Algorithm of constructing hybrid effective modules for elastic isotropic composites

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Abstract. The algorithm of constructing of new effective elastic characteristics of two-component composites based on the superposition of the models of Reiss and Voigt, Hashin and Strikman, as well as models of the geometric average for effective modules. These effective characteristics are inside forks Voigt and Reiss. Additionally, the calculations of the stress-strain state of composite structures with new effective characteristics give more accurate prediction than classical models do.

1. Introduction

In this paper, we consider the problem of constructing new constitutive equations of elastic isotropic two-component composites. Most of modern publications focus on mathematical models that take into account the geometry of heterogeneities in an elastic body, while the creation of mathematical models of effective modules is neglected. In this paper, we used a model based on the formulation of effective relations for the hybrid module. For the first time, effective modules, called hybrid, were introduced in [1] as a Crisher model [2]. In the present paper, this model was further developed: implemented new types of hybrid modules, in particular the geometric average effective modules and their combinations with upper and lower bounds of the models of Voigt, Reiss, Hanshin and Strikman. Numerical and graphical analysis of the new effective characteristics have been implemented. Also, to address the problems in effective modules used [6-11].

2. Analysis of the Crisher model

The research work of Krisher [2] can be attributed to the number of the first jobs, in which hybrid models of the effective modules elastic composites have been received and have been used. We will give the basic idea of this model as an example.

The next shear module of the two-component composite was introduced by Krisher during modeling of the effective characteristics:

$$G_{eff} = \left(\frac{1-a}{G_V} + \frac{a}{G_R} \right)^{-1}. \quad (1)$$

According to Crisher, a – experimentally determined coefficient, G_V, G_R – effective modules of Voigt (V) and Reiss (R)

$$G_V = \gamma_1 G_1 + \gamma_2 G_2,\; G_R = \left(\frac{\gamma_1}{G_1} + \frac{\gamma_2}{G_2} \right)^{-1}. \quad (2)$$

G_1, G_2 – phases shift modules, γ_1, γ_2 – volume content.

If the ratio of (1) is replaced by $1-a$ γ_1, we get:

$$G_{eff} = \left(\frac{\gamma_1}{G_V} + \frac{\gamma_2}{G_R} \right)^{-1}. \quad (3)$$

A new expression of G_{eff} allowed us to obtain more accurate performance characteristics in comparison with the forecast defined by relations (2).

Let us analyze (3). In fact, this is a ratio of a new effective module of Reiss type, wherein classic modules Voigt and Reiss are used as phase modules. Thus, a new module (3) prepared as a function of two classic modules Voigt and Reiss. If the new module of Voigt is built in a similar way, we obtain:

$$G_V^1 = \gamma_1 G_V + \gamma_2 G_R. \quad (4)$$

The question arises, whether all functions are applicable for getting more accurate effective characteristics? It turns out [3], [4] that functions are applicable, if they produce compressive transformations, in which new modules satisfy the inequalities:

$$G_1 \geq G_V \geq G_V^1 \geq G_{eff} \geq G_R \geq G_2. \quad (5)$$

For transformations (3), (4) as we can easily verify, these inequalities are fulfilled.

3. Mixed-type hybrid modules

Let us consider the effective shear modules of the Hanshin, Shtrikman models (H-S) [5] of a two-component elastic composite.

$$G^{"} = G_2 + \frac{\gamma_1 (G_1 - G_2)}{1 + \gamma_2 (G_1 - G_2)/(G_2 - G_U)},$$
$$G^{'} = G_2 + \frac{\gamma_1 (G_1 - G_2)}{1 + \gamma_2 (G_1 - G_2)/(G_2 - G_L)}. \quad (6)$$

Here:

$$G_L = \frac{3}{2}\left(G_2 + \frac{10}{9/K_2 + 8/G_2} \right)^{-1},\; G_U = \frac{3}{2}\left(G_1 + \frac{10}{9/K_1 + 8/G_1} \right)^{-1}.$$

K_1, K_2 – the modules of volume elasticity of the phases. For $G^{"}$, $G^{'}$, fork H-S is performed:

$$G^{"} > G^{*} > G^{'}.$$

We form new effective shear moduli obtained as a superposition of effective modules V-R and H-S:

$$\tilde{G}_V = \gamma_1 G^{"} + \gamma_2 G^{'},\; \tilde{G}_R = \left(\frac{\gamma_1}{G^{"}} + \frac{\gamma_2}{G^{'}} \right)^{-1}, \quad (7)$$

and let us consider the following ratio:

$$\tilde{\tilde{G}}'' = G_R + \frac{\gamma_1 (G_V - G_R)}{1 + \gamma_2 (G_V - G_R)/(G_R - \tilde{\tilde{G}}_U)},$$
$$\tilde{\tilde{G}}' = G_R + \frac{\gamma_1 (G_V - G_R)}{1 + \gamma_2 (G_V - G_R)/(G_R - \tilde{\tilde{G}}_L)}. \quad (8)$$

