# Adaptive interpretation of gas well deliverability tests with generating data of the IPR curve

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**Abstract.** The paper considers topical issues of improving accuracy of estimated parameters given by data obtained from gas well deliverability tests, decreasing test time, and reducing gas emissions into the atmosphere. The aim of the research is to develop the method of adaptive interpretation of gas well deliverability tests with a resulting IPR curve and using a technique of generating data, which allows taking into account additional a priori information, improving accuracy of determining formation pressure and flow coefficients, reducing test time. The present research is based on the previous theoretical and practical findings in the spheres of gas well deliverability tests, systems analysis, system identification, function optimization and linear algebra. To test the method, the authors used the field data of deliverability tests of two wells, run in the Urengoy gas and condensate field, Tyumen Oblast. The authors suggest the method of adaptive interpretation of gas well deliverability tests with the resulting IPR curve and the possibility of generating data of bottomhole pressure and a flow rate at different test stages. The suggested method allows defining the estimates of the formation pressure and flow coefficients, optimal in terms of preassigned measures of quality, and setting the adequate number of test stages in the course of well testing. The case study of IPR curve data processing has indicated that adaptive interpretation provides more accurate estimates on the formation pressure and flow coefficients, as well as reduces the number of test stages.

## 1. Introduction

Deliverability tests with the resulting inflow performance relationship (IPR) curve, run in the gas wells with steady-state filtration, are one of the most informative and common methods well tests to characterize the behavior of well and the bottomhole conditions. Currently, the data obtained via deliverability tests are interpreted using the methods described in [1-3], which are based on Forchheimer binomial equation for gas filtration:

$$p_{\mathrm{r}}^2 - p_{\mathrm{w}}^2 = aq + bq^2 \ , \tag{1}$$

where $p_{\mathrm{r}}^2, p_{\mathrm{w}}^2$ are formation pressure and bottomhole pressure, respectively; $a$ and $b$ - flow coefficients dependent on bottomhole zone parameters and bottomhole structure; $q$ - flow rate. Coefficients $a$ and $b$ for IPR curve model (1) should be estimated using the least square method, with the formation pressure being known [2-4]. IPR interpretation based on model (1) and the least square method is challengeable as a field method, which is attributed to the following facts: the initial formation pressure is difficult to determine, estimates should be robust and accurate, the number of test stages (a number of "cycles" characterized by a stabilized flow when the pressure and the flow rate are recorded) is reduced.

To ensure that the estimates are accurate and robust, in the work [5] we suggest interpreting the IPR curve using integrated IPR curve models with account of additional a priori data on the formation pressure and flow coefficients. However, the question is how to provide additional a priori data on the formation pressure and flow coefficients and to determine the adequate number of test stages to secure preassigned estimate accuracy.

In this research the method of adaptive interpretation of deliverability tests with resulting IPR curve and generating data with variable parameters, additional a priori data on the formation pressure and flow coefficients is suggested and investigated. The method implies that the parameters depend on the number of test stages. To define the formation pressure and to generate data of bottomhole pressure and the flow rate at different test stages, we use the empirical power law for gas filtration, which is widely applied in deliverability test analysis over the years. [6-8]

$$q = \lambda (p_{\mathrm{r}}^2 - p_{\mathrm{w}}^2)^{\gamma} \tag{2}$$

where $\lambda$ - productivity index; $\gamma$ - constant factor with theoretical value ranging from 0.5 (turbulent flow) to 1.0 (laminar flow).

