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Parametric identification of the controlled object based on transfer characteristics

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Abstract. Classical identification techniques are shown; their strengths and shortcomings are analyzed. The correlations associating direct indexes of transfer characteristic quality of the controlled object and its pole-zero configuration are received. The identification technique of the linear dynamic object based on correlations is developed. The numerical model is given.

1. Introduction

According to the classification criteria automation objects identification techniques are divided into several groups – in accordance with scope of initial information, type of experiment, type of object of research, mathematical tool, etc.

The most widespread and commonly occurring identification techniques are used to be a frequency method and a method with the help of transfer function [1-10].

As a rule, the frequency identification technique is used in laboratory setting. It implies a possibility of artificial influence on the controlled object by means of a sinusoidal signal of different frequencies. It is assumed that the object structure is known, it is necessary to define its parameters, b_m, b_{m-1},… b_1, a_n, a_{n-1} ,… a_0:

$$W(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\ldots+1}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_0}. \tag{1}$$

Based on (1), we obtain complex harmonic locus as a sum of the real frequency response and the imaginary frequency response:

$$W(j\omega)=\frac{b_m(j\omega)^m+b_{m-1}(j\omega)^{m-1}+\ldots+1}{a_n(j\omega)^n+a_{n-1}(j\omega)^{n-1}+\ldots+a_0}=P(\omega)+jQ(\omega). \tag{2}$$

In order to receive test values, real frequency response $P(j\omega_1)$ and imaginary frequency response $Q(j\omega_1)$ influence the controlled object by means of sinusoidal signal with frequency ω_1. Equating the theoretical real frequency response and the imaginary frequency response (2) and test values, we obtain two equations

$$P_{th}(\omega_1)=P_{ex}(\omega_1);$$
$$Q_{th}(\omega_1)=Q_{ex}(\omega_1).$$

However, in (1), and correspondingly in (2), $m+n$ of unknown parameters is contained, for that reason to define all the parameters of system transfer function, it is essential to conduct $k/2$ experiments, where $k=m+n+1$, m, n are the order of a numerator and a denominator, correspondingly. Thus, having solved a set of k equations, all the parameters of transfer function are defined.

Together with the frequency technique identification method based on transfer characteristics is widely used. The controlled object is influenced with unit step function and the object reaction is obtained - transfer characteristic in the form of continuous curve or values array with incremental sampling. Approximating the transfer function by analytic expression and differentiating this expression, we shall obtain impulse response. In engineering grapho-analytical methods have become a frequent practice, thus allowing to calculate the parameters for the given controlled object structure.

In accordance with the form of the transfer characteristic curve the additional constructions for approximation are provided and approximation formulas for calculation are applied. So, for identification of the controlled object described by differentiation equation $T\frac{d}{dt}y(t)+y(t)=kx(t)$, it is essential to define the parameters k, T. Let us consider fig. 1 showing determination methods of system parameters.

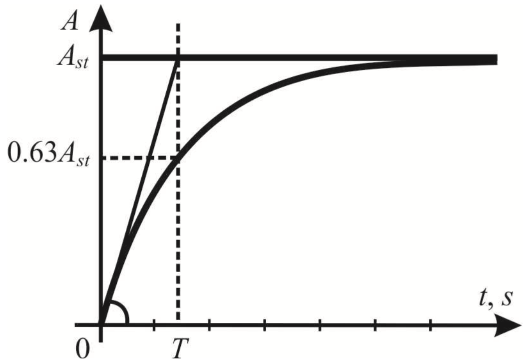

Figure 1. Transfer characteristic of the first-order object.

As the transfer characteristic is the reaction of the controlled object to the step function, the coefficient of object transfer $k=\frac{A_{st}}{x}$, where A_{st} is a stationary value, x is step impact on the object.

Time constant T can be defined in two ways. While defining T by the first method, a tangent to the curve of transfer characteristic in the point t=0 is plotted, from a cross point of tangent and direct $A=A_{st}$ a vertical to time axis is drawn. Using the second method, it is assumed that T corresponds to the time of achievement by the curve of transfer characteristic 63% from value A_{st}, fig. 1.

In a similar way, delay systems and vibration systems are identified, fig. 2.

