$$t = c_1,$$
$$x = \frac{1}{\alpha}\sin(\alpha y) + c_2,$$
$$z = -\frac{1}{\alpha}\cos(\alpha y) + c_3 \qquad (20)$$

and

$$t = c_1,$$
$$x = -\frac{1}{\alpha}\sin(\alpha y) + c_2,$$
$$z = \frac{1}{\alpha}\cos(\alpha y) + c_3. \qquad (21)$$

From (20) and (21) we see that curvature lines of the 1-st kind, going through the point $M \in \boldsymbol{E}_4$ and corresponding to the principal curvatures of the 1-st kind not equal to zero belong to one tree-dimensional plane, lie on two circular cylinder of the same radius $\frac{1}{\alpha}$ with common generator and common two-dimensional diametral plane. The curvatures k of all curvature lines of the 1-st kind are the same ($k = \frac{1}{\alpha}$). Torsions of these lines $\kappa = -\frac{1}{\alpha}$, i.e. are also the same in all points.

REFERENCES

1. Vershik A.M., Gershkovich V.Ya. Non-holonomic dynamic systems. Geometry of distribution and variation problems // Results of Science and Technology. Modern problems of mathematics. – V. 16. – Moscow: VINITI, 1987. – P. 5–85.
2. Slukhayev V.V. Geometry of vector fields. – Tomsk: Tomsk State University Press, 1982. – 96 p.
3. Dubrovin B.A., Novikov S.P., Fomenko A.T. Modern geometry. – Moscow: Nauka, 1979. – 759 p.
4. Onishchuk N.M. Geometry of vector field in four-dimensional Euclidian space // International conference on mathematics and mechanics. – Tomsk, 2003. – P. 60–68.

Arrived on 26.05.2006

UDC 330.43

APPLICATION OF ONE-DIMENSION STS-DISTRIBUTION FOR MODELLING MAGNITUDES OF STOCK INDEXES

O.A. Belsner, O.L. Kritskiy

Tomsk Polytechnic University
E-mail: olegkol@mph.phtd.tpu.edu.ru

Modified method STS-GARCH(1,1) has been considered. Modification consisted in rejection of the statement on normal low of lagarithm distribution of time series day increment and in their application for the description of Smoothly Truncated a-Stable (STS)-distribution (smoothly abridged a-stable). The method parameters were found by the technique of maximum likelihood. Statistic investigation of the suggested algorithm accuracy was carried out and decrease of autocorrelation in data structure used for the analysis was shown. The method was used to predict share prices of lag 5.

1. Introduction

Study of properties, calculation of parameters and determination of distribution type of some stochastic process underling the market fluctuations is a main problem of econometrics. Information on distribution is necessary for designing econometric methods (ARCH, GARCH, EGARCH, FIGARCH, FIEGARCH etc., see on the methods in details in [1]), estimation of risk limit value VAR, calculation of probable values of dynamic series in future as well as defining asymptomatic behaviour of distribution function densities. The latter is of particular importance since rare events determining forms and type of their tails correspond to making the most possible profit or suffering the most probable losses.

In most cases logarithms of day increments in financial tool quotations (shares, bonds, swaps, options etc.) do not have normal distribution [2–4]. It is connected with the fact that empirical density distribution function designed on such logarithms has a non-zero excess and asymmetry; there is an oblongness of density function in ε-suburb of mathematical expectation point as well as the so-called «thick tails» (in case when probability of significant changes in prices is higher than that of normal distribution) are observed. All these factors make difficult or impossible to apply the common econometric methods: ARCH(p), GARCH(p,q) and others, which are initially based on the assumption of normal increment and remainder distribution.

Dissatisfaction of financial market participants with the results obtained by normal approximation make the researchers search for new distributions and develop new approaches to empirical financial data processing. Thus, in the works [5–7] to describe time series the Pareto generalized distribution, in [8, 9] the Student generalized t-distribution, in [3] the Laplace distribution, in [10] α-stable distribution has been used. However, at present the idea of combination of all mentioned distributions with normal one is developing increasingly (see, for example, [11]). The idea consists in cutting off the tails of initial density function and their replacement into the tails of normal distribution.

In the given paper a modified model GARCH (1,1) is considered. Modification has consisted in refusal from the assumption on normal law of logarithm distribution of time series day increments and using Smoothly Truncated α-Stable (STS)-distribution for their description [11]. The construction of STS-distribution is performed by finding parameters of normal distributions forming tails and calculating the first and the second initial time. Efficiency of the STS-GARCH(1,1) modification proposed has been shown by means of simulating share prices of RAO UES RTS. In this case 371 values of dollar quotations within the period from January 4, 2003 to June 30, 2004 (the data are given by RTS Company, http://www.rts.ru) are used.