## 2. Models and Algorithms for adaptive interpretation of the IPR curve with generating data

The basis to develop algorithms for gas well deliverability test data interpretation is an integrated system of IPR curve models (1) with variable parameters dependent on the number of the test stage, takes into account of generated values of squared bottomhole pressure $\hat{y}_j$, additional a priori data on the formation pressure $\bar{p}_{\mathrm{r},n}^2$ and flow coefficients $\bar{a}_n, \bar{b}_n$ :

$$\begin{cases} y_i^* = p_{\mathrm{r},i}^2 - a_i q_i - b_i q_i^2 + \xi_i = y_i + \xi_i, i = \overline{1,n}, \\ \hat{y}_j = y_j + \eta_j, \bar{p}_{\mathrm{r},n}^2 = p_{\mathrm{r},n}^2 + \nu_n, j = \overline{1,l}, \\ \bar{a}_n = a_n + \varepsilon_{1,n}, \bar{b}_n = b_n + \varepsilon_{2,n}, n = \overline{2,nk}, \end{cases} \tag{3}$$

where $y_i^* = p_{\mathrm{w},i}^2, q_i$ are values of squared bottomhole pressures and flow rates obtained at test stage number $i$; $\bar{p}_{\mathrm{r},n}^2, \hat{y}_j = \hat{p}_{\mathrm{w},j}^2$ are estimates of formation pressure and generated values of squared bottomhole pressures; $y_j = p_{\mathrm{r},j}^2 - a_j \hat{q}_j - b_j \hat{q}_j^2$ are values of squared bottomhole pressures given by model (1) with generated values of flow rate $\hat{q}_j$; $nk$ is unknown parameter defines the number of test stages $n$ appropriate to secure preassigned estimates accuracy for the formation pressure and flow coefficients $p_{\mathrm{r}}^2, a, b$ ; $\xi_n, \eta_l, \nu_n, \varepsilon_{1,n}, \varepsilon_{2,n}$ are random variables, i.e. errors in measurements, recovery data, and estimates of flow coefficients, as well as deficiencies of gas filtration models (1),(2) etc.

The additional data on formation pressure $\bar{p}_{\mathrm{r},n}^2$ and parameters estimates $\bar{\lambda}_n$ and $\bar{\gamma}_n$ of model (2) can be obtained by solving the following optimization problem:

$$\bar{\boldsymbol{\alpha}}_n = \arg\min_{\boldsymbol{\alpha}} \sum_{i=1}^{n} f(q_i^* - \alpha_1 (\alpha_2 - y_i^*)^{\alpha_3}) \tag{4}$$

where $\arg\min_x f(x)$ is minimum point $x^*$ of function $f(x)$ $(f(x^*) = \min_x f(x))$; $\bar{\boldsymbol{\alpha}}_n = (\bar{p}_{\mathrm{r},n}^2, \bar{\lambda}_n, \bar{\gamma}_n)$ is a vector of estimates; $f$ is a known function. The additional data on flow coefficients $\bar{a}_n, \bar{b}_n$ can be obtained from the system of linear equations:

$$z_i = a q_i + b q_i^2, i = \overline{1,n}, \tag{5}$$

which is the result of grouping models (1),(2) for depression $p_{\text{r}}^2 - p_{\text{w}}^2$ where $z_i = \sqrt[\bar{\gamma}_n]{q_i / \bar{\lambda}_n}$ , $q_i$ is a value of the flow rate obtained at test stage number $i$; $\bar{\lambda}_n, \bar{\gamma}_n$ are optimal estimates obtained by solving problem (4).

The optimal values of squared formation pressure $p_{\text{r},n}^2$ and flow coefficients $a_n, b_n$ of model (3) represented for convenience as matrix

$$\begin{cases} \tilde{\mathbf{y}}_k = F_k \boldsymbol{\alpha}_k + \boldsymbol{\xi}_k, k = \overline{1, n+l}, \\ \bar{\boldsymbol{\alpha}}_{k(n)} = \boldsymbol{\alpha}_k + \boldsymbol{\eta}_k, n = \overline{3, nk}, \end{cases} \tag{6}$$

are calculated using the method of adaptive identification by solving optimization problems (7),(8)

$$\boldsymbol{\alpha}_k^*(\boldsymbol{\beta}_k, h_k) = \arg\min_{\boldsymbol{\alpha}_k} (J_0(\boldsymbol{\alpha}_k, h_k) + J_a(\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k)), \tag{7}$$

