The analysis of the root quality factors of a power unloading system

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Abstract. The article focuses on the development of the technique for the analysis of the robust quality of the interval automatic control systems with the affine uncertainty of their characteristic polynomial coefficients. This technique is based on the edge route of the polyhedron of the system with interval parameters and its mapping on the root plane. Theoretical results are used for the analysis of the root robust quality factors of the power unloading system.

1. Introduction

The automatization of modern manufacturing processes with mechanical and electrically-powered equipment requires the consideration of the control objects features, namely, the variation of their parameters during operation. These objects can have integral parameters which vary within certain limits on the beforehand unknown laws [1, 2]. Systems with such kind of objects are called the interval automatic control systems (IACS).

The designer of IACS has to solve the problem associated with the analysis of the system stability at any variations of the object parameters within known ranges. If the interval system of automatic control is stable, we can state that it has robust stability. Kharitonov's theorem is used for the analysis of the robust stability of the interval automatic control systems [3]. This theorem states that for stability of the interval automatic control systems with a polynomial, it is sufficient when four polynomials formed from the extreme values of coefficients, alternating in pairs (two lower and two upper values), are steady.

The estimation of the robust stability can be root indices: the degree of robust stability α and the degree of robust oscillation $\mu = tg\varphi$. These indices can be found on the basis of the arrangement of poles localization $\Gamma([\alpha])$ of the interval automatic control systems (Figure 1).

It is well known that in order to define the values of robust stability and an oscillation degree at interval uncertainty of the polynomial coefficients, it is sufficient to find its roots in all 2^m vertices of the polyhedron of the interval coefficients, where m is the number of interval coefficients. After that, we choose the desired values [4].

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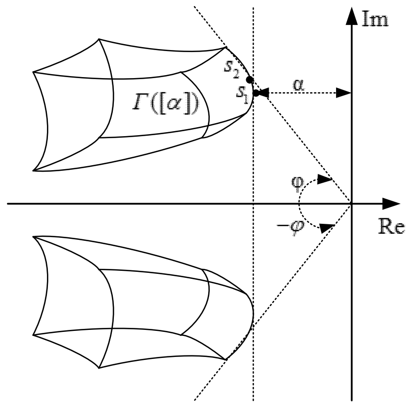

Figure 1. The domain of poles localization of the interval automatic control system

2. Problem statement

If the real parameters of the interval automatic control system are linearly included in the polynomial coefficients, there is a special method based on the edge theorem [5]-[9] to perform the quantitative analysis of the robust stability of the interval automatic control system more precisely. According to this theorem, the interval automatic control system has robust stability on all edges of the polyhedron of the interval parameters of the interval automatic control system. By mapping these edges on the root plane, we can define quantitative assessment of robust quality factors by the values of the interval parameters corresponding to the worst modes of the interval automatic control system operation. However, it is known that the boundary of the poles localization domain of the interval automatic control system is defined not by all edges, but only by exterior ones, which specify the boundary edge route [9]. Therefore, it is important to solve the problem associated with the route construction.

3. The definition of root quality factors of the interval automatic control system on the basis of the edge route of the polyhedron of the interval parameters

Let us consider the procedure of the boundary edge route construction. It consists of the following stages.

1) The calculation of the characteristic polynomial of the IACS of the following image:

$$\sum_{i=1}^{m} T_i \cdot A_i(s) + B(s) = 0\,, \tag{1}$$

where $T_{i\min} \le T_i \le T_{i\max}$, $T_{i\min} = \underline{T_i}$, $T_{i\max} = \overline{T_i}$.

2) The definition of vertices coordinates V_q ($q \in \overline{1,2^m}$ – the number of the vertex) of the polyhedron of interval parameters $P_T = \{T_i \,|\, \underline{T_i} \le T_i \le \overline{T_i}, i = \overline{1,m}\}$.

