figure 7. Figure 2 shows that an increase in the quantization interval leads to an increase in the ISE, i.e. there is a directly-proportional relationship between these parameters.

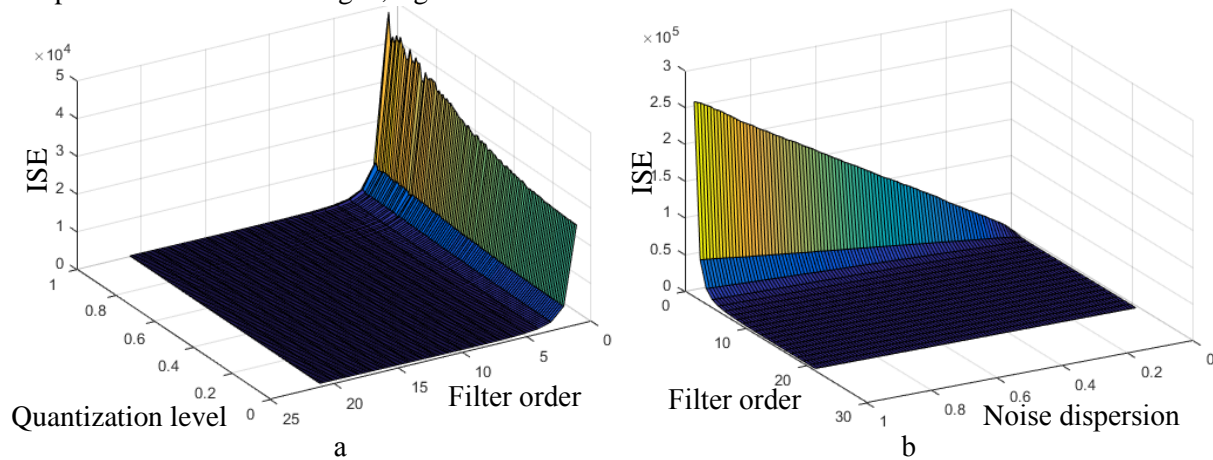


**Figure 2.** Dependence of the integral square error on the filter order at different quantization level values.

The diagrams show that an increase in the quantization level significantly increases the integral error in the region of filter order values of 1-4. In the region of values over 5, the effect of the quantization level on the error is negligibly small.

Let us check the results for multiple values of the quantization level [0.01 1] and dispersion [0.01 1] at 0.01 intervals. In order to achieve this, two experiments were carried out; in the first one, the dispersion was equal to 0.5 and maintained as constant, and the filter order was changed [1 22], figure

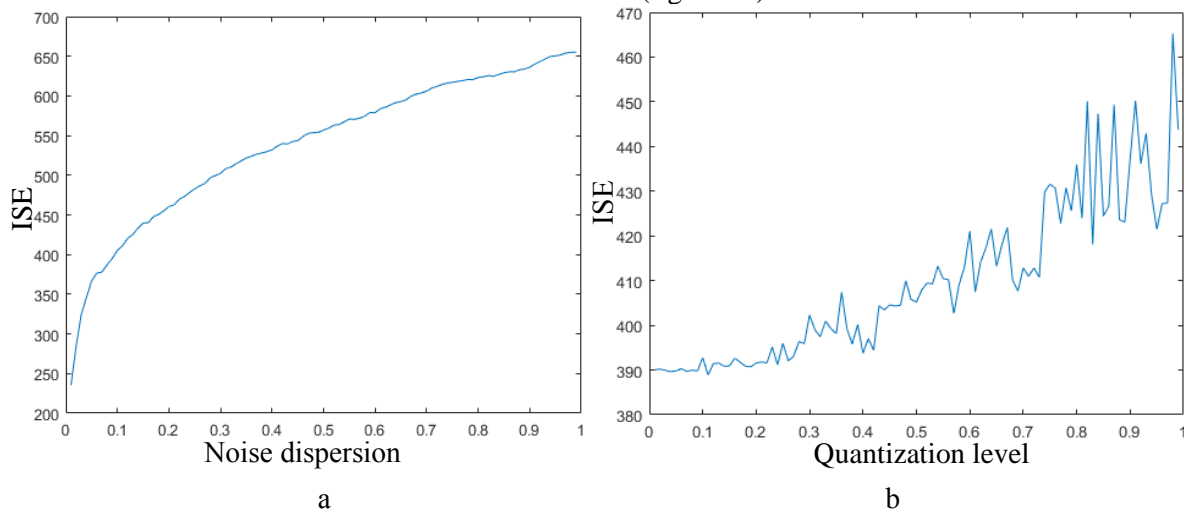
3a. On the contrary, the quantization level value was equal to 0.01, and the filter order and the dispersion value were changed, figure 3b.