2. General statements

Consider the GARCH (1,1) classical method [12]. Let h_n, n=1,2,..., be some time series. Assume that the autoregressive dependence is true:

$$\bar{\sigma}_n^2 = \gamma V + \bar{\alpha} u_{n-1}^2 + \bar{\beta}\bar{\sigma}_{n-1}^2 = \omega + \bar{\alpha} u_{n-1}^2 + \bar{\beta}\bar{\sigma}_{n-1}^2, \quad (1)$$

where $\gamma>0$, $\alpha>0$, $\beta>0$ are some model coefficients, $V>0$ is the long-term average deviation in the data structure, $\gamma+\bar{\alpha}+\bar{\beta}=1$, $\omega=\gamma V$, $u_n=\ln(h_n)-\ln(h_{n-1})$ are the increment logarithms of time series values h_n, $\bar{\sigma}_n$ is the day volatility, n=1,2,...

Instead $u_n=\ln(h_n)-\ln(h_{n-1})$ one may use, for example, estimation $u_n=\ln(h_n-h_{n-1})h_{n-1}^{-1}$. In both cases u_n are dependable random qualities. Therefore u_n demands for existence of at least two first conditional initial and central moments. To calculate correctly the latter let us assume that in probabilistic space (Ω,F,P), where F is the σ-algebra of subsets Ω, filtration $F=(F_n)_{n\geq 0}$ consisting of σ-subalgebras F_n so that $F_m\subset F_n\subset F$, if $m\leq n$ is set. In doing so the events from F_n are interpreted as available information at the moment of time $(n-1)$.

Then $E(u_i|F_{i-1})=\bar{a}_i$ and $D(u_i|F_{i-1})=\bar{\sigma}_i^2=E(u_i^2|F_{i-1})-\bar{a}_i^2$, i=1,2,... If one assumes that $u_n\sim N(a_n,\sigma_n)$, then, according to GARCH (1,1) methodology the relation is met:

$$u_n = \bar{\sigma}_n\varepsilon_n + \bar{a}_n,\ n = 0,1,2,..., \quad (2)$$

where $\varepsilon_n\sim N(0,1)$ is the standard random magnitude. It allows us to perform simulation of further values of time series h_n by the volatilities $\bar{\sigma}_{n+1}$ calculated according to (1):

$$\ln(h_{n+1}) = \ln(h_n) + \bar{\sigma}_{n+1}\varepsilon_{n+1} + \bar{a}_{n+1},\ n = 0,1,2,...$$

However in random case, if the distribution law u_n is unknown, it is difficult to apply the formula (2). Let us plot the distribution function for u_n, assuming that it refers to STS-distribution with some parameters.

Determination. Let

$$g_\theta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp[i\,x(\mu - t)]\times$$

$$\times\exp\left(-|c\cdot t|^\alpha\left(1-\beta\cdot i\cdot\mathrm{sign}(t)\cdot tg\left(\frac{\pi\alpha}{2}\right)\right)\right)dt$$

be the density function of α-stable distribution with parameter vector (α, β, c, μ), where α is the characteristic exponent, β is the skewness coefficient, c is the scale, μ is the average. Let $h_1(x)$, $h_2(x)$ be densities of normal distribution with parameters (a_i,σ_i^2), i=1,2. Let two real numbers a, $b\in\Re$ be chosen so that $a<\mu<b$, and the relations are satisfied:

$$h_1(a) = g_\theta(a),\ h_2(b) = g_\theta(b),$$

$$\int_{-\infty}^{a} h_1(x)dx = \int_{-\infty}^{a} g_\theta(x)dx,\ \int_{b}^{\infty} h_2(x)dx = \int_{b}^{\infty} g_\theta(x)dx. \quad (3)$$

Name the function of $f(x)$ type density of STS-distribution:

$$f(x) = \begin{cases} h_1(x), x < a; \\ g_\theta(x), x\in[a,b]; \\ h_2(x), x > b. \end{cases} \quad (4)$$

Mathematical expectations and variances (a_i,σ_i^2), i=1,2, are explicitly determined from the equalities (3).