Here:

$$\tilde{\tilde{G}}_U = \frac{3}{2}\left(G_R + \frac{10}{9/K_R + 8/G_R} \right)^{-1}, \ \tilde{\tilde{G}}_L = \frac{3}{2}\left(G_V + \frac{10}{9/K_V + 8/G_V} \right)^{-1}.$$

For volumetric modules, K_V, K_R perform usual ratio V-R:

$$K_V = \gamma_1 K_1 + \gamma_2 K_2, \ K_R = \left(\frac{\gamma_1}{K_1} + \frac{\gamma_2}{K_2} \right)^{-1}.$$

As we can see from (7), (8), the new hybrid modules are produced by inserting the *H, S* modules in the classical *V, R* model instead of G_1, G_2 and by inserting the effective *V, R* modules in the *H, S* model.

It can be shown that for effective new modules (7), (8), the inequalities that provide compressibility conditions of the form are:

$$G_1 > G_V > \tilde{G}_V > \tilde{G}_R > G_R > G_2, \ G_1 > G'' > \tilde{\tilde{G}}_V'' > \tilde{\tilde{G}}_V' > G' > G_2.$$

Let us consider two more pairs of new effective hybrid modules. We will introduce into consideration geometric mean modules for pairs of upper and lower estimations of models (V-R) and (H-S):

$$G_{S,VR} = \sqrt{G_V G_R}, \ G_{S,\mathrm{HS}} = \sqrt{G' G''}. \quad (9)$$

Then, new effective characteristics will be:
a) based on model (V- R):

$$G_{V,S} = \gamma_1 G_V + \gamma_2 G_{S,VR}, \ G_{R,S} = \left(\frac{\gamma_1}{G_V} + \frac{\gamma_2}{G_{S,VR}} \right)^{-1}. \quad (10)$$

b) based on model (H-S):

$$G_{V,S}' = G_{S,HS} + \frac{\gamma_1 (G_V - G_{S,HS})}{1 + \gamma_2 (G_V - G_{S,HS})/(G_{S,HS} - G_{L,S})},$$
$$G_{V,S}'' = G_{S,HS} + \frac{\gamma_1 (G_V - G_{S,HS})}{1 + \gamma_2 (G_V - G_{S,HS})/(G_{S,HS} - G_{V,S})}. \quad (11)$$

Here: $G_{L,\mathrm{S}} = \frac{3}{2}\left(G_{S,HS} + \frac{10}{9/K_{S,HS} + 8/G_{S,HS}} \right)^{-1}$, $G_{V,\mathrm{S}} = \tilde{\tilde{G}}_V$, $K_{S,\mathrm{HS}} = \sqrt{K' K''}$.

4. The discussion of the results

As we noted earlier, O Krisher's work was the first productive experience of using the hybrid effective modules in the mechanics of composites. Here the expressions of the upper and lower estimations of the classical model of Voigt and Reuss were introduced into the expression of the effective module of Reuss type instead of the modules shear phases. In this paper, the given method of constructing effective hybrid characteristics was further developed.

The first hybrid model is structured as follows. The expressions are taken for the upper and lower evaluations of models Hashin-Shtrikman and Voigt-Reuss. Then, the upper and lower assessments of model Hashin-Shtrikman are inserted in the ratios of Voigt and Reuss model instead of phase shift modules. As a result, we obtain (7). The second hybrid model has been obtained by reverse substitution – the effective modules of Voigt and Reuss have been put into the expressions of model Hashin-Shtrikman instead of phase shift modules. This model is represented by expressions (8).

The following hybrid modules were obtained by the superposition of estimates Hashin and Shtrikman, Voigt-Reuss jointly with the geometric mean effective modules, have been introduced in [6]. These models are represented by relations (9-11).

The requirement of compressibility of the function that produces a transition from one pair of effective modules to another is the criterion for the correctness of the procedure construction new effective characteristics. Mathematically, this requirement is expressed as an inequality of type (5).

For the purpose of clarity, a graphical representation of relations for the hybrid modules is shown separately for each pair of found effective characteristics in comparison with modules of Voigt and Reuss or with modules of Hashin and Shtrikman. The hybrid effective modules (7) are shown in figure 1. The graph shows that the values of the new effective characteristics fully lay within bounds of the upper and lower estimates of the model of Voigt-Reuss. Similarly, the values of new effective characteristics of model (9) lay within bounds of fork Voigt-Reuss (figure 2).