3) The choice of any complex root node U_q from the poles of the transfer functions of edge branches

$$W_i^{Vq}(\Delta T_i, s) = \frac{\Delta T_i \cdot A_i(s)}{Q^{Vq}(s) + B(s)}\;, \tag{2}$$

where $Q^{Vq}(s) = \sum_{i=1}^{m} T_i^{V_q} \cdot A_i(s)$; $T_i^{V_q}$ – is the value of the interval parameter of the IACS in the V_q vertex.

4) The definition of the outlet angle of the edge branches by all interval parameters from the U_q node found in paragraph 3. This definition is based on (2).

These angles can be found from the phase equation [10]. Let a characteristic polynomial be of n power and polynomial A_i be of r power. We can write down outlet angles Θ_{Ti}^{Vj} of the edge branch from node U_q by the increase of T_i:

$$\Theta_{T_i}^{V_q} = 180° - \sum_{k=1}^{n-1} \Theta_k + \sum_{l=1}^{r} \Theta_l \ , \tag{3}$$

and the reduction of T_i:

$$\Theta_{T_i}^{V_q} = -\sum_{k=1}^{n-1} \Theta_k + \sum_{l=1}^{r} \Theta_l \ , \tag{4}$$

where Θ_k and Θ_l are angles defined by the vectors from node U_q to the k pole and l zero of function (2). If $\Theta_{T_i}^{V_q} \notin [0°;180°]$, then the resultant angle is $\Theta_{T_i} = \Theta_{T_i}^{V_q} - 360^0$.

5) The conditional test is

$$\Theta_{\max}^{V_q} - \Theta_{\min}^{V_q} < 180° \ , \tag{5}$$

where $\Theta_{\max}^{V_q}, \Theta_{\min}^{V_q}$ - relatively maximal and minimal outlet angles. If the statement of the problem is (5), then U_q is on the boundary of the poles localization.

6) Arranging $\Theta_{T_i}^{V_q}$, $i = \overline{1,m}$ in an increasing order and in accordance with this arrangement to write down the sequence of edges $R_{T_i}^{V_q}$, $i = \overline{1,m}$, forming the direct edge route.

7) Finding faces G_{ij}, edge images $R^q{}_i$ and $R^q{}_j$ which can intersect.

It is necessary to get conditions of these intersections. Let us write down the equations of face plane reflection G_{ij}:

$$T_i A_i(s) + T_j A_j(s) + \sum_k T_k^q A_k(s) + B(s) = 0 \ . \tag{6}$$

If in (6), we specify $s = s_r = \alpha + j\beta$, $r \in \overline{1,n}$ and distinguish real and imaginary components we will get the combined equations consisting of two linear equations which combine s_r with two variables T_i and T_j:

$$\begin{cases} \operatorname{Re} A_i(\alpha,\beta) \cdot \operatorname{Im} A_j(\alpha,\beta) - \operatorname{Re} A_j(\alpha,\beta) \cdot \operatorname{Im} A_i(\alpha,\beta) = 0; \\ \operatorname{Re} A_j(\alpha,\beta) \cdot \operatorname{Im}\left[\sum_k T_k^q \cdot A_k(\alpha,\beta) + B(\alpha,\beta)\right] - \operatorname{Im} A_j(\alpha,\beta) \cdot \operatorname{Re}\left[\sum_k T_k^q \cdot A_k(\alpha,\beta) + B(\alpha,\beta)\right] = 0, \end{cases} \tag{7}$$

If the system of the image as in (7) has solutions for all combinations of the interval parameters by $\beta \neq 0$ then boundary S_r consists of intersecting edge branches $R^q{}_i$ and $R^q{}_j$. If the images of two successive edges $R^q{}_i$ and $R^q{}_j$ of face G_{ij} intersect, it is required to add two opposite edges for this face in the direct edge route.

8) Mapping of the obtained boundary edge route on the root plane and the definition of root robust quality factors of the system on its basis.

4. The quality analysis of the power unloading system with interval parameters

Power unloading systems (PUS) should be capable of ensuring smooth and accurate shifting of the cargo within the working area by direct manual operator's action over the cargo applying small directed stress, which steers the cargo along the path and assigns the velocity of cargo shifting. PUS is a two mass system with a flexible connection. The rope is used for connecting the cargo and an electric drive.