Therefore, $S(x) = \int_{-\infty}^{x} f(t)dt = S(x,a,b,\alpha,\beta,c,\mu)$ will depend on six parameters: a, b, α, β, c, μ. Different possible parameterizations are considered in the monograph [13], where their advantages and disadvantages are discussed.

Let us denote the cutting off probability through p_1 and p_2:

$$p_1 = \int_{-\infty}^{a} h_1(t)\,dt;\ p_2 = \int_{b}^{\infty} h_2(t)\,dt.$$

To determine p_1 and p_2 it is necessary to employ some quadrature formula and calculate integrals correspondingly $\int_{-\infty}^{a} g_\theta(x)dx, \int_{b}^{\infty} g_\theta(x)dx$.

The probabilities p_1 and p_2 play an important part in determination of normal distribution parameters with densities $h_1(x)$, $h_2(x)$. The following theorem is true.

Theorem 1. Let $S(x,a,b,\alpha,\beta,c,\mu)$ be the function of STS-distribution with density $f(x)$, satisfying (4). Then the parameters of the normal distributions with densities $h_1(x)$, $h_2(x)$ are calculated by the formulas:

$$\sigma_1 = \varphi(\Phi^{-1}(p_1))\cdot[g_\theta(a)]^{-1},$$

$$\sigma_2 = \varphi(\Phi^{-1}(p_2))\cdot[g_\theta(b)]^{-1},$$

$$a_1 = a - \sigma_1\Phi^{-1}(p_1),\ a_2 = b + \sigma_2\Phi^{-1}(p_2), \quad (5)$$

where $\varphi(x)$, $\Phi(x)$ are the density and the distribution function of standard normal random value correspondingly.

The theorem proving is based on the use of equalities (3) and satisfaction of the relations:

$$p_1 = \int_{-\infty}^{a} h_1(t)dt = \Phi\left(\frac{a-a_1}{\sigma_1}\right);$$

$$p_2 = \int_{b}^{\infty} h_2(t)dt = 1 - \Phi\left(\frac{b-a_2}{\sigma_2}\right).$$

For the task in (2) of standard STS-distributed random magnitude ε_n it is required to know the first and the second initial moments. The following theorem is true.

Theorem 2. Let $\xi\sim S(x,a,b,\alpha,\beta,c,\mu)$. Then the first and the second initial moments are calculated by the formulas:

$$E\xi = ap_1 - \sigma_1[\Phi^{-1}(p_1)p_1 + \varphi(\Phi^{-1}(p_1))] +$$
$$+\int_a^b xg_\theta(x)\,dx + bp_2 + \sigma_2[\Phi^{-1}(p_2)p_2 + \varphi(\Phi^{-1}(p_2))],$$
$$E\xi^2 = (\sigma_1^2 + a_1^2) - \sigma_1(a+a_1)\varphi(\Phi^{-1}(p_1)) +$$
$$+\int_a^b x^2 g_\theta(x)\,dx + (\sigma_2^2 + a_2^2)p_2 + \sigma_2(a_2+b)\varphi(\Phi^{-1}(p_2)).$$

The theorem proving is based on direct integration of the equality (4).

Let us compare the plotted according to (3)–(5) density of STS-distribution with the densities of normal and α-stable distribution with similar parameters, for this purpose let us represent them in fig. 1.

As it follows from the analysis of fig.1, densities of α-stable and STS distributions coincide with each other at $x\in[a,b]$. Curve 2 is different from curves 1, 3 owing to non-zero coefficient of asymmetry β. Besides, characteristic exponent α is not equal to two (the case of normal probabilistic law). Then, for $x>\max\{|a|,|b|\}$ the tails of curve 1 are between curves 2 and 3. This suggests that rare events at their description by means of STS-distribution take place with higher probability than at using the normal law.

The density dependence of STS-distribution on a, b parameters is presented in fig. 2. As it is evident from the analysis of fig. 2, with the growth of absolute value of cutting off level a the tail thickness falls, but the density function concentrates its values near the mode μ. Besides, at definite parameter values the density of STS-distribution can have thicker tails than that of normal and α-stable distributions. Therefore, one can state that the function $f(x)$ plotted according to (3)–(5) possesses high adaptive capacity for description of empirical data achieved by variation of six parameters.

3. Econometric data analysis

Let us use STS-distribution for GARCH(1,1) modification plotted above. Assume that in the expression (1) $u_n\sim S(x,a,b,\alpha,\beta,c,\mu)$. Choose the standard STS-distributed random value $\varepsilon_n\sim S(x;-5{,}92;3{,}33;1{,}85;0{,}6;-0{,}1;0)$, having zero mathematical expectation and unit variance.

