Maximization of the robust stability degree of interval systems by means of a linear controller in the presence of limits

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Abstract. The authors of this article developed the technique of combined parametric synthesis of a linear controller on the basis of the coefficient method and the method of mathematical programming capable of ensuring the maximization of the degree of robust stability of a control system. The article also presents the numerical illustration of the PI controller synthesis of the position stabilization system of an underwater object.

1. Introduction

The issue of intensification of the parameterization of a controller by unfavorable system modes of operation is urgent by designing robust controllers for control systems with interval parameters. One of the methods to solve this problem can be the acquisition of such robust stability margin which would be able to ensure the maximal operation speed in the system by most unfavorable parameters of the control object. The research of domestic and foreign scientists [1-10] is of interest for the solution of the above-specified problem. The approach [3] is mainly employed in these works, where standardized polynomials on a certain parameter are used. By applying the coefficient methods for the solution of this problem on the basis of the interval coefficient stability and oscillation indices in [11], the sufficient conditions for the parameterization of the controller are obtained, which ensures the maximal lower estimate of the degree of robust stability. Since such approach allows defining only a quasi maximal degree of the robust system stability, it is desirable to maximize the stability till the maximal degree increases the operation speed of the integral control system. To cope with this problem, we suggest using the theory of mathematical programming, as it was done for the steady-state system in [4]. Thus, it is necessary to solve the problem associated with the definition of the maximal degree of stability of the linear system and parameters of its controller on the basis of the interval characteristic polynomial if there are restrictions on a Q factor and system oscillation.

2. Robust expansion of the mathematical programming technique

Let the interval characteristic polynomial be specified as $D(s,\vec{k})=\sum_{i=0}^{n}[d_i(\vec{k})]s^i$, where $[d_i(\vec{k})]=[\underline{d_i(\vec{k})};\overline{d_i(\vec{k})}]$, $\vec{k}$ – the vector of parameterization of the controller . It is required to carry out robust expansion for employing the method of mathematical programming to solve the assigned task. It is suggested using the vertex interval characteristic polynomial:

$$D^q(s,\vec{k}) = \sum_{i=0}^{n} d_i^q(\vec{k}) s^i \ , \tag{1}$$

which corresponds to the worst mode of the interval control system operation. When $s = \alpha + j\beta$, the vertex interval characteristic polynomial has the following image:

$$D^q\left(\vec{k},\alpha,\beta\right) = \operatorname{Re} D^q\left(\vec{k},\alpha,\beta\right) + \operatorname{Im} D^q\left(\vec{k},\alpha,\beta\right). \tag{2}$$

Based on (2), let us work out combined equations:

$$\begin{cases} \operatorname{Re} D^q\left(\vec{k},\alpha,\beta\right) = 0; \\ \operatorname{Im} D^q\left(\vec{k},\alpha,\beta\right) = 0; \\ \dfrac{\partial \operatorname{Re} D^q\left(\vec{k},\alpha,\beta\right)}{\partial \alpha} = 0; \\ \dfrac{\partial \operatorname{Im} D^q\left(\vec{k},\alpha,\beta\right)}{\partial \alpha} = 0; \\ \dots \\ \dfrac{\partial^c \operatorname{Re} D^q\left(\vec{k},\alpha,\beta\right)}{\partial \alpha^c} = 0; \\ \dfrac{\partial^c \operatorname{Im} D^q\left(\vec{k},\alpha,\beta\right)}{\partial \alpha^c} = 0. \end{cases} \tag{3}$$

The solution of the given system is the parameterization of the $\vec{k}$ controller and the value of the maximal degree of α stability in the worst mode of the interval control system. The number of system equations is defined by the number of the decision variable.

3. The synthesis technique of a robust controller

Employing the coefficient method [11] of the controller, the synthesis of a quasi maximal degree of $\eta_{\max}$ robust stability, the definition procedure of the corresponding vertex of the polyhedron of interval coefficients of the interval characteristic polynomial and combined equations (3), the technique allowing maximizing the controller parameters of a quasi maximal degree of $\eta_{\max}$ robust stability was developed. The technique includes the following stages:

To find the parameterization of the $\vec{k}$ controller, ensuring the quasi maximal degree of robust stability $\eta_{\max}$.

To construct the vertex route of the polyhedron of interval coefficients of the interval characteristic polynomial [13] based on the found parameterizations of the controllers.

