

# Feasibility of the Davidenkov method for investigation of hoop residual stresses in cold expanded cylinders

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**Abstract.** Cold expansion of holes in hollow cylinders is accompanied by raise of residual stresses in the bulk of the cylinder, the largest of which are hoop residual stresses. Since hoop residual stresses affect critical performance characteristics, assessment of their nature and magnitude is one of the most significant engineering tasks. The most wide spread mechanical methods to define magnitude of hoop residual stresses are G Sachs and N N Davidenkov methods. The paper presents result of experimental studies by Sachs and Davidenkov methods of hoop residual stresses generated in hollow cylinders made of carbon steel AISI 1050 after cold expansion. It is shown that the shape of curves of hoop residual stresses calculated by the mentioned methods, depending on the hoop residual strain generated on the outer surface of the cylinder, can differ (strain is elastic) or coincide (strain is elastic-plastic). It was established that the absolute values of the stresses defined by the mentioned methods in the area adjacent to the hole differ by an average of 15%, and in the area adjacent to the outer surface differ by an average of 20%.

## 1. Introduction

Cold expansion is one of the most effective methods for finishing and hardening of holes in hollow cylinders [1-6]. However, with the ratio of cylinder external diameter to hole diameter  $D/d \geq 3$ , cold expansion forms significant residual stresses in entire volume of a cylinder [1, 2, 7]. At the same time, the largest in absolute value stresses are the hoop residual stresses, which define the most important performance characteristics of components, such as fatigue strength [8-10], wear resistance [3, 7], dimensional stability [11, 12], etc. In this regard, determination of hoop residual stresses generated in a cold expanded cylinder is an important engineering problem.

Review has shown [13-16] that most of the existing methods for determination of residual stresses in cylinders are mechanical methods (the use of which is usually accompanied by destruction of the product). The most widespread mechanical methods include methods of G Sachs [17] and N N Davidenkov [18]. Sachs method consists in successive removal of layers of metal from the internal or external surface of the samples and measurement of hoop and axial strains on the outer or inner radius, respectively [14]. The method helps to define hoop, radial and axial residual stresses simultaneously. It is noted [13] that this method is very labor-intensive, time-consuming, and places high demands on the means of measurement due to the small quantities of strains to be measured. In cases only hoop residual stresses have to be determined, it is reasonable to apply Davidenkov method in order to save time and make it less labour intensive. This method implies cutting the cylinder along its axis and measuring the change in the median diameter without removal of metal from the internal surface of the cylinder [13, 14]. In [13], it is noted that the magnitude of hoop residual stresses



determined with this method (by cutting the cylinder) is approximate, since its calculation is made on the assumption that the stresses are distributed in the cylinder wall hyperbolically as in bending of a curved beam. In this regard, it is reasonable to expect that the stress distribution may be different and the results obtained will have an error, but the magnitude of this error is obscure.

The purpose of this paper is to investigate experimentally hoop residual stresses in the cold expanded hollow cylinders using methods of G Sachs and N N Davidenkov (reduced to cutting the cylinder along its axis) and to compare the results.

## 2. Methods of research

Experiments were carried out on samples (figure 1) made of steel AISI 1050 with a hole diameter of  $d = 5$  mm. External diameter  $D$  of the samples was 15 mm ( $D/d = 3$ ) and 25 mm ( $D/d = 5$ ). Length  $L$  of the sample was equal to 10 and 40 mm. Samples were machined on a lathe 'DMG Mori CTX 310 ecoline'; holes were drilled and then reamed with hand reamers. The total number of samples was 72. Three samples were used in each experiment.

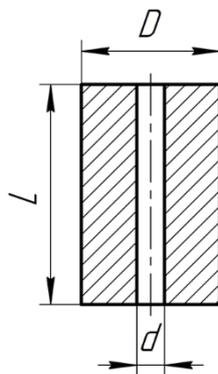


Figure 1. Sample

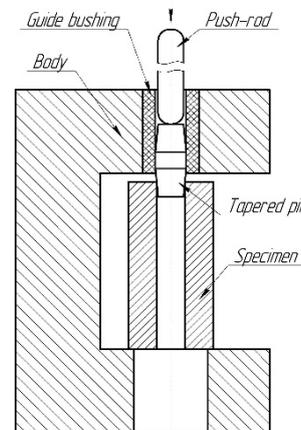


Figure 2. Direct cold expansion

Direct cold expansion was used in this study (figure 2). To achieve cold expansion, a tapered pin was forced through the hole. Interference was 0.9%, 3.4% and 7.1%. It was defined by the following equation:

$$\text{Interference \%} = \frac{d_p - d}{d} \times 100,$$

where  $d_p$  represents the diameter of the cylindrical part of the pin, and  $d$  represents the specimen hole diameter.