Assuming independence of logarithmic increments, let us apply the method of maximum likelihood for estimation of coefficients $\gamma,\overline{\alpha},\overline{\beta}$ of the model (1) (the problems of applicability of the method in the case of conditional densities have been considered in [14] in details) and calculate the function maximum L:

$$L = \prod_{i=1}^{m} f_i,$$

or the functions ln L:

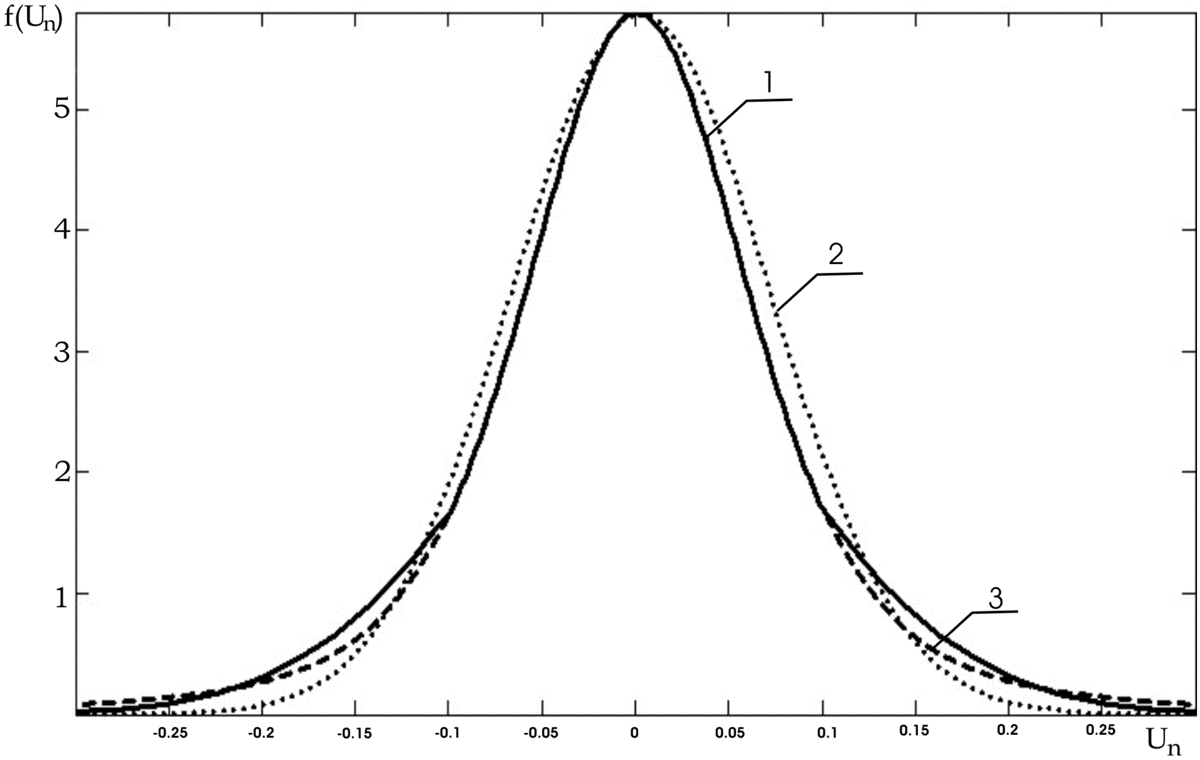

Fig. 1. *Densities of normal, α-stable and STS-distributions at $a=-0{,}1$, $b=0{,}1$ ($p_1=p_2=0{,}105$), $\alpha=1{,}5$, $\beta=0{,}1135$, $c=0{,}05$, $\mu=0{,}00191$: 1) density of STS-distribution; 2) density of normal distribution with mathematical expectation $\mu=0{,}00191$ and variance 0,004761; 3) density of α-stable distribution*