To represent the found vertices on the rooted plane and to define vertex V^*, which image is the nearest to the imaginary axes.

To find vertex V^*, it is required to solve nonlinear equations (1) and define the limited values of robust quality factors and the parameterizations of controller $\vec{k}$ which ensure these quality factors.

4. Numerical example

Let us carry out the parametric synthesis of the robust PI controller $W_C(s) = (k_1 + k_2 s)/s$ for the position stabilization system of a descent underwater vehicle (DUV) on the basis of the developed technique. The mathematical model in the form of the block diagram illustrated in fig. 1 is created on the basis of the differential equations describing the dynamics of the certain components of the stabilization system of a DUV. The system has the following constant parameters [1]: k_{m2}=0,3 (Nm/A) the transfer constant of a shock-absorbing hoist (SAH) drive on torque; k_{e2}=1 (Vs/rad) is a back EMF coefficient of the motor of SAH ; J_2=0,5 (kgm^2) is the moment of inertia of SAH; R_2=0,1 (m) is a

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drum radius of SAH and interval parameters, $\chi=[35;200]$ (Ns/m) is an equivalent value of the damping factor of the cable; $C=[800;4.25\times10^{3}]$ (N/m) is an equivalent value of stiffness of the cable; $m_{DUV}=[4.47\times10^{3};4.65\times10^{3}]$ (kg) is mass of the DUV.

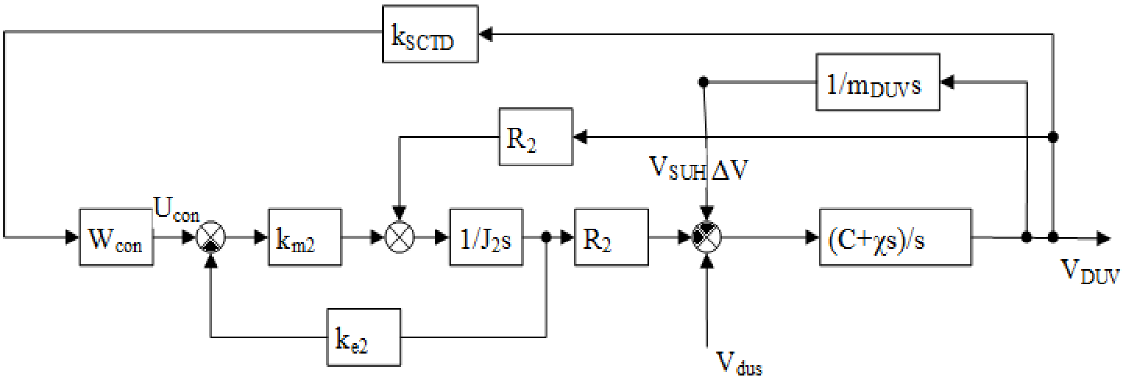

Figure 1. The structure diagram of the stabilization system of a DUV

The transfer function on the extraneous signal is obtained on the basis of the system structure: $W_{зам}(s)=F_{н}/V_{ЗАМС}=[b_3]s^3+[b_2]s^2+[b_1]s/[a_3]s^3+[a_2]s^2+[a_1]s^1+[a_0]$, where $[b_1]=[1072800;\ 5928750]$; $[b_2]=[762135;4371000]$; $[b_3]=[31290;279000]$. $[a_0]=[240+214560k_1;1275+1185750k_1]$; $[a_1]=[35930.5+214560k_2+9387k_1;198565+1185750k_2+83700k_1]$; $[a_2]=[2912.5+9387k_2;15405+83700k_2]$; $[a_3]=[894;930]$.

Let us specify at the first stage the synthesis of the tolerable index of oscillation $\delta_{Д}=4.8\times10^{-2}$ from the expression in [11]. We will get relation $k_1(k_2)=\dfrac{(15405+83700k_2)^2-8863941.6+52931880k_2}{3736368}$.