The analysis of dynamic properties of the properties of power unloading systems (PUS) is a very urgent problem since the availability of the rope in the system can cause the oscillation in this system. The system is required to be a high-performance one and be able to shift the cargo of any mass and length of the rope from the known ranges.

The analysis of the transfer function of the PUS on disturbance under the steady-state condition with different transfer functions of the controller has shown that it is reasonable to use the PI-controller, which ensures astaticism of the first order. Its transfer function is as follows: $W_c(s) = (k_1 + k_2 s)/s$. The given controller has two setting parameters k_1, k_2, determining the quality of the transient processes of the PUS. As a result of mathematical description of PUS [12], the block diagram (Figure 2) is constructed.

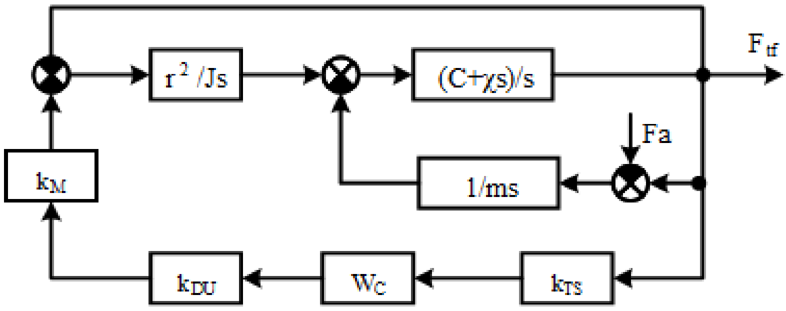

Figure 2. The block diagram of the PUS

k_M is the motor gain coefficient; k_{DU} is the transfer coefficient of the amplifying device; k_{TS} is the tension sensor transfer coefficient; $J=0.7\ \text{kgm}^2$ is the moment of inertia of the electric drive of PUS; $C=C_{sr}/l$ is the coefficient of the cable rigidity, $C_{sr}=1.8\cdot 10^4\ N/m$ is the specific coefficient of the cable rigidity; $\chi=\chi_{sd}/l$ is the coefficient of the rope resilience losses, $\chi_{sd}=0.8\cdot 10^4\ N\cdot s/m$ is the specific coefficient of the rope damping; $r=0.15\,m$ is the block radius; $k_1=1, k_2=0.03$ are parameterizations of the PI controller, $m=[40;200]\,kg$ is the cargo mass; $l=[90;130]\,m$ is the rope length; $k=k_{DU}\cdot k_M\cdot k_{TS}=[3;35]$ is the gain coefficient of the electric part of the electric drive. The input signal of the PUS is external action force F_a which drives the section of cargo in the weightlessness environment. The output signal of the PUS is the tighting force of cable F_t.

Thus, we will get the transfer function of the closed PUS, which input signal is the operator's stress and the output signal is the variation of the rope tensile stress:

$$W(s)=\frac{J\chi_{y\partial}\cdot s^2+C_{y\partial}J\cdot s}{J[l][m]\cdot s^3+(J+[m]r^2(1+k_2k))\cdot\chi_{y\partial}s^2+(C_{y\partial}(J+[m]r^2)+k[m]r^2(\chi_{y\partial}k_1+k_2\text{C}_{y\partial}))\cdot s+[m]\text{C}_{y\partial}kk_1r^2}.$$

A characteristic polynomial of PUS with affine uncertainty is as follows: $[T_1]\cdot A_1(s)+\frac{1}{[T_2]}\cdot A_2(s)+[T_3]\cdot A_3(s)+B(s)=0$, where $[T_1]=[l]$; $[T_2]=[m]$; $[T_3]=[k]$; $A_1(s)=Js^3$; $A_2(s)=J(\chi_{y\partial}s+C_{y\partial})s$; $A_3(s)=r^2(\chi_{y\partial}k_2s^2+(\chi_{y\partial}k_1+C_{y\partial}k_2)s+C_{y\partial}k_1)$; $B(s)=r^2s(\chi_{y\partial}s+C_{y\partial})$. It is necessary to construct the boundary edge route of the polyhedron of SUP interval parameters on the basis of the developed algorithm of robust stability and find the values of root robust quality factors on its reflection.