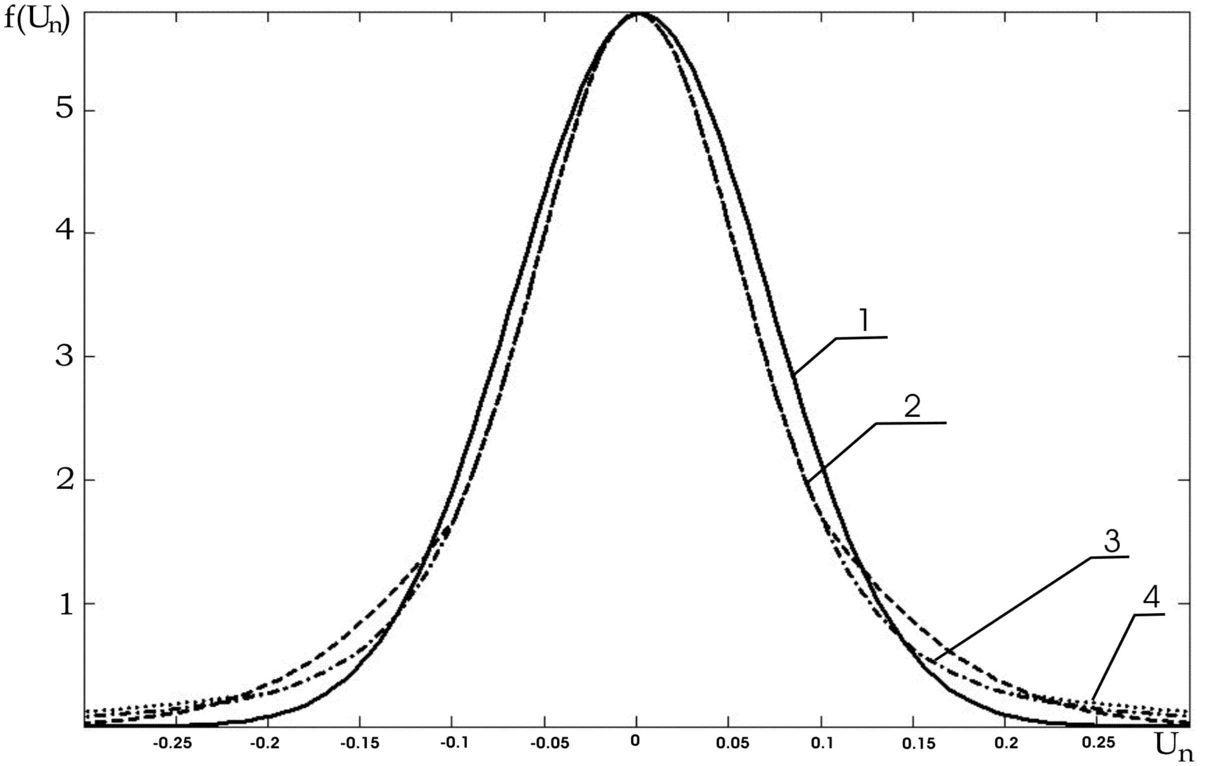

Fig. 2. *Densities of STS-distribution at α=1,5, β=0,1135, c=0,05, μ=0,00191 and density function of normal distribution with mathematical expectation μ and variance 0,00476: 1) density of the normal probabilistic law; 2) density of STS at a=−0,1, b=0,1, p_1=p_2=0,105; 3) density of STS at a=−0,2, b=0,2, p_1=p_2=0,031; 4) density of STS at a=−0,3, b=0,3, p_1=p_2=0,015*

$$\ln L = \sum_{i=1}^{m} f_i, \qquad (6)$$

where $f_i=f(x_i|F_{i-1})$ are the functions of the conditional density of STS-distribution, determined by the equality (4), m is the number of observations, L is the likelihood function.

Search for the maximum (6) is performed according to the necessary condition of extremum existence of the tree variables function:

$$\frac{\partial L}{\partial \omega} = 0, \ \frac{\partial L}{\partial \overline{\alpha}} = 0, \ \frac{\partial L}{\partial \overline{\beta}} = 0. \qquad (7)$$

Solution of nonlinear system (7) in assumption of extremum uniqueness in some design region can be made by any iteration method: the quickest descent method, conjugate gradients method, etc.

Having estimated the coefficients $\omega,\overline{\alpha},\overline{\beta}$ and substituted them in (1) statistical examination of reliability of GARCH(1,1) method suggested is left to carry out for volatility prediction. For this purpose we use the well-known Ljung-Box test to check the hypothesis H_0 on equality to zero of the first m autocorrelations [15], where $m<n$. Since at the condition of existing the fourth initial moment $E|u_n|<\infty$ the process GARCH(1,1) can be written down in the form of ARMA process with the parameters p=1 and q=1, it is reasonable to consider the normalized sample autocorrelation function of remainders $\hat{a}_n$ of the view

$$\hat{r}_k = \sum_{i=1}^{n-k} \hat{a}_i \hat{a}_{i+k} \Big/ \sum_{i=1}^{n} \hat{a}_i^2, \ k = 1,2,3,...$$

Then, write down the statistics

$$\overline{\gamma} = n(n+2)\sum_{k=1}^{m} \frac{\hat{r}_k^2}{n-k},$$

that, as it is known, at sufficiently large n will have χ^2-distribution with (m–p–q) degree of freedom, if theoretical values of the model parameters (1) are unknown. At last, calculation of $\hat{r}_k$ should be performed for the remainders $\hat{a}_n=u_n^2$ and $\hat{a}_n=(u_n-\overline{a})/\overline{\sigma}_n^2$ correspondingly before and after application of GARCH(1,1).