Then, it is required from the expression in [11] to find dependence k_2 on $\eta_{\max}$:

$$k_2(\eta_{\max})=\sqrt[3]{-\frac{Q1(\eta_{\max})}{2}+\sqrt{-\frac{Dis(\eta_{\max})}{108}}}+\sqrt[3]{-\frac{Q1(\eta_{\max})}{2}+\sqrt{-\frac{Dis(\eta_{\max})}{108}}},\quad Dis(\eta_{\max})=-4e(\eta_{\max})^3-27Q1(\eta_{\max})^2,$$

$$e(\eta_{\max})=\frac{\gamma(\eta_{\max})^3}{3\nu(\eta_{\max})^2}-\frac{\upsilon(\eta_{\max})}{\nu(\eta_{\max})},\qquad \nu(\eta_{\max})=-\frac{\lambda}{\delta_{Д}\overline{a_2}}\underline{b_1}^{3},\qquad Q1(\eta_{\max})=\frac{(2\gamma(\eta_{\max}))^3}{27\nu(\eta_{\max})^3}-\frac{\upsilon(\eta_{\max})\gamma(\eta_{\max})}{3\nu(\eta_{\max})}+\frac{\vartheta(\eta_{\max})}{\nu(\eta_{\max})},$$

$$\gamma(\eta_{\max})=\frac{1}{\delta_{Д}\overline{b_1}}\underline{b_1}^{2}\overline{b_0}-\lambda(\overline{b_1}(\underline{b_0}-\eta_{\max}\overline{b_1})+\frac{\underline{b_1}^{2}}{\delta_{Д}\overline{a_2}}(2\underline{a_1}+\frac{1}{\overline{b_1}}\underline{b_1}\overline{a_1})-\underline{b_1}\overline{b_0}),$$

$$\upsilon(\eta_{\max})=\frac{\overline{b_0}}{\overline{b_1}}(\frac{2\underline{a_1}\underline{b_1}}{\delta_{Д}}-\underline{b_0}\underline{a_2})-\lambda(\frac{\underline{b_1}\underline{a_1}}{\delta_{Д}\overline{a_2}}(\underline{a_1}-\frac{2\underline{b_1}\overline{a_1}}{\overline{b_1}})+\overline{b_1}(\underline{a_0}-2\eta_{\max}\overline{a_1})+\overline{a_1}(\underline{b_0}-\frac{\underline{b_1}\overline{b_0}}{\overline{b_1}})),$$

$$\vartheta(\eta_{\max})=\frac{\overline{b_0}}{\overline{b_1}}(\frac{\underline{a_1}^{2}}{\delta_{Д}}-\overline{a_2}\,\overline{a_0}))-\lambda(\overline{a_1}(\underline{a_0}-\eta_{\max}\overline{a_1}))-\frac{\underline{b_1}\overline{a_1}}{\overline{b_1}}(\frac{\underline{a_1}^{2}}{\delta_{Д}\overline{a_2}}-\overline{a_0})).$$

After that, on the basis of dependences $k_1(k_2(\eta_{\max}))$ and $k_2(\eta_{\max})$ we will form a set of inequalities [11]

$$\begin{cases} \underline{F_g(k_1,k_2,\eta_{\max})} = \underline{d_1}(k_1\ (k_4(\eta_{\max})), k_4(\eta_{\max})) - \overline{d_2(k_4(\eta_{\max}))}(n-m-1)\eta_{\max} \geq 0; \\ \underline{F_0(k_1,k_2,\eta_{\max})} = \underline{d_0(k_1\ (k_2(\eta_{\max})))} - \overline{d_1(k_1\ (k_2(\eta_{\max})), k_4(\eta_{\max}))}\eta_{\max} + 2\underline{d_2(k_2(\eta_{\max}))}\dfrac{\eta_{\max}{}^2}{3} \geq 0; \\ \underline{\delta_1(k_1,k_2,\eta_{\max})} = \dfrac{\underline{d_1^2(k_1\ (k_2(\eta_{\max})), k_2(\eta_{\max}))}}{\overline{d_0(k_1\ (k_2(\eta_{\max})))}\,\overline{d_2(k_2(\eta_{\max}))}} \geq \delta_{Д}. \end{cases}$$

Solving this system, we will find $\eta_{\max} = 0.056$ and define $k_2(\eta_{\max}) = 0.078$, $k_1\ (k_2(\eta_{\max})) = 0.079$.

By these parameterizations of the controller, let us find the vertex coordinates in which the system has its minimal degree of stability. Let us construct the marginal edge route of the polyhedron of the interval characteristic polynomial coefficients of the position stabilization system of a descent underwater object and reflect it on its rooted plane. To construct this route, we will use the algorithm developed for the interval characteristic polynomial with the interval uncertainty of coefficients [13]. The obtained route has an image shown in Figure 2.