$$\boldsymbol{\beta}_k^*, h_k^* = \arg\min_{\boldsymbol{\beta}_k, h_k} J_0(\boldsymbol{\alpha}_k^*(\boldsymbol{\beta}_k, h_k)), \tag{8}$$

where $\tilde{\mathbf{y}}_k = (\tilde{y}_k, k = \overline{1, n+l}, n = \overline{3, nk}) = (y_i^*, \hat{y}_j, i = \overline{1, n}, j = \overline{1, l}, n = \overline{3, nk})^T$ is a combined vector of initial data and generating data on squared bottomhole pressures; $F_k = (\varphi_n, \varphi_l)^T, k = \overline{1, n+l}, n = \overline{3, nk}$ is a combined matrix of vectors $\varphi_n = (1, q_i, q_i^2), i = \overline{i, n}$ and $\varphi_l = (1, \hat{q}_j, \hat{q}_j^2), j = \overline{1, l}$; $\boldsymbol{\alpha}_k = (\alpha_{1,k} = p_{r,k}^2, \alpha_{2,k} = a_k, \alpha_{3,k} = b_k), k = \overline{1, n+l}$ is a combined vector of parameters $(\boldsymbol{\alpha}_i, \boldsymbol{\alpha}_j, i = \overline{1, n}, j = \overline{1, l})^T$ of initial data and generating data; $\bar{\boldsymbol{\alpha}}_{(n)} = (\bar{\alpha}_{1,k} = \bar{p}_{\text{r},k}^2, \bar{\alpha}_{2,k} = \bar{a}_k, \bar{\alpha}_{3,k} = \bar{b}_k)_{(n)}$ is a vector of additional a priori data obtained at stage number $n$; $J_0(\boldsymbol{\alpha}_k) = \sum_{k=1}^{n+l} \omega_k(h_k) \cdot \psi_0(\tilde{y}_k - \tilde{\varphi}_k^T \boldsymbol{\alpha}_k)$, $J_a(\boldsymbol{\alpha}_k^*, \boldsymbol{\beta}_k) = \sum_{j=1}^{3} \beta_{j,k} \psi_a(\omega_{j,k} \bar{\alpha}_{j,k} - a_{j,k})$ are measures of model (6); $\boldsymbol{\beta}_k = (\beta_{j,k}, j = \overline{1,3})$ is a vector of control parameters defining the importance (weight) of additional a priori data $\bar{\alpha}_{j,k}, j = \overline{1,3}$; $\psi_0, \psi_a$ are known functions; $\omega_k(((k-i)/h_k), i = \overline{1, n+l-1}, k = \overline{1, n+l})$ is a weighting function with decay parameter $h_k$ to secure adaptive identification and interpretation $(\omega(x_1) < \omega(x_2), x_1 < x_2)$; $kr_{j,k}$ is an adjustment parameter for additional data $\bar{\boldsymbol{\alpha}}_{k(n)}$.

The solution on the time for deliverability test with the resulting IPR curve to be completed can be taken via visual analysis of the graph (see figures 2-4) or using the criterion for estimates stabilization, where $nk$ is such test stage $k$ that

$$\left| (\alpha_{j,k}^*(\beta_{j,k}^*, h_k^*) - \alpha_{j,k-1}^*(\beta_{j,k-1}^*, h_{k-1}^*)) / \alpha_{j,k}^*(\beta_{j,k}^*, h_k^*) \right| \le \varepsilon_j, \ j = \overline{1,3}, k = k_0, k_0 + 1, .. \tag{9}$$

is a valid inequality, where $\varepsilon_j$ is preassigned accuracy.