The hypothesis is accepted, if $\overline{\gamma}<\chi^2_{1-s}(m-p-q)$, where s is the level of criteria significance, and it is rejected in the other case. Consequently, GARCH(1,1), defined by the expression (1) with the coefficients meeting (7), is a statistically reliable one with the significance level s, if $\overline{\gamma}<\chi^2_{1-s}(m-p-q)$.

To simulate the probable values of the time series for the future h_n together with calculation of the Ljung-Box test one needs to check if increments u_n, found according to (2), would have STS-distribution. Adjustment of empirical distribution to STS according to sampled data can be carried out in different ways, for example, by adjustment of characteristic function or density approximation using FFT. In the given paper the writing of STS distribution density through the Zolotarev integral is used [13] with its subsequent numerical integration by the Simpson quadrature formula. The quality of adjustment was checked by χ^2-test [14].

4. Analysis of empirical data

Let us apply the method of STS-GARCH(1,1) to simulate the time series h_n of dollar quotations of RAO UES RTS shares, for this purpose use 371 values during the period from January 04, 2003 to June 30, 2004.

Note that $u_n=\ln(h_n)-\ln(h_{n-1})$, $n=1,2,...$, possess the following parameters: average is $1{,}9\cdot10^{-3}$, variance is $8{,}2\cdot10^{-4}$, the third central moment is $2{,}6\cdot10^{-6}$, the fourth one is $4{,}1\cdot10^{-6}$, the asymmetry coefficient is $1{,}1\cdot10^{-1}$, kurtosis (excess) is 6,15.

The numerical calculations show that $u_n \sim S(x,a,b,\alpha,\beta,c,\mu)$ with probability 0,99, and $a=-4\cdot10^{-2}$, $b=4{,}5\cdot10^{-2}$, $\alpha=1{,}9$; $\beta=1{,}1\cdot10^{-1}$, $c=1{,}8\cdot10^{-2}$, $\mu=1{,}9\cdot10^{-3}$. In this case the value of χ^2-test is equal to 21,67.

The day volatility σ_n in (1) is calculated according to the last k observations:

$$\bar{\sigma}_n^2 = \frac{1}{k-1}\sum_{i=1}^{k}(u_{n-i}-\bar{u})^2,$$

where $\bar{u} = \frac{1}{k}\sum_{i=1}^{k} u_{n-i}$ is the sample average, k is the lag of time series. The model coefficients (1) are estimated by the maximum likelihood method with logarithmic likelihood function (6). The system (7) is solved by the quickest descent method with the error $\varepsilon=10^{-3}$. The initial approximation is chosen to be zero.

The parameter values of STS-GARCH(1,1) at different lags k are presented in the table.

Table. *Coefficient values of STS-GARCH (1,1) at different lags*

k	ω	$\bar{\alpha}$	$\bar{\beta}$	γ	V
2	10^{-4}	0,20	0,77	0,03	$3\cdot10^{-2}$
5	10^{-4}	0,21	0,78	10^{-2}	10^{-2}
10	10^{-3}	0,16	0,83	10^{-2}	10^{-1}

Let us present the values of the Ljung-Box test of $\bar{\gamma}$ before and after application of GARCH(1,1). Thus, at $k=3$ before and after application it was equal to 29,58 and 0,09, correspondingly, at $k=5$ it is 48,6 and 1,11, at $k=10$ it is 60,13 and 6,78. The threshold values of distribution $\chi^2_{1-s}(m-2)$ at the significance level $s=0{,}05$ and $m=3$, $m=5$, $m=10$ are equal to 3,8414; 7,8147; 15,5073. Hence, the STS-GARCH(1,1) method, designed on the basis of (1) with the table coefficients, is statistically reliable with the probability 0,95.