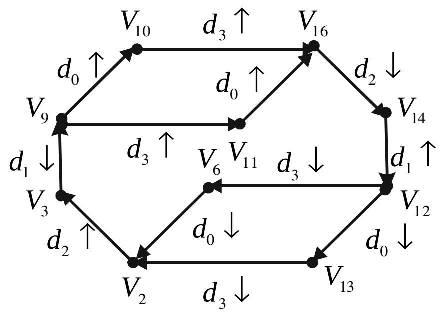

Figure 2. The vertex-edge route

Since all the edge branches by the reflection of the route are convex relative to imaginary axes, in order to define α it is sufficient to reflect only the vertices of the route on the rooted plane. Figure 3 illustrates only those vertices which define the mapped parameter.

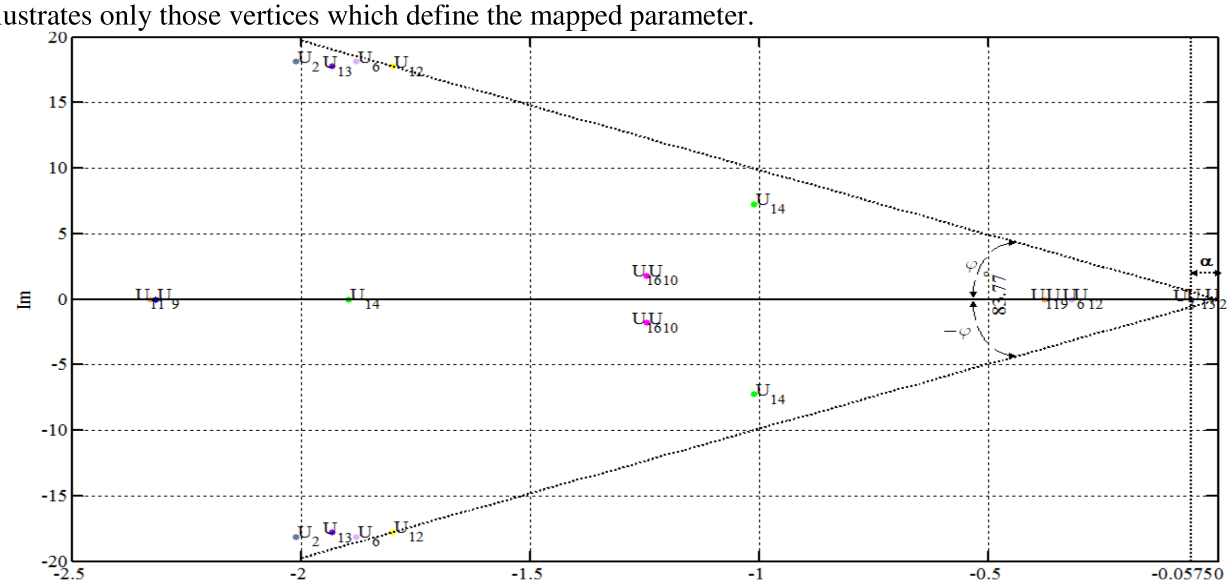

Figure 3. The representation of the vertices of the marginal route

As a result, we will get $\alpha = 0.057$ in vertex $V_2(\underline{d_0};\overline{d_1};\underline{d_2};\underline{d_3})$. At the third stage of the synthesis, let us substitute vertex coordinates V_2 in (2) and get system (3)

$$\begin{cases} \operatorname{Re} D^2(k_1, k_2, \alpha, \beta) = 0; \\ \operatorname{Im} D^2(k_1, k_2, \alpha, \beta) = 0; \\ \dfrac{\partial \operatorname{Re} D^2(k_1, k_2, \alpha, \beta)}{\partial \alpha} = 0; \\ \dfrac{\partial \operatorname{Im} D^2(k_1, k_2, \alpha, \beta)}{\partial \alpha} = 0. \end{cases}$$

The solution of the given system is the parameterization of PI controller $k_1 = 0.011$, $k_2 = 0.16$ and the maximal robust degree of stability $\alpha = 0.07$ of the position stabilization system of a descent underwater object.

5. Conclusion

To increase the operation speed of the integral control system, the technique of a combined parametric synthesis of a robust controller was developed. This technique allows maximizing the quasi maximal robust stability on the basis of two-stage parametric synthesis employing the coefficient methods and the method of mathematical programming. The technique was tested on a numerical illustration.

Acknowledgments

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