**Figure 3.** Dependence of the integral error on the filter order: a – quantization level; b – noise dispersion

The diagrams only confirm the results obtained earlier.

Now we will analyze the opportunity to compensate effects of dispersion and the quantization level in relation to the ISE. To achieve this, a dependence between noise dispersion and the ISE was found for the best filter order (figure 4a) and the relationship between the quantization level and the ISE, as well as for the best filter order values was established (figure 4b).



**Figure 4.** Dependence of the ISE (with the best filter order settings) on a – noise dispersion; b – quantization level.

An increase in the filter order reduces the integral square error only to a certain value, after which the ISE gradually increases.

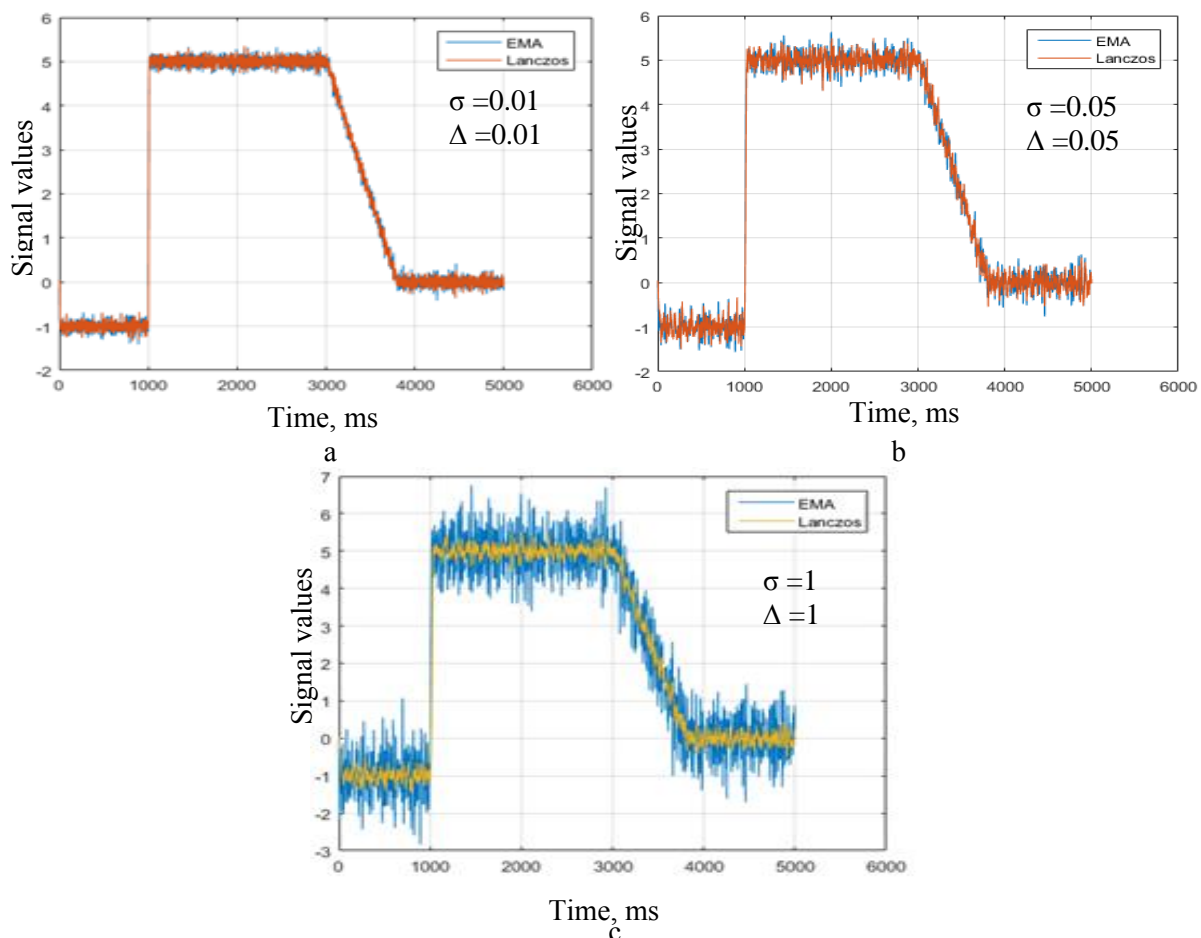
The result of the research carried out is as follows:

- Noise dispersion has a significant effect on the ISE at filter order values under 6; at higher values, effects of noise dispersion are compensated.
- The quantization level has a significant effect on the ISE at filter order values under 4; at higher values, effects of the quantization level are compensated.

- As shown in the diagrams, it is not possible to compensate the effect of noise dispersion and the quantization level on the ISE completely; however, the increase in the error is not very significant.

#### 4. Comparison of the results between Lanczos filtering and exponential moving average methods

As described in [7], the ISE has a directly-proportional dependence on dispersion and the quantization interval, as well as the Lanczos filter. Three experiments were conducted to compare the filters: with dispersion equal to 0.01 and the quantization interval equal to 0.01 (figure 5a); with dispersion equal to 0.5 and the quantization interval — to 0.5 (figure 5b); with dispersion equal to 1 and the quantization interval — to 1 (figure 5c). The diagrams showing the output signals of the filters are below.



**Figure 5.** Filtered signals with dispersion and the quantization level equal to: a –  $\sigma = 0.01$ ,  $\Delta = 0.01$ ; b –  $\sigma = 0.05$ ,  $\Delta = 0.05$ ; c –  $\sigma = 1$ ,  $\Delta = 1$ .

According to the diagrams of transformed signals, it is evident that with the values of dispersion and the quantization interval equal to 0.01, the filters work with close mean-square error values; however, when increasing the values of dispersion and the quantization interval, the Lanczos filter performs better with a lower integral mean-square error.

#### 5. Conclusion:

1. The use of the Lanczos filter for filtering digital signals is more appropriate for systems with higher noise dispersion.

2. The use of the Lanczos filter of the 6th or 7th order allows one to virtually compensate effects of noise dispersion and the quantization level on the ISE.

## References

- [1] Kim J and Kim T 2009 *Proc. 6th Int. Conf. on Computer Graphics, Imaging and Visualization: New Advances and Trends* (Tianjin: IEEE) pp 367-74
- [2] Long H, Qu Z, Fan X, and Liu S 2012 *Proc. 10th World Congress on Intelligent Control and Automation* (Beijing, IEEE) pp 4315-9
- [3] Yudan H, Zhihua C, Wenyin G, and Qin L 2014 *IEEE Congress on Evolutionary Computation* (Beijing, IEEE) pp 2893-900
- [4] Shelomentcev E and Alexandrova T 2014 *IOP Conf. Series: Mat. Sci. and Eng.* (Tomsk) vol 66 (IOP Publishing) 012027
- [5] Pandey C and Nemade S 2014 *Int. IEEE India Conf.* (Pune, IEEE) pp 1-6
- [6] Lycashin Y 2003 *Adaptivie metodi kratkosrochnogo prognozirovaniy vremennih rydov* (Moscow: Finance and statistica)
- [7] Belyaev A, Tutov I and Butuzov D 2016 *Int. Siberian Conf. on Control and Communications* (Moscow) *Electronic Materials* (IEEE) 2380-6516
- [8] Hamming R 1977 *Digital filters* (Englewood Cliffs, NJ: Prentice-Hall)
- [9] Jonathan Taquet and Claude Labit 2010 *Multimedia Signal Proc. IEEE Int.Workshop* (ST. Malo, IEEE) pp 122-7
- [10] Amina K, Nejmeddine B, Nouri M, Amine S and Mohamed A 2014 *1st Int. Conf. on Adv. Techn. for Signal and Image Proc.* (Sousse-Tunisia, IEEE) pp 148-52