Apply STS-GARCH(1,1) for simulation of dollar quotations of RAO UES RTS shares. In this case use the volatilities σ_n obtained before and generate the random sequence $\varepsilon_n=\{y_n\}_{n=0}^{370}$, having STS-distribution with zero average and variance one:

$$\varepsilon_n \sim S(x; -5{,}92;\ 3{,}33; 1{,}85;\ 0{,}6;\ -0{,}1;\ 0).$$

The value assignment ε_n was performed according to the classical scheme [17]: in the interval [0,1] the sequence of uniformly distributed random values was generated $\{x_n\}_{n=0}^{370}$, after which at the fixed n the transcendental equation with respect to y_n was solved:

$$x_n = S(y_n), \qquad (8)$$

where

$$S(y) = S(y; -5{,}92; 3{,}33; 1{,}85; 0{,}6;\ -0{,}1; 0) = \int_{-\infty}^{y} f(t)\,dt$$

is the function of standard STS-distribution. Solution of the equation (8) was made by the method of tangents with zero initial approximation and accuracy 10^{-4}. The relative error of root calculation (8) did not exceed 10^{-6}.

The sequence ε_n was used in (2) to determine the increment logarithms u_n and to calculate the probable values h_{n+1}^{STS} of dollar quotations of RAO UES RTS shares:

$$h_{n+1}^{STS} = h_n \cdot \exp(\bar{\sigma}_{n+1}\varepsilon_{n+1} + \bar{a}_{n+1}), \quad n=0,1,2,3...$$

The obtained h_n^{STS}, calculated with lag 5, and initial historical data h_n are presented in fig. 3. Besides, in fig. 3 there is the relative error δ_n between them:

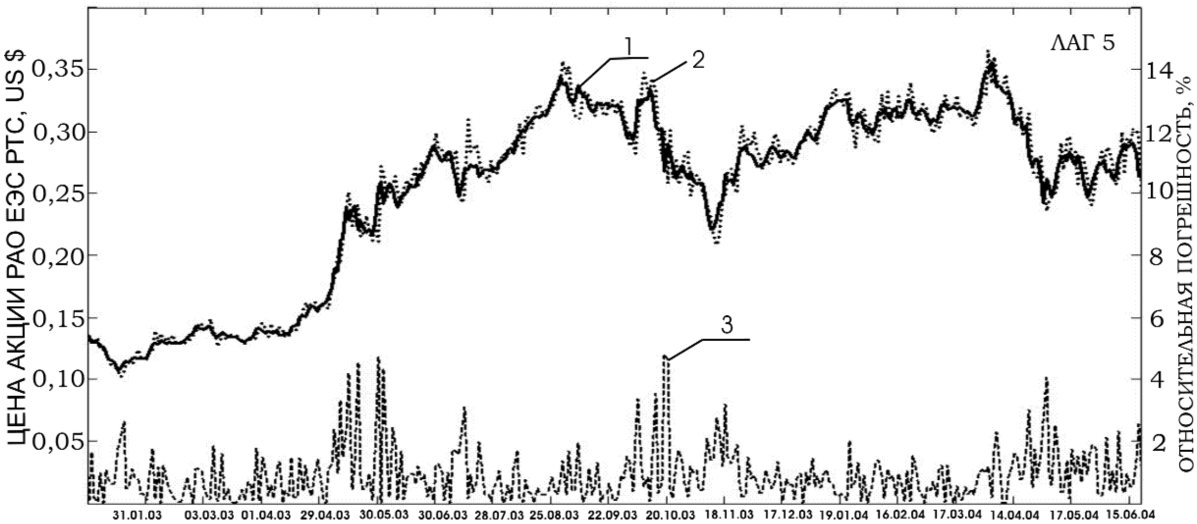

Fig. 3. *Comparison of historical and model quotation values of RAO UES RTS shares from January 04, 2003 to June 30, 2004: 1) time series of historical data, 2) time series, calculated by STS-GARCH(1,1) with lag 5, 3) relative error, %*

$$\delta_n = \left| h_n^{STS} - h_n \right| h_n^{-1}, \quad n = 1, 2, ...$$

As it follows from the analysis of fig. 3, the relative error δ_n did not exceed 5 %. The maximum was achieved on 206 auction day and was equal to 4,8 %. The average error value amounted 2,28 %.