The parametric polyhedron is formed by three interval parameters and includes eight vertices: $V_1(\underline{T_1},\underline{T_2},\underline{T_3})$, $V_2(\underline{T_1},\overline{T_2},\underline{T_3})$, $V_3(\underline{T_1},\overline{T_2},\overline{T_3})$, $V_4(\overline{T_1},\underline{T_2},\underline{T_3})$, $V_5(\overline{T_1},\overline{T_2},\underline{T_3})$, $V_6(\overline{T_1},\overline{T_2},\overline{T_3})$, $V_7(\overline{T_1},\underline{T_2},\overline{T_3})$, $V_8(\underline{T_1},\underline{T_2},\overline{T_3})$. The poles of SUP $[-2.04-j3.36\ \ -2.04+j3.36\ \ -1.24]$ are defined in the first vertex of the polyhedron. Three sets of zeroes are also defined in the first vertex of the polyhedron (roots $A_1(s)=0$ have values $[0\ \ 0\ \ 0]$, roots $A_2(s)=0$ have values $[-2.25\ \ 0]$ and roots $A_3(s)=0$ have values $[-33.3\ \ -2.25]$). Outlet angles $\Theta_{T_2}^{V1}=13.48^\circ$, $\Theta_{T_1}^{V1}=-8.19^\circ$, $\Theta_{T_3}^{V1}=78.26^\circ$ for root node $U_1=-2.04+j3.36$ were also found. By these angle parameters, boundary condition U_1 is fulfilled, therefore V_1 belongs to the edge route. Since $\Theta_{T_1}^{V1}<\Theta_{T_2}^{V1}<\Theta_{T_3}^{V1}$, we can write down the consequence of the interval parameters variation from vertex V_1: $T_1\to T_2\to T_3\to T_1\to T_2\to T_3$. This sequence corresponds to the direct edge route illustrated in Figure 3.

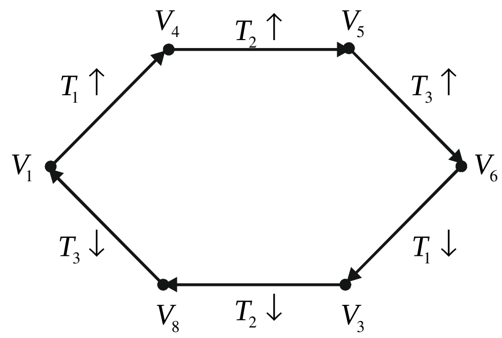

Figure 3. The direct edge route

Let us check whether the boundary edge route has intersecting edges. Three faces G_{31}, G_{21} and G_{32}, converge in vertex V_1, where the indices correspond to the indices of the interval parameters. Let us work out combined equations for each face (7). Since there is no solution for combined equations G_{31}, G_{21} and G_{32}, it means that there is no intersection for the edge. As a result, the boundary edge route preserves the image illustrated in Figure 3. Its mapping on the root plane is shown in Figure 4