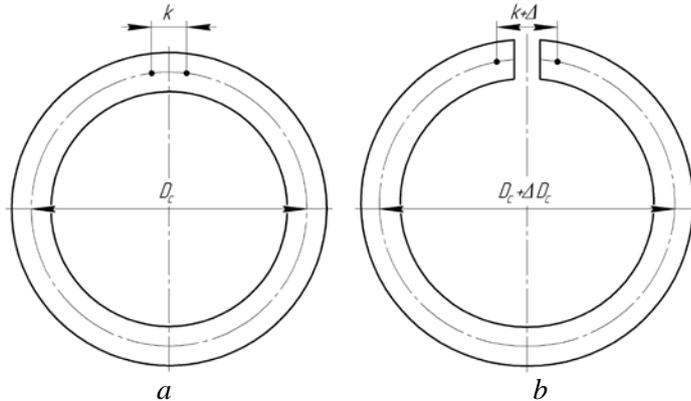
The tapered pin (figure 2) used for cold expansion was made of WC-8%Co (92% of tungsten carbide and 8% of cobalt) with a cones angle equal to  $6^\circ$  and a width of the cylindrical margin equal to 3 mm. Oil fluid MR-7 was used as a lubricant. Tests were performed on the universal testing machine 'IR5082-500'. The cold expansion speed was equal to 0.008 m/s. The sample was held in the device shown in figure 2.

According to Davidenkov method, hoop residual stresses  $\sigma_\theta$  were determined by the change in the median diameter of the sample (a hollow cylinder) after cutting it along the axis (figure 3). The calculation was performed using formula [13], which in our notation (figure 4) is as follows:

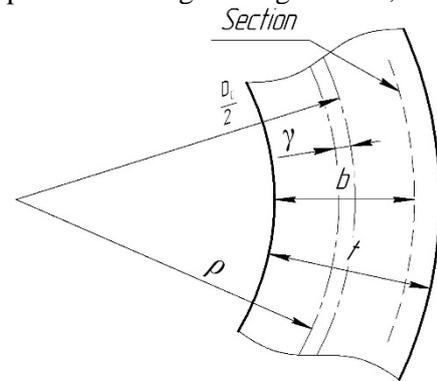
$$\sigma_\theta(b) = \frac{1}{1-\nu^2} \cdot \frac{E}{D_C - 2\gamma} \cdot \left( \frac{b - \frac{t}{2} + \gamma}{b - \frac{t}{2} + \frac{D_C}{2}} \right) \cdot \Delta D_C, \quad (1)$$

where  $E$  is the Young's modulus of the sample material, MPa;  $\nu$  is the Poisson's ratio of the sample material;  $D_C$  is the diameter of the median surface of the sample, mm;  $\gamma$  is the distance from the

neutral axis of the sample wall section to the sample center of gravity, mm;  $t$  is the specimen wall thickness, mm;  $b$  is the distance from the given section to the internal surface of the sample, mm;  $\Delta D_C$  is the increment of the median surface diameter of the sample after cutting it along the axis, mm.



**Figure 3.** Change in the median diameter of the sample:  
a – before cutting; b – after cutting



**Figure 4.** Coordinates of the sample section

Distance  $\gamma$  was found by the formula [13]:

$$\gamma = R_C - \rho = \frac{D_C}{2} - \frac{t}{\ln \frac{D}{D-2t}}, \quad (2)$$

where  $R_C$  is the radius of the sample median surface, mm;  $\rho$  is the radius of the sample neutral layer, mm;  $D$  is the external diameter of the sample, mm (considered to be the same after cutting). The  $\Delta D_C$  value was determined by increment  $\Delta$  of distance  $k$  between the indentations of the conical indenter left on the median circle of the sample end, using ratio  $\Delta D_C = \Delta/\pi$ . The specified distance was measured before and after the cutting using the microscope ‘UIM 21’ (the distance before cutting was about 4 mm). Cutting was performed on a CNC wire-EDM machine, model DK 7725.