According to fig. 3, there observed three peaks of values δ_n: from 29.04.03 to 30.05.03, 20.10.03 and from 14.04.04 to 17.05.04. It is explained by the sudden jumps in share quotations in the given time intervals, as price volatility amounted from 7 to 15 % per day. Note that such sudden changes in prices are one of the reasons for refusal from the assumption on the normal distribution ε_n in calculating u_n in the expression (2). As the calculations show, if we take instead $\varepsilon_n \sim S(x;-5,92;3,33;1,85;0,6;-0,1;0)$ – $\varepsilon_n \sim N(0,1)$, then the maximum of relative error is equal to 17,1 % at the average error value 7,3 %. The facts mentioned above give the ground to conclude that STS-GARCH(1,1) describes the initial data satisfactorily and makes possible to simulate their values with small error.

5. Conclusion

On the basis of STS-distribution the modified econometric method STS-GARCH(1,1) has been designed. The parameters of normal distributions forming the tails of STS-distribution are found, the expressions of finding the first and the second initial moments are obtained. Modification of the classical algorithm GARCH(1,1) is carried out. It consists in refusal from the assumption on normal logarithm distribution law of day increment of time series. Estimation of STS-GARCH(1,1) coefficients is performed by the maximum likelihood method, the coefficient values are found by solving the nonlinear system (7). By means of the Ljung-Box test the statistic examination of algorithm reliability suggested is performed and the decrease in autocorrelation of data structure employed for the analysis is shown. The STS-GARCH(1,1) method is applied for simulation of equity prices of RAO UES RTS with lag 5. The higher accuracy is shown in comparison with the classical model of GARCH(1,1) in which the assumption on logarithm normality of day increments is made.

REFERENCES

1. Ser-Huang P. A Practical Guide to Forecasting Financial Market Volatility. – Chichester: John Wiley & Sons, 2005. – 123 p.
2. Fama E. The behavior of stock market prices // Journal of Business. – 1965. – V. 38. – № 1. – P. 34–105.
3. Haas M., Mittnik S., Paolella M.S. Modeling and Predicting Market Risk With Laplace-Gaussian Mixture Distributions // Center for Financial Studies (J.W. Goethe University), 2005. – № 11. – P. 36.
4. Mantegna R.N., Stanley H.E. Introduction to Econophysics, Correlations and Complexity in Finance. – Cambridge: Cambridge University Press, 2000. – 148 p.
5. Basrak B., Davis R.A., Mikosch T. Regular variation of GARCH processes // Stochastic Processes and their Applications. – 2002. – V. 99. – P. 95–115.
6. Brooks C., Clare A.D. a. o. A comparison of extreme value theory approaches for determining value at risk // Journal of Empirical Finance. – 2005. – № 12. – P. 339–352.
7. Mittnik S., Paolella M.S., Rachev S.T. Stationarity of stable power-GARCH processes // Journal of Econometrics. – 2002. – V. 106. – P. 97–107.
8. Jondeau E., Rockinger M. Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements // Journal of Economic Dynamics and Control. – 2003. – V. 27. – P. 1699–1737.
9. Wagner N., Marsh T.A. Measuring tail thickness under GARCH and an application to extreme exchange rate changes // Journal of Empirical Finance. – 2005. – V. 12. – P. 165–185.
10. Menn C., Rachev S.T. A GARCH option pricing model with alpha-stable innovations // European Journal of Operational Research. – 2005. – V. 163. – P. 201–209.
11. Rachev S.T., Menn C., Fabozzi F.J. Fat-tailed and Skewed Asset Return Distribution. Implications for Risk Management, Portfolio Selection and Option Pricing – John Wiley & Sons, 2005. – 370 p.
12. Hull J. Options, futures and other derivatives – Prentice-Hall, 2002. – 745 p.
13. Zolotarev V.M. One-dimension stable distributions. – Moscow: Nauka, 1983. – 304 p.
14. Ayvazyan S.A., Mhitaryan V.S. Applied statistics. Bases of econometrics. – Moscow: UNITI-DANA, 2001 – V. 1. – 656 p.
15. Ljung G.M., Box G.E.P. On a measure of lack of fit in time series models // Biometrika. – 1978. – V. 65. – № 2. – P. 297–303.
16. Chernobai A., Rachev S.T., Fabozzi F. Composite Goodness-of-Fit Tests for Left-Truncated Loss // Samples Technical Report: Department of Statistics and Applied Probability, University of California, USA – 2005. – 23 p. http: // www.pstat.ucsb.edu/research/papers/KSmissing20050604-JFE.pdf
17. Demidovich B.P., Maron I.A. Bases of computational mathematics. – Moscow: Phys.-math. Literature Pub. House, 1960. – 661 p.

Arrived on 27.09.2006