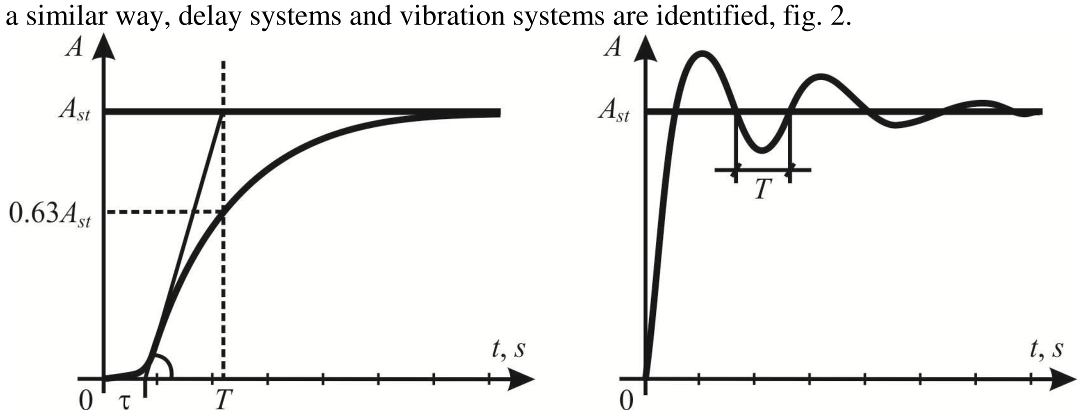

Figure 2. Transfer characteristics: the delay object (on the left); the vibration object (on the right).

Suggested identification techniques have their own strengths and shortcomings. The frequency technique is suitable for identification of not high order objects; otherwise it leads to a large number of experiments that is not always acceptable in virtue of the controlled object specific character. The method gives reasonably exact results in the case when the controlled object structure is a priori known, but in this connection a large number of experiments and calculations for identification process are required.

In its turn, identification technique based on transfer characteristic identifies the objects less accurately by reason of approximation and approximate formulas application. The main shortcoming of that technique is that it does not provide the occurrence of zeros in the controlled object, i. e. the transfer function of such object shall be $W(s)=\dfrac{ke^{\tau s}}{a_n s^n + a_{n-1}s^{n-1}+...+a_0}$. This technique is fairly simple in calculations and it is recommended to apply for the systems of not higher than the second order.

It is known that system transfer characteristic $h(t)$ on unit step function is described by the Heaviside formula:

$$h(t)=\frac{G(0)}{H(0)}+\sum_{k=1}^{n}\frac{G(s_k)}{s_k H'(s_k)}e^{s_k t}, \tag{3}$$

where $G(s)$, $H(s)$ are a nominator and a denominator of the controlled object, s_k is a k-pole of the object.

However (3) can be presented as:

$$h(t)=\frac{G(0)}{H_0(0)}+\frac{\prod_{i=1}^{n}|s_i|}{\prod_{j=1}^{m}|N_j|}\sum_{k=1}^{n}\frac{\prod_{j=1}^{m}(s_k-N_j)}{s_k\prod_{i=1,i\neq k}^{n}(s_k-s_i)}e^{s_k t}, \tag{4}$$

where s_k and N_j are the poles and zeros of the object.

Relationship (4) connects the pole-zero configuration of the object with transfer characteristic curve $h(t)$.

Thus, the problem solution of the pole-zero configuration, corresponding to the given curve of transfer characteristic $h(t)$ is of great interest.

2. Problem statement

Let us assume that unit step impact X is given to the input of the controlled object. In the output, the reaction of the object to this impact is received, fig. 3.

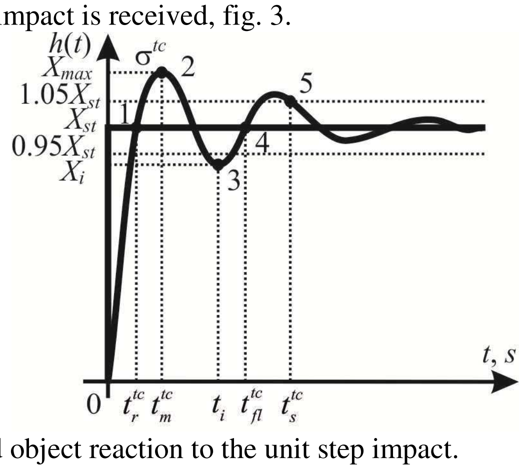

Figure 3. The controlled object reaction to the unit step impact.

It is necessary to identify the transfer characteristic curve according to a probable pole-zero configuration of the system on the root flat.

3. The solution of the identification problem

It is known that the system dynamics is influenced both by all system poles and zeros [11]. However, a degree of impact of one or another pole and zero is different. The type of transfer function depends on the system pole-zero configuration: steady, aperiodic, vibrational.