Hoop residual stresses were calculated by Sachs method with the use of I A Birger formula [14], which in our notation is as follows:

$$\sigma_\theta(r) = \frac{E}{1-\nu^2} \cdot \left[ \frac{R^2 - r^2}{2r} \cdot \left( \frac{d\varepsilon_\theta}{dr}(r) + \nu \frac{d\varepsilon_z}{dr}(r) \right) - \frac{R^2 + r^2}{2r^2} \cdot (\varepsilon_\theta(r) + \nu\varepsilon_z(r)) \right], \quad (3)$$

where  $R$  is the radius of the external surface of the sample, mm;  $r$  is the radius of the internal surface of the sample (cutting radius), mm;  $\varepsilon_\theta(r)$  is the hoop strain at the outer radius as a result of cutting internal surface to radius  $r$ ;  $\varepsilon_z(r)$  is the axial strain at the outer radius as a result of cutting the internal surface to radius  $r$ . Residual stresses were calculated using the experimental technique described in [19, 20].

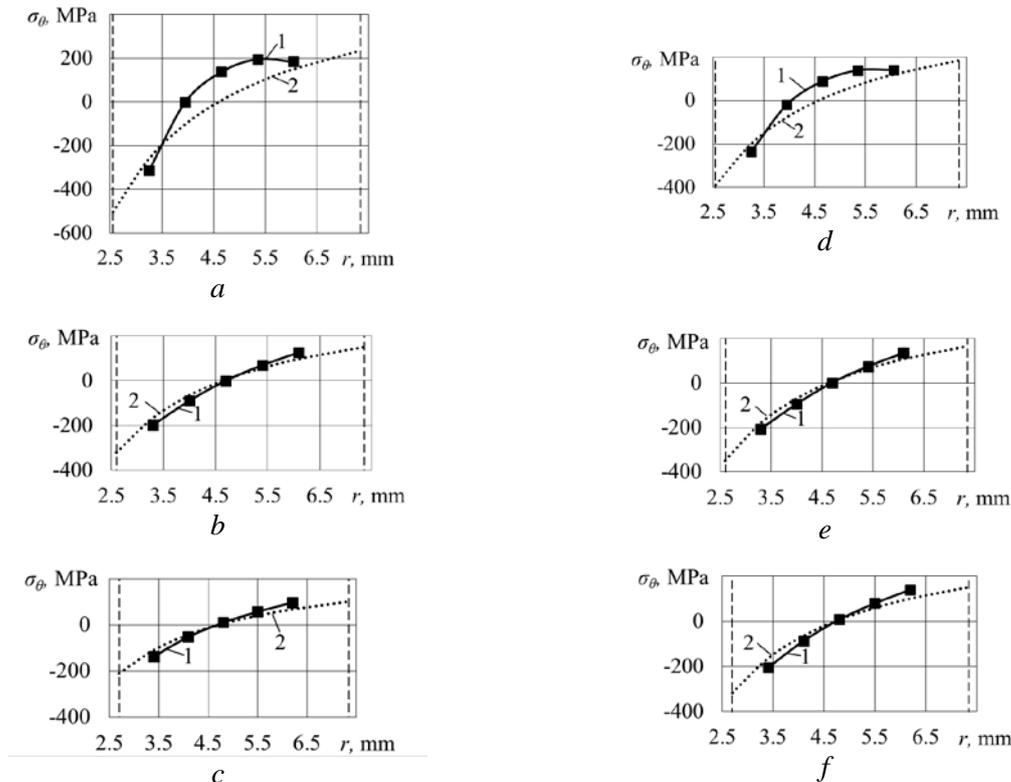
Young's modulus  $E = 210$  GPa and Poisson's ratio  $\nu = 0.3$  were used in calculations.

### 3. Results and discussion

Distribution of hoop residual stresses  $\sigma_\theta$  along radius  $r$  of the cold expanded samples (stress curves) is shown in figure 5 ( $D = 15$  mm) and figure 6 ( $D = 25$  mm). We can see that both methods show compressive hoop residual stresses in the zone adjacent to the hole.

