UDC 658.512.2.012.122

QUADRATIC FLAT INVOLUTIONS AS A BASIC METHOD OF OBTAINING CURVES IN THE SYSTEMS OF CAE

I.F. Borovikov, E.G. Fisochenko

Yurga Technological Institute of TPU
E-mail: bif1986@mail.ru

The way of setting up non-central quadratic flat involutions, based on application of pencils of circles is suggested, transformation operators are introduced, and the examples of rational circular curves constructed by this method are given.

Design of engineering curves (all possible aero- and hydrodynamic profiles, axes of pipe lines, frames, etc.) is reduced to construction of curves mating points of given discrete array with meeting some set of edge conditions (fixed tangents, curvature circles and etc.). In its turn, engineering surfaces (different fairings, inlets, longerons, turbine blades) can be interpreted as surfaces mating initial curves according to definite smoothness order. At present, engineering curves are presented in most cases in the form of constituent bypasses of the definite order of smoothness. In spite of the fact that in this case a large function selection is used (exponential, power, logarithmic functions and etc.) bypass constituents are often chosen without necessary geometrical ground. As a result, the bypass does not correspond to its function, but the number of its constituents is overstated. Besides, in different calculations it is important to have the curves which are described by one equation. Therefore the search for a basic method of obtaining curves in the systems of CAE, which would be universal and simple enough, is actual.

When constructing curves it is necessary to apply non-linear transformations. Moreover, it is desirable for mechanism of transformation to include simple geometrical images, e.g. lines and circles. In this respect the suggested quadratic transformations, for setting up of which pencils of circles are of great interest.

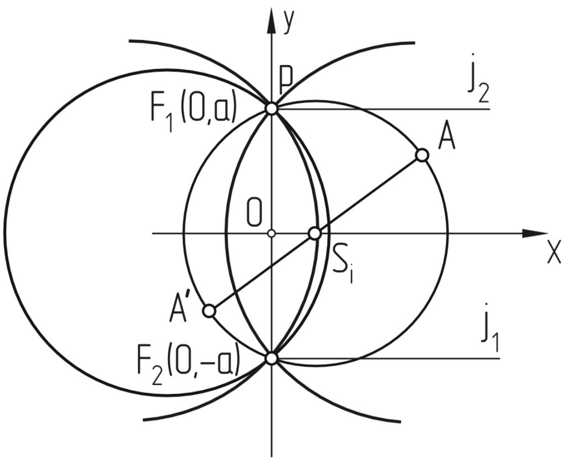

Fig. 1. *Setting involution by means of elliptic circle pencil*

Let elliptic pencil of circles be set up on the plane by two base points $F_1(0,a)$, $F_2(0,-a)$ (fig. 1). Then an arbitrary point $A(x_A,y_A)$ marks up in the pencil a single circle k, the equation of which has the view:

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ a^2 & 0 & a & 1 \\ a^2 & 0 & -a & 1 \\ x_A^2+y_A^2 & x_A & y_A & 1 \end{vmatrix} = 0.$$

After transformation we obtain:

$$\left(x + \frac{a^2 - x_A^2 - y_A^2}{2x_A} \right)^2 + y^2 = a^2 + \frac{(a^2 - x_A^2 - y_A^2)^2}{4x_A^2}.$$

The centre of the circle is point

$$S_i\left(\frac{x_A^2 + y_A^2 - a^2}{2x_A}, 0 \right),$$

its radius is defined by the expression

$$R_i = \sqrt{a^2 + \frac{(a^2 - x_A^2 - y_A^2)^2}{4x_A^2}}.$$

Diametrically opposite point $A'(x_A',y_A')$ is considered to be corresponding to point A. Thus, nonlinear transformation, splitting into central symmetries in the circle pencils is induced on the plane. It is not difficult to demonstrate that the coordinates of point A' are determined by the expressions:

$$x_A{}' = \frac{y_A^2 - a^2}{x_A}, \quad y_A{}' = -y_A.$$

Since any point of may be chosen as a preimage point A on the plane, the indexes of the latter expression can be omitted. As a result, we obtain operators of direct transformation:

$$x' = \frac{y^2 - a^2}{x}, \quad y' = -y.$$

Reverse transformation operators have the symmetrical view.