It is noteworthy that there are many strategies of generating data of IPR curve $y_i^* = p_{\text{r},i}^2, q_i, i = \overline{1, n}$ to define values of additional flow rates $\hat{q}_j$ and bottomhole pressures $\hat{y}_j = \hat{p}_{w,j}^2, j = \overline{1, l}$ (3). In this research we use a simple method to generate data of the IPR curve that the total number of data $n+l$ is doubled $n=l$,

$$\hat{P}_{\text{w},j} = P_{\text{w},i} + \Delta_i = P_{\text{w},i} + (P_{\text{w},i+1} - P_{\text{w},i})/2 \ , \ \hat{q}_j = \bar{\lambda}_n (\bar{p}_{\text{r},n}^2 - \hat{p}_{\text{w},j}^2)^{\bar{\gamma}_n}, j = \overline{2, n} \tag{10}$$

Where $\bar{p}_{\text{r},n}^2, \bar{\lambda}_n, \bar{\gamma}_n$ are the estimates of power law parameters for gas filtration (2) obtained by solving problem (4).

The algorithm given below represents the method of adaptive interpretation of the IPR curve with generating data:

1. Forming combined vector $\tilde{\mathbf{y}}_k$ (3) of initial and generating data $\hat{q}_j, \hat{p}^2_{w,j}$ (10), beginning with the minimum number of test stage *n* = *2*.

2. Defining the vector of additional data $\bar{\boldsymbol{\alpha}}_{k(n)} = (\bar{\alpha}_{1,k} = \bar{p}^2_{r,n}, \bar{\alpha}_{2,k} = \bar{a}_k, \bar{\alpha}_{3,k} = \bar{b}_k)_{(n)}$ by solving problem (4) and system of linear equations (5).

3. Selecting measures of model (6) quality $J_0(\boldsymbol{\alpha}_k, h_k), J_a(\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k)$.

4. Solving problems (7), (8) using the appropriate method of function optimization.

5. Checking condition (9): if the condition is fulfilled, the test is completed; if condition (9) fails to be fulfilled, the next test stage *n*+1 is arranged, and one should start new research with step 1 of the algorithm.

## 3. Results of IPR curve interpretation for gas wells.

The results of a case study of the deliverability test with the resulting IPR curve run in wells 1 and 2 of the Urengoy gas and condensate field are given in figures 1-4 and tables 1, 2.

For example, figure 1 shows the IPR curves for wells 1 and 2, initial data on five test stages and generating data on bottomhole pressure and the flow rate.

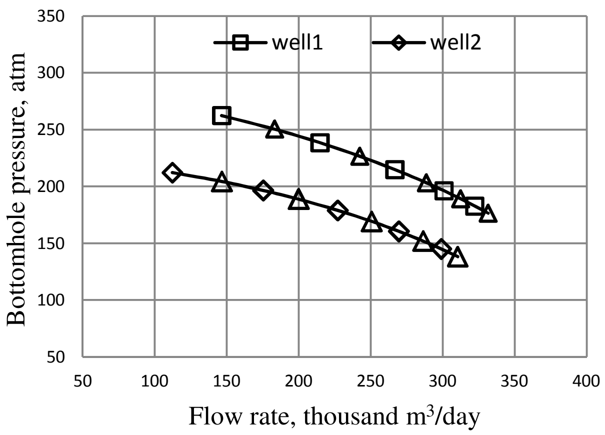


**Figure 1**. *Initial and generating data (Δ) of IPR curves for wells 1 and 2.*

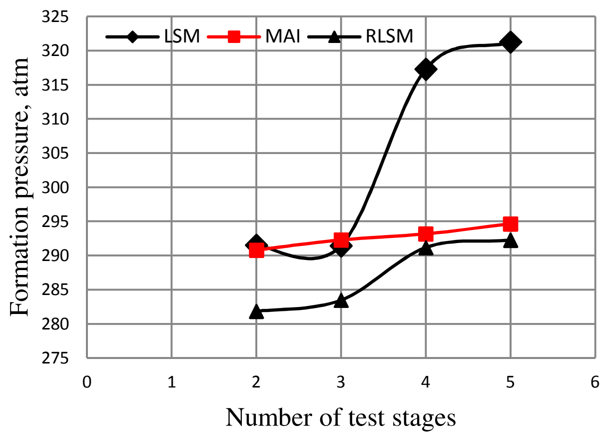


**Figure 2**. Formation pressure estimates for well 1.

Figures 2-4 show the estimates of formation pressure and flow coefficients of well 1, which are obtained using the following techniques:

1. the method of adaptive interpretation (MAI) (7) with quadratic measures of quality $\psi_0(x) = \psi_a(x) = x^2$ by solving the system of linear equations when $\mathbf{kr}_k = (kr_{j,k}, j = \overline{1,3})$ and $\beta_{j,k} = \beta_k, j = \overline{1,3}$ [9].

$$(F_k^T W_k(h_k^*) F_k + \beta_k \mathbf{I})\, \boldsymbol{\alpha}_k^*(\beta_k^*, h_k^*) = (F_k^T W_k(h_k^*) \tilde{\mathbf{y}}_k + \beta_k^* \cdot \mathbf{kr}_k \cdot \bar{\boldsymbol{\alpha}}_{k(n)}), k(n)=2n,\ k = 4,5,\ldots, nk \quad (11)$$

where the estimates of control parameter $\beta_k^*$ and decay parameter $h_k^*$ are defined by solving problem (8) using the downhill simplex method [10]; $W(h_k^*) = diag(\exp((k-i)/h_k^*), i = \overline{1,2n-1})$ is a diagonal matrix of weighting function values;

2. the least squares method (LSM) from (11) with $\beta_k^* = 0, \bar{\boldsymbol{\alpha}}_{k(n)} = 0$

3. the regularized least squares method (RLSM) from (11), with $\bar{\boldsymbol{\alpha}}_{k(n)} = 0$.

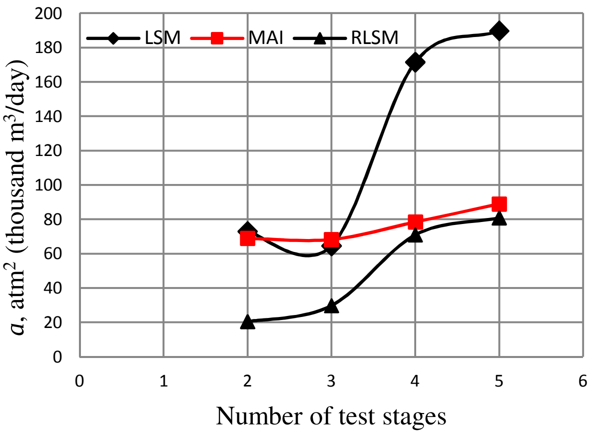


**Figure 3**. Estimates of flow coefficients *a* in well 1.

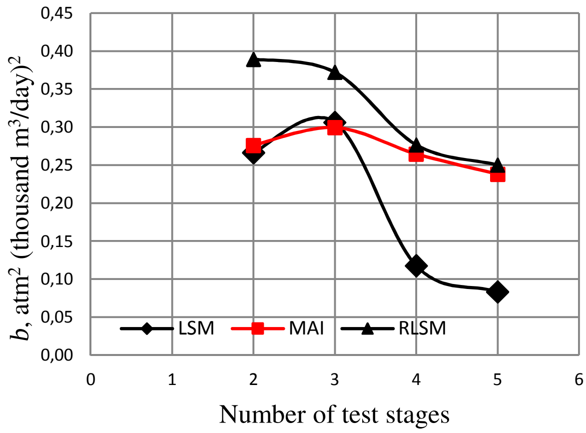


**Figure 4**. Estimates of flow coefficients *b* in well 1.

Table 1 shows the estimates of flow coefficients and the formation pressure of well 1 given by different methods.