Samples with  $D = 15$  mm cold expanded with 0.9% interference (figure 5, a and d), which leads to elastic hoop residual strain  $\Delta D/D$  at the external surface equal to 0.04% for the samples with the length of 40 mm and 0.03% for the samples with the length of 10 mm, show different forms of the curves obtained by different methods. With the increase in the interference to 3.4% (figure 5, b and e), hoop residual strain becomes elastic-plastic (for the length of 40 mm, it is equal to 0.33%; for the length of 10 mm, it is 0.29%), and the forms of curves obtained by different methods coincide. With further increase in the interference to 7.1% (figure 5, c and f), hoop residual strain continues to increase (to

0.79% for 40 mm long samples, to 0.74% for 10 mm long samples), and the forms of curves obtained by different methods also coincide.



**Figure 5.** Curves for hoop residual stresses obtained by Sachs method (1) and by Davidenkov method (2), in the cold expanded samples with  $D = 15$  mm:

*a* – Interference = 0.9%,  $L = 40$  mm; *b* – Interference = 3.4%,  $L = 40$  mm;

*c* – Interference = 7.1%,  $L = 40$  mm; *d* – Interference = 0.9%,  $L = 10$  mm;

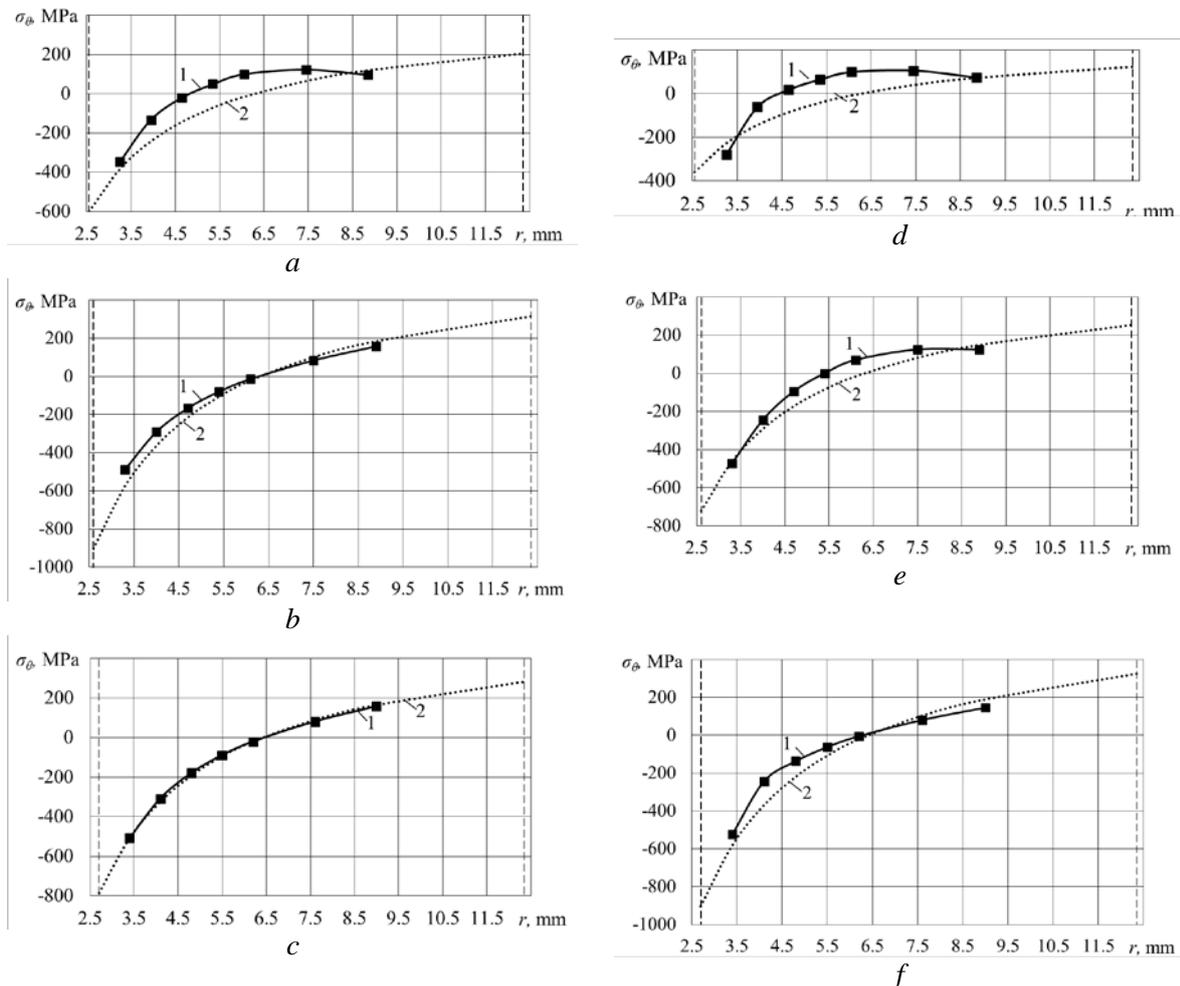
*e* – Interference = 3.4%,  $L = 10$  mm; *f* – Interference = 7.1%,  $L = 10$  mm.