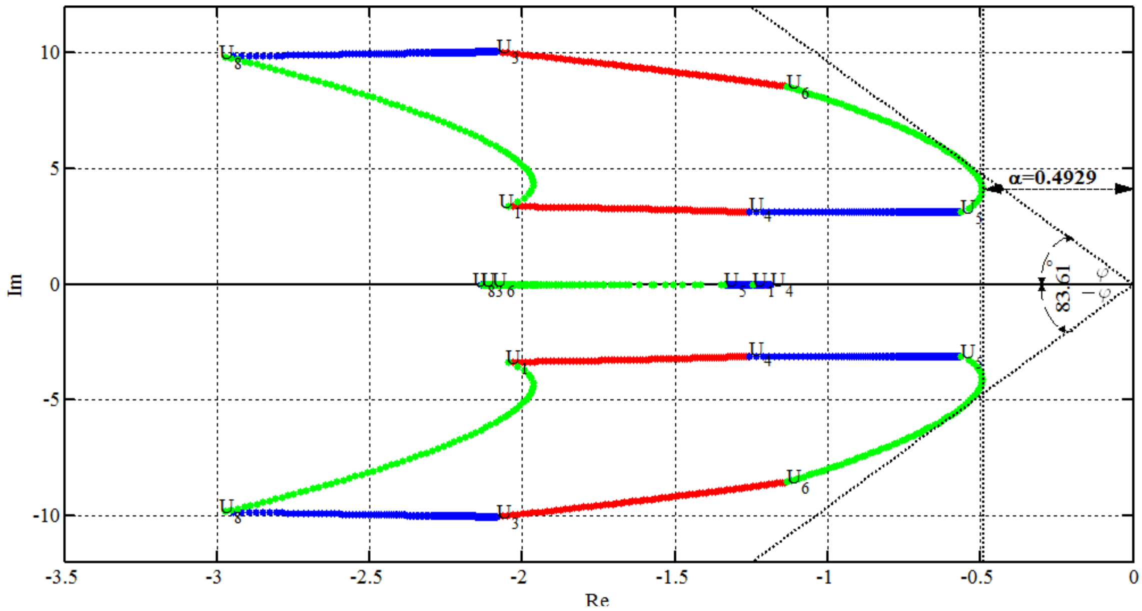

Figure 4. The interval root locus

As we can see from Figure 4, the image of the edge route defines the degree of robust stability $\alpha = 0.49$ and the degree of robust oscillation $\mu = 9.6$, which corresponds to the sector with angle $\varphi = \pm 84^\circ$. It is to be noted that these quality factors are defined by the image of one edge T_3.

Let us construct the transient processes of the power unloading system by different values of parameters T_1, T_2, T_3 on the edge route (Figure 5). From Figure 5, we can define their maximal decay time $t_{\max} = 7.15$ s and the value of overcontrol $\sigma_{\max} = 46\%$. The maximal decay time corresponds to the desired degree of robust stability α.

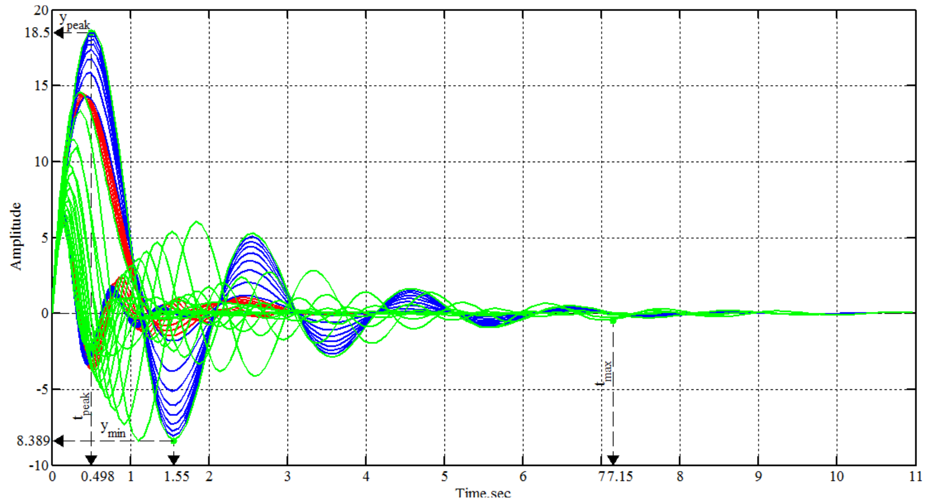

Figure 5. Transient processes on the boundary edge route

5. Conclusion

The algorithm of the boundary edged route of the polyhedron of the interval parameters of the system was developed in the given work. The algorithm was developed on the basis of the edge theorem. It was established that in order to define the rooted robust quality factors, it is sufficient to reflect only the edges which form the boundary edged route on the root plane. The developed algorithm was tested on a real power unloading system with three interval parameters.

Acknowledgments

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