The first step in identification is the choice of the controlled object structure. The designer has to define a number of poles and zeros. The transfer function object order can be predetermined by an iterative method sophisticating it until we arrive at the satisfactory result. Taking into account the form of the transfer function curve, it is possible to reduce calculations. For example, the object with the transfer function curve, fig. 3, as vibration has minimum two complex conjugate poles, we can exclude versions with one or two real poles, as a steady-state process will take place. Then, if the obtained results of identification with the given structure turn out to be unsatisfactory, the transfer function object order increases, poles and/or zeros are added.

For the given object, the field equation is derived:

$$H(s,s_1,s_2...s_n)=(s-s_1)(s-s_2)...(s-s_n), \tag{5}$$

where s is a Laplace operator; s_i are the object poles. The zeros form equation $G(s)=k(s–N_1)(s–N_2)\ldots(s–N_m)$, where k is a constant factor of the transfer function, N_j are predetermined zeros.

Those equation poles (5) which influence the summation curve are called dominant. The information on the choice and predetermination of dominant poles is given in [11].

Let us assume that for the transfer function curve, fig. 3, the dominant poles will be $s_{1,2}=\delta_1\pm\omega_1 j$. Then (5) is as follows:

$$H(s,\delta_1,\omega_1,s_3...s_n)=(s-(\delta_1+\omega_1 j))(s-(\delta_1-\omega_1 j))(s-s_3)...(s-s_n), \tag{6}$$

Then, on the transfer function curve we define main direct indexes of quality: overregulation σ^{tc}; overregulation time t_s^{tc}; characteristic time t_r^{tc}.

For overregulation the following relationship is true:

$$\sigma^{tc}=\frac{\xi}{X_{st}}, \tag{7}$$

where overshoot $\xi=X_{max}– X_{st}$; X_{st} is a steady state value; X_{max} is maximum.

In its turn, for the vibration system overshoot, ξ is defined in accordance with [8] by the expression:

$$\xi=2A_1\frac{\omega_1}{\omega_0}\exp\left[\frac{\delta_1}{\omega_1}\left(\pi+\sum_3^n\varphi_k-\sum_1^m\Phi_j\right)\right]+\sum_3^m B_k e^{s_k t_m}, \tag{8}$$

where $W(s)=\dfrac{G(s)}{H(s)}$; $s_{1,2}=\delta_1\pm\omega_1 j$ are dominant poles; φ_k and $\Phi_j\in[-\pi;\pi]$ are angles formed by dominant pole s_1, other system pole or zero and positive direction of an X-axis of the root flat; $\omega_0=\sqrt{\delta_1^2+\omega_1^2}$; $A_1=\left|\dfrac{G(s_1)}{s_1H'(s_1)}\right|$; $B_k=\left|\dfrac{G(s_k)}{s_kH'(s_k)}\right|$; $t_m=\dfrac{1}{\omega_1}\left(\pi+\sum_3^n\varphi_k-\sum_1^m\Phi_j\right)$.

Having substituted expression (8) into (7), we shall obtain:

$$\sigma^{tc}=\frac{2A_1\dfrac{\omega_1}{\omega_0}\exp\left[\dfrac{\delta_1}{\omega_1}\left(\pi+\sum_3^n\varphi_k-\sum_1^m\Phi_j\right)\right]+\sum_3^m B_k e^{s_k t_m}}{X_{st}}. \tag{9}$$

According to the cosine theorem, the angle formed by dominant pole $s_1=\delta_1+\omega_1 j$ of the root flat, one of the zeros N_j and positive direction of the X-axis is defined by formula [12]:

$$\Phi_j = \operatorname{sign}\left(\delta_1 - \operatorname{Re}(N_j)\right) \cdot \arccos\frac{l^2\left(s_1, N_j\right) + l^2\left(N_j, \operatorname{Im} N_j\right) - l^2\left(s_1, \operatorname{Im} N_j\right)}{2l\left(s_1, N_j\right) \cdot l\left(N_j, \operatorname{Im} N_j\right)}, \tag{10}$$

where l is a distance between the points on the root flat. For example, $l(a,b)$ is a distance from point a to point b.

In the same way, for the angle formed by point $s_1=\delta_1+\omega_1 j$, one of non-dominant poles of the system s_k and positive direction of X-axis of the root flat:

$$\varphi_k = \operatorname{sign}(\delta_1 - \operatorname{Re}(s_k)) \cdot \arccos\frac{l^2(s_1, s_k) + l^2(s_k, \operatorname{Im} s_k) - l^2(s_1, \operatorname{Im} s_k)}{2l(s_1, s_k) \cdot l(s_k, \operatorname{Im} s_k)}. \tag{11}$$

Having substituted (10) and (11) into (9), we obtain:

$$f\left(\delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m, k\right) = \sigma^{tc}. \tag{12}$$

Relationship (12) connects overregulation σ^{tc} estimated on the transfer function curve, with the predetermined by designer poles $s_1,s_2…s_n$, zeros $N_1,N_2…N_m$ and constant factor k of transfer function of the controlled object.