Here and below, dashed lines show internal and external surfaces of the samples

Only in one of the experiments on samples having  $D = 25$  mm (interference is 7.1% and sample length is 40 mm), hoop residual strain on the external surface was elastoplastic (0.24%), and the form of curves of hoop residual stresses was the same for both methods (figure 6, *c*). In other experiments, hoop residual strain was elastic ( $\Delta D/D < 0.2\%$ ). Particularly, when the interference is 0.9% (figure 6, *a* and *d*), the strain is 0.05% for 40 mm long samples, and it is equal to 0.03% for 10 mm long samples. With the increase in the interference to 3.4% (figure 6, *b* and *e*), hoop residual strain increases to 0.12% for 40 mm long samples, and it increases to 0.08% for 10 mm long samples. When interference is 7.1% and the sample length is 10 mm (figure 6, *f*), hoop residual strain is 0.18%. Curves obtained by different methods for samples with  $\Delta D/D < 0.2\%$  are different (figure 6, *a*, *b*, *d*, *e* and *f*). It is obvious that with increasing hoop residual strain, the difference in the form of curves becomes less significant. This is probably due to the fact that distribution of hoop residual stresses in the wall of the sample becomes close to the hyperbolic law under the whole plastic deformation ( $\Delta D/D > 0.2\%$ ).

The absolute value of hoop residual stresses in the area adjacent to the hole (at a distance of 0.7 mm from the surface) for the samples of both diameters determined by Davidenkov method differs from the value defined by Sachs method on the average by 15% (less than 50 MPa). In the region adjacent to the external surface (at a distance of about 1 mm from the surface for the samples with

$D = 15$  mm and at a distance of about 3 mm for the samples with  $D = 25$  mm), these stresses differ on the average by 20% (less than 40 MPa).



**Figure 6.** Curves for hoop residual stresses obtained by Sachs method (1) and by Davidenkov method (2), in the cold expanded samples with  $D = 25$  mm:

- $a$  – Interference = 0.9%,  $L = 40$  mm;  $b$  – Interference = 3.4%,  $L = 40$  mm;  
 $c$  – Interference = 7.1%,  $L = 40$  mm;  $d$  – Interference = 0.9%,  $L = 10$  mm;  
 $e$  – Interference = 3.4%,  $L = 10$  mm;  $f$  – Interference = 7.1%,  $L = 10$  mm

#### 4. Conclusions

1. The form of curves for hoop residual stresses generated during cold expansion of hollow cylinders made of AISI 1050 (the hole diameter is 5 mm, the outer diameter is 15 mm and 25 mm, the length is 10 mm and 40 mm, the interferences from 0.9% to 7.1%) obtained under elastic-plastic hoop residual strain on the external surface of the cylinder ( $\Delta D/D > 0.2\%$ ) by the method of Davidenkov (reduced to cutting of the cylinder along the axis) coincides with the form of curves obtained by Sachs method. When  $\Delta D/D < 0.2\%$ , the form of the curves obtained by these methods differs, and the smaller the value of  $\Delta D/D$  is, the greater the difference is.
2. The nature of the hoop residual stresses (their sign) defined by different methods is the same. Thus, the absolute value of the stresses in the region adjacent to the hole (at the distance of 0.7 mm from the surface) differs on the average by 15% (not more than 50 MPa), and in the region adjacent to the external surface (at a distance of about 1 mm from the surface for the

samples with  $D = 15$  mm and at a distance of about 3 mm for the samples with  $D = 25$  mm) these stresses differ on the average by 20% (not more than 40 MPa).

3. The method of N N Davidenkov, reduced to cutting of the cylinder along its axis, without further incremental removal of metal layers from the internal surface, is a simple and effective method to determine hoop residual stresses with the required accuracy and can be used instead of the method of G Sachs.

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