Table 2 gives the estimates of the formation pressure and flow coefficients $\bar{\boldsymbol{\alpha}}_{k(n)} = (\bar{\alpha}_{1,k} = \bar{p}^2_{r,n}, \bar{\alpha}_{2,k} = \bar{a}_k, \bar{\alpha}_{3,k} = \bar{b}_k)_{(n)}$ of wells 1 and 2, which are used as additional data in (3) and obtained by solving optimization problem (4) using Gauss-Newton method with $f(x)=x^2$ [9] and the system of linear equations (5).

As can be seen in figures 2–4 and table 1, the suggested method of adaptive interpretation of the IPR curve with generating data as well as taking into account additional data allows obtaining more accurate estimates of the formation pressure and flow coefficients with less amount of field data, compared to the method of least squares. For example, for the adaptive interpretation method, three test stages are enough (see figures 2–4 and table 1).

**Table 1**. Flow coefficients and formation pressure estimates of well 1.

| Number of the test stage ($n$) | The total amount of data ($n+l$) | Method | Flow coefficient estimate | | Formation pressure estimate |
|---|---|---|---|---|---|
| | | | $a_n^* = \alpha_{2,n}^*(\beta_n^*, h_n^*)$, atm²/(thousand m³/day) | $b_n^* = \alpha_{3,n}^*(\beta_n^*, h_n^*)$, atm²/(thousand m³/day)² | $p_{r,n}^* = \alpha_{1,n}^*(\beta_n^*, h_n^*)$, atm |
| 2 | 4 | MAI | 68.95 | 0.2760 | 290.80 |
| | | LSM | 73.02 | 0.2664 | 291.53 |
| | | RLSM | 20.44 | 0.3893 | 281.85 |
| 3 | 6 | MAI | 68.21 | 0.2995 | 292.27 |
| | | LSM | 64.58 | 0.3063 | 291.43 |
| | | RLSM | 29.75 | 0.3721 | 283.46 |
| 4 | 8 | MAI | 78.43 | 0.2646 | 293.12 |
| | | LSM | 171.57 | 0.1177 | 317.29 |
| | | RLSM | 70.96 | 0.2766 | 291.17 |
| 5 | 10 | MAI | 88.97 | 0.2381 | 294.63 |
| | | LSM | 189.76 | 0.0833 | 321.26 |
| | | RLSM | 80.81 | 0.2500 | 292.25 |

**Table 2**. Additional data.

| Number of the test stage ($n$) | Well | Formation pressure $\bar{p}_{r,n}$ atm | Flow coefficient | |
|---|---|---|---|---|
| | | | $\bar{a}$, atm$^2$/(thousand m$^3$/day) | $\bar{b}$, atm$^2$/(thousand m$^3$/day)$^2$ |
| 2 | 1 | 286.8 | 46.85 | 0.3277 |
| | 2 | 226.3 | 29.87 | 0.2385 |
| 3 | 1 | 285.2 | 31.32 | 0.3807 |
| | 2 | 225.7 | 29.00 | 0.2341 |
| 4 | 1 | 286.8 | 49.19 | 0.3205 |
| | 2 | 225.3 | 23.88 | 0.2555 |
| 5 | 1 | 289.0 | 61.94 | 0.2904 |
| | 2 | 225.4 | 24.81 | 0.2511 |

## 4. Conclusion

To overcome the challenges of interpreting deliverability tests with the resulting IPR curve of gas wells, the method of adaptive interpretation has been suggested. This method allows:

1. Generating field data with the resulting IPR curve, using the power law for gas filtration.
2. Accounting additional a priori data on the formation pressure and flow coefficients.
3. Defining estimates of the formation pressure and flow coefficients within the period of test time.
4. Setting the number of test stages adequate for efficient well testing.

The case study of IPR curve interpretation for two wells of the Urengoy gas and condensate field has indicated that adaptive interpretation provides robust and more accurate estimates of the formation pressure and flow coefficients, as well as allows reducing the number of test stages.

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