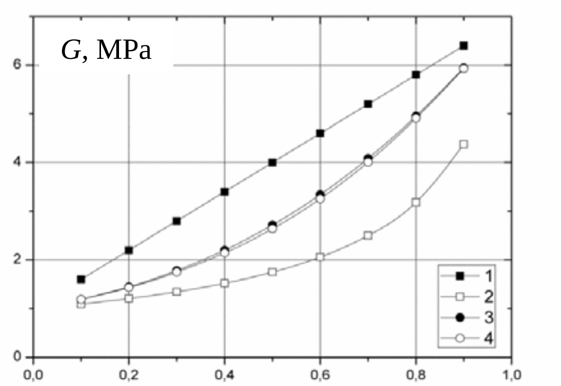

Figure 1. Curves of values of effective characteristics depending on γ_1. 1, 2 – modules Voight and Reuss; 3, 4 – $\tilde{G}_V$, $\tilde{G}_R$.

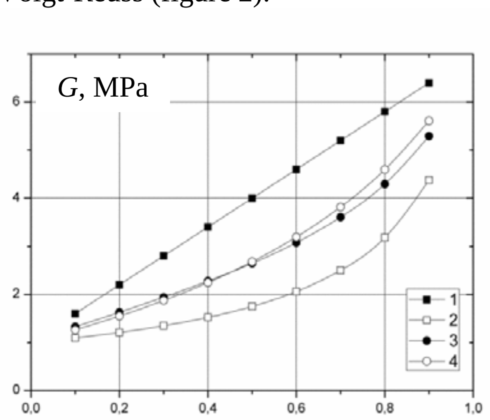

Figure 2. Effective characteristics depending on γ_1.1, 2 –modules Voight and Reuss, 3, 4 – $G_{S,VR}$, $G_{S,HS}$.

The hybrid effective modules (11) are shown in figure 3. The graph shows that the values of the new effective characteristics fully lay within bounds of the upper and lower estimates of the model of Hashin-Shtrikman. Similarly, the values of new effective characteristics of model (9) lay within bounds of the fork Voigt-Reuss (figure 4).

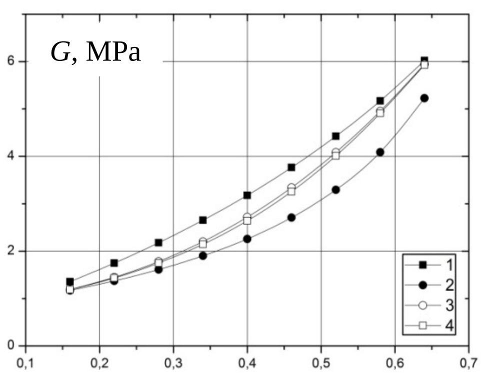

Figure 3. Curves of values of effective characteristics depending on γ_1. 1, 2 – modules Hashin and Shtrikman; 3, 4 – $\tilde{\tilde{G}}''$, $\tilde{\tilde{G}}'$.

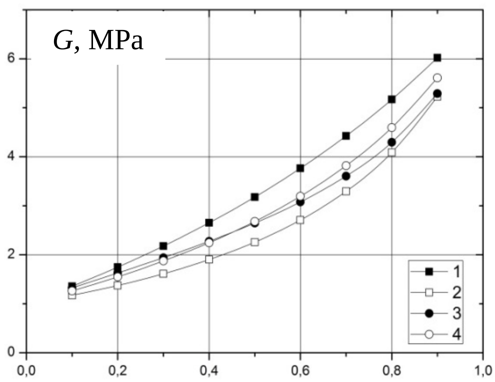

Figure 4. Effective characteristics depending on γ_1.1, 2 –modules Hashin and Shtrikman, 3, 4 – $G_{S,VR}$, $G_{S,HS}$.

Table 1 gives comparison of the values for effective modules (9). The table shows that the values of effective characteristics of models (9) are practically the same in the range of 5-7 %.

Table 2. Effective characteristics depending on volume of fraction γ_1

γ_1	G_V ,MPa	G_R , MPa	$G^{''}$, MPa	$G^{'}$, MPa	$G_{S,VR}$, MPa	$G_{S,\mathrm{HS}}$, MPa
0.1	1.621	1.094	1.357	1.172	1.346	1.291
0.2	2.214	1.207	1.748	1.373	1.719	1.659
0.3	2.887	1.346	2.178	1.613	2.138	2.106
0.4	3.415	1.522	2.652	1.902	2.622	2.638
0.5	4.254	1.750	3.178	2.258	3.185	3.258
0.6	4.696	2.059	3.765	2.708	3.843	3.960
0.7	5.288	2.500	4.425	3.294	4.591	4.728
0.8	5.841	3.182	5.170	4.089	5.420	5.531
0.9	6.416	4.375	6.021	5.229	6.266	6.315

Only practical calculation of a structure made of composite material with the help of a particular model and a comparison of this calculation with the exact solution can serve as a criterion for the selection of a type for new effective characteristics

5. Conclusion

Some new characteristics of effective two-component elastic composites had performed. Couples of new effective characteristics are defined by (7)–(11) and, so, we have 10 efficient modules expressions. Numerical analysis showed that the values of all found effective characteristics are within the scope of external evaluations Voigt-Reiss models. The values of effective characteristics of the models (8-11) are practically the same in the range of 7 %.

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