The Heaviside formula presented in (4) allows setting actual values of output signal at the particular period of time according to values of the controlled object zeros and poles.

Let us set peak time X_{max} by variable t_m^{tc}. Overregulation is calculated from point 2 (fig. 3) of the transfer function curve, where the output signal reaches value X_{max}, so with regard to (4) we obtain:

$$h(t_m^{tc}, \delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m, k) = X_{max}. \tag{13}$$

On the other hand, the point of the transfer function curve is the extreme point, thus

$$\frac{dh(t_m^{tc}, \delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m, k)}{dt} = 0. \tag{14}$$

In a similar way to (13), at time period t_s^{tc} the Heaviside formula will be as follows:

$$h(t_s^{tc}, \delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m, k) = X_{st} \pm 0.05 X_{st}. \tag{15}$$

For time period $t= t_r^{tc}$:

$$h(t_r^{tc}, \delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m, k) = X_{st}. \tag{16}$$

In its turn, it is obvious that for the constant factor of the transfer function of the controlled object it is true:

$$\left| \frac{k \prod_{i=1}^{m} N_i}{\left(\delta_1^2 + \omega_1^2\right) \prod_{j=1}^{n} s_j} \right| = X_{st}. \tag{17}$$

For the controlled object, the complementary equations can be predetermined, which define species of predetermined zeros and poles (real or complex conjugate). Let us assume that the designer predetermines a couple of complex conjugate zeros, then

$$\begin{cases} \operatorname{Re} N_k = \operatorname{Re} N_{k+1}, \\ \operatorname{Im} N_k = -\operatorname{Im} N_{k+1}, \end{cases} \tag{18}$$

and for the real zero or pole it is true:

$$\begin{cases} \operatorname{Im} N_j = 0, \\ \operatorname{Im} s_i = 0. \end{cases} \tag{19}$$

In accordance with (4), when the output signal of the controlled object reaches values X_i at time periods t_i (for example, point 3 in fig. 3), the following relationship takes place:

$$h(t_i, \delta_1, \omega_1, s_3 ... s_n, N_1 ... N_m) = X_i. \tag{20}$$

Let us consider the flex point 4 of the transfer function curve in fig. 3. It is known that the second derived function at the flex point of the curve is equal to zero, so if we assume that at point 4 $t=t_{fl}^{tc}$, we obtain

$$\frac{d^2h(t_{fl}^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m)}{dt^2}=0. \tag{21}$$

Let us connect (12)–(21) into the set of equations:

$$\begin{cases} f\left(\delta_1,\omega_1,s_3...s_n,N_1...N_m,k\right)=\sigma^{tc}, \\ h(t_m^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m,k)=X_{max}, \\ \dfrac{dh(t_m^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m,k)}{dt}=0, \\ h(t_s^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m,k)=X_{st}\pm 0.05X_{st}, \\ h(t_r^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m,k)=X_{st}, \\ \left|\dfrac{k\prod_{i=1}^{m}N_i}{\left(\delta_1^2+\omega_1^2\right)\prod_{j=1}^{n}s_j}\right|=X_{st}, \\ \operatorname{Re}N_k=\operatorname{Re}N_{k+1}, \\ \operatorname{Im}N_k=-\operatorname{Im}N_{k+1}, \\ \operatorname{Im}N_j=0, \\ \operatorname{Im}s_i=0, \\ h(t_i,\delta_1,\omega_1,s_3...s_n,N_1...N_m)=X_i, \\ \dfrac{d^2h(t_{fl}^{tc},\delta_1,\omega_1,s_3...s_n,N_1...N_m)}{dt^2}=0. \end{cases} \tag{22}$$

The set of equations (22) connects predetermined by the designer zeros, poles and the constant factor of the controlled object transfer function with direct indexes of quality: overregulation, characteristic time, setting time, and with values X_{max} and X_{st} defined on the transfer function curve obtained while signaling to the object input in a stepped configuration.

Solution (22) allows defining zeros $N_1…N_m$, poles $s_{1,2}=\delta_1\pm\omega_1 j$, $s_3…s_n$ and the constant factor of object transfer function k relating to defined direct indexes of quality on the transfer function curve.

By reason of complexity and the nonlinear nature of the set of equations (22), the solution is performed numerically.

The equations (18) and (19), the sets (22) allow finding a solution with predetermined zeros and poles, thus reducing calculations.

In its turn, relations (20) and (21) are not basic and are added to system (22) by the designer who wants to improve identification accuracy under the given transfer function object order. The number of given relations as in (20) and (21) is defined by the designer.

4. Accuracy evaluation of identification

After identification, it is very important to evaluate the error of the obtained results. On calculated poles $s_{1,2}=\delta_1\pm\omega_1 j$, $s_3…s_n$ and zeros $N_1…N_m$, the transfer characteristic graph is constructed. To evaluate accuracy, it is supposed to discretize the predetermined and the obtained on roots transfer characteristics at time intervals Δt. Let us set values of the given transfer function at time periods in $h^{tc}(t_i)$, and calculated – $h^{cc}(t_i)$. Then at each time step t_i error is estimated. Absolute or ratio error, root-mean-square error, etc. can be taken as an assessment criterion.

Let us take a ratio error as the assessment identification criterion. So,

$$\delta_i=\frac{\left|h^{tc}\left(t_i\right)-h^{cc}\left(t_i\right)\right|}{h^{cc}\left(t_i\right)}100\ \%.$$

In its turn, identification results are considered to be satisfactory under condition of

$$\max\left(\delta_i\right) \le \delta_{req},$$

where $\max(\delta_i)$ is a maximum ratio error at time period t_i, δ_{req} is the given ratio identification error.

5. The example

The transfer function curve of the controlled object is given, fig. 4a. Find its transfer function.

As there is oscillation transient, at first, the object structure is given as $W(s) = \dfrac{k}{\left(s-(\delta_1+\omega_1 j)\right)\left(s-(\delta_1-\omega_1 j)\right)}$. For the given object structure, the set of equations (22) was produced containing relationships which connect overregulation, characteristic and setting time with the predetermined poles and constant k, the derivative at the maximum point and equations setting the species of poles. However, identification with such structure gave unsatisfactory results, fig. 4b, thus the object order increased. Satisfactory results were obtained with the transfer function of the fifth-order object with zeros $N_{1,2}$=–9.412±0.944j, N_3=–9.81, poles $s_{1,2}$=–0.281±2.414j, $s_{3,4}$=–2.167±0.337j, s_5=–17.194, and constant factor k=3.34. The maximum ratio error reached 4.88%.

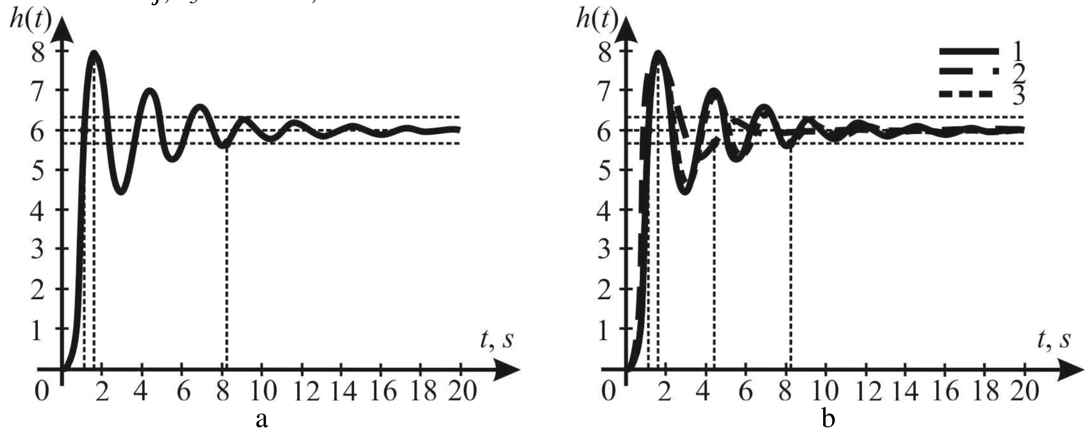

Figure 4. Step response: a – basic identification data; b – the identification results: 1) the given one, 2) the object of the 2nd order, 3) the object of the 5th order.

6. Conclusion

The analysis of classical identification techniques such as frequency identification and with the help of transfer function has been carried out.

The equations connecting quality indexes of transient with the values of zeros, poles and the constant factor of the controlled object transfer function have been obtained.

The object identification technique on the basis of transfer characteristics has been presented.

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