The image of line m', described by the equation $Ax'+Bx'+1=0$ is a curve of the second order m, the equation of which has the view: $Ay^2 - Bxy + x - Aa = 0$. Thus, we obtain quadratic involution with a pencil of light-invariant circles. The points F_1, F_2 are simple F-points. They are matched by p-lines, the equations of which have the view: $y=a$, $y=-a$. The limiting line q is Oy axis. Even at the stage of setting up the preimage one can have a conception about form of the designed curve. Thus, for example, multiplicity of curve points in the fundamental F-points are defined by the number of preimage intersection points with p-lines, the presence of improper points

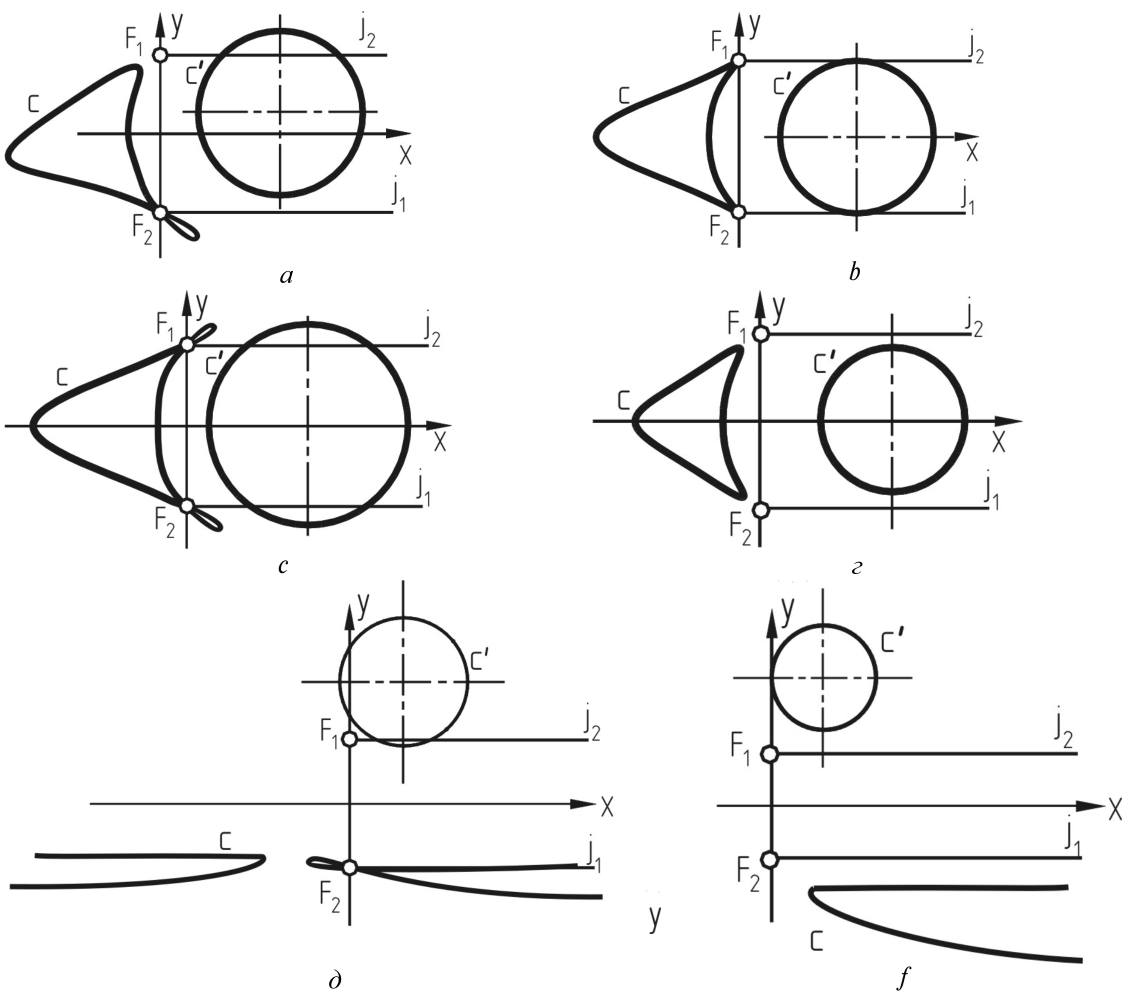

Fig. 2. *Different forms of curves for the elliptic circle pencil*

is done by preimage location relative to limiting line. If we take a circle as a preimage, its image will be a curve of the fourth order, the equation of which has the view:

$$y^4 + y^2x^2 - 2x_0xy^2 + 2y_0yx^2 - 2a^2y^2 +$$
$$+y_0^2x^2 - R^2x^2 + x_0^2x^2 + 2a^2x_0x + a^4 = 0,$$

where x_0, y_0 are the coordinates of the circle centre, R_0 is the circle radius.

In the homogeneous form it can be presented in the following way:

$$x_2^4 + x_1^2x_2^2 - 2x_0x_1x_2^2x_3 + 2y_0x_2x_1^2x_3 - 2a^2x_2^2x_3^2 +$$
$$+y_0^2x_1^2x_3^2 - R^2x_1^2x_3^2 + x_0^2x_1^2x_3^2 + 2a^2x_0x_1x_3^3 + a^4x_3^4 = 0.$$

The coordinates of the cyclic points $I_1(1,i,0)$, $I_2(1,-i,0)$ corresponds to this equation, hence, the obtained curve is a circular curve. Moreover, the form of the curve depends on the location of image with respect to the fundamental points, principle curves and limiting line (fig. 2).

In the case of parabolic pencil of circles, when a=0 (fig. 3) the transformation operators have the view:

$$x' = \frac{y^2}{x}, \quad y' = -y.$$

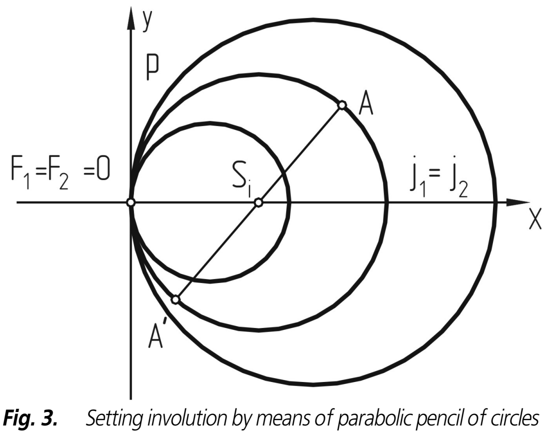

Fig. 3. *Setting involution by means of parabolic pencil of circles*

All homoloids in this case in the point $F_1 \equiv F_2 \equiv 0$ are tangent to Oy axis. Both principle p-lines coincide with Ox axis. The images of the circles having the equation $(x'-x_0)^2+(y'-y_0)^2=R^2$, are the curves of the fourth order. They are described by the equation:

$$y^4 + y^2x^2 - 2x_0xy^2 + 2y_0yx^2 + y_0^2x^2 - R^2x^2 + x_0^2x^2 = 0$$

or in the homogeneous form:

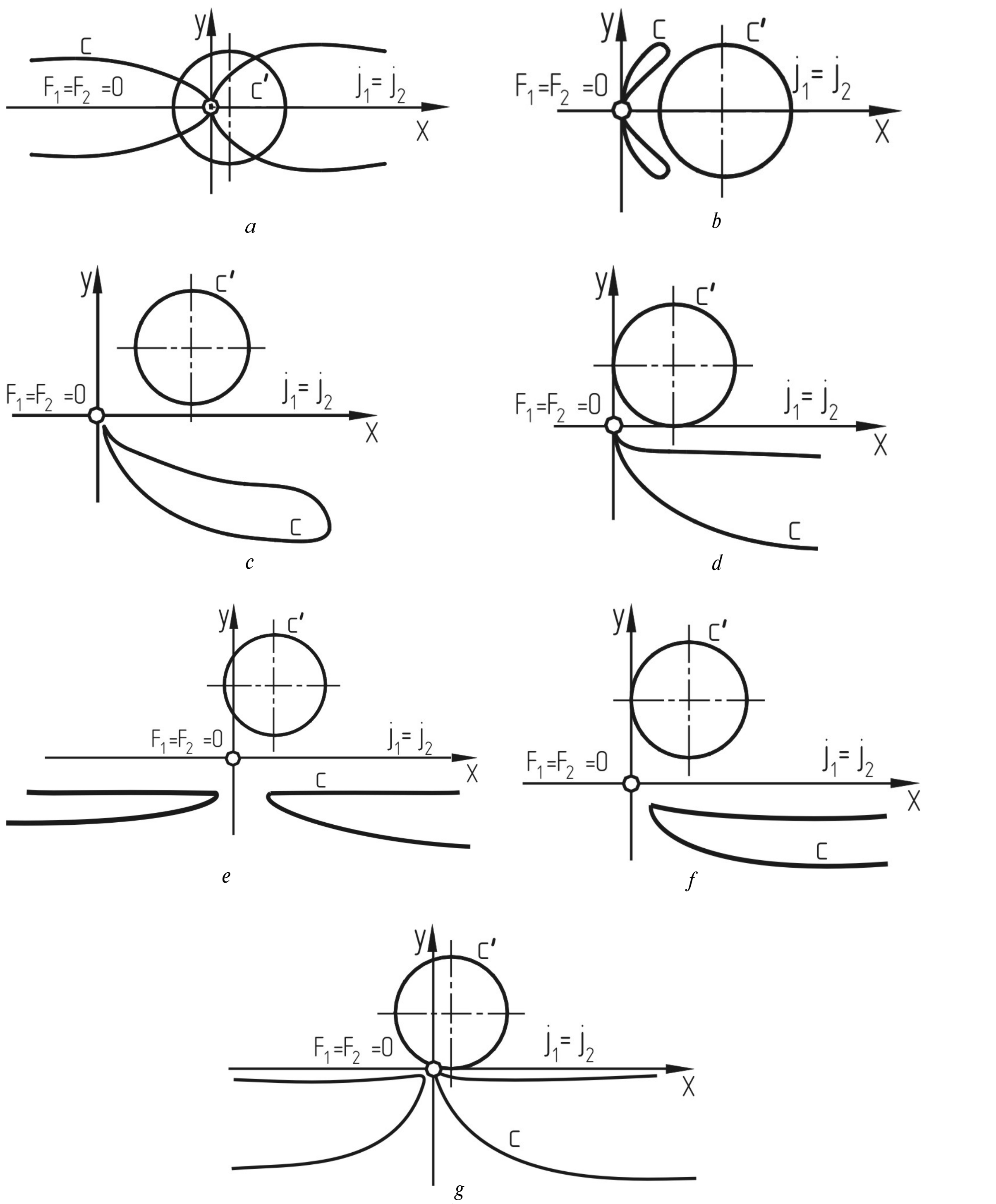

Fig. 4. *Different forms of curves for parabolic pencil of circles*

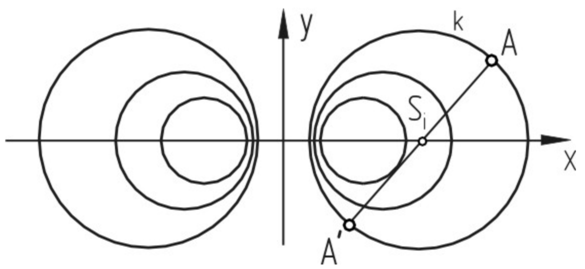

Fig. 5. *Setting up involution by means of hyperbolic pencil of circles*

$$x_2^4 + x_1^2x_2^2 - 2x_0x_1x_2^2x_3 + 2y_0x_2x_1^2x_3 +$$
$$+ y_0^2x_1^2x_3^2 - R^2x_1^2x_3^2 + x_0^2x_1^2x_3^2 = 0.$$

Possible forms of the curves depending on the location of the circle-preimage with regard to elements of fundamental and principle systems as well as to *Oy* axis being a limiting line are presented in fig. 4.

Let us consider the transformations induced by hyperbolic pencil of circles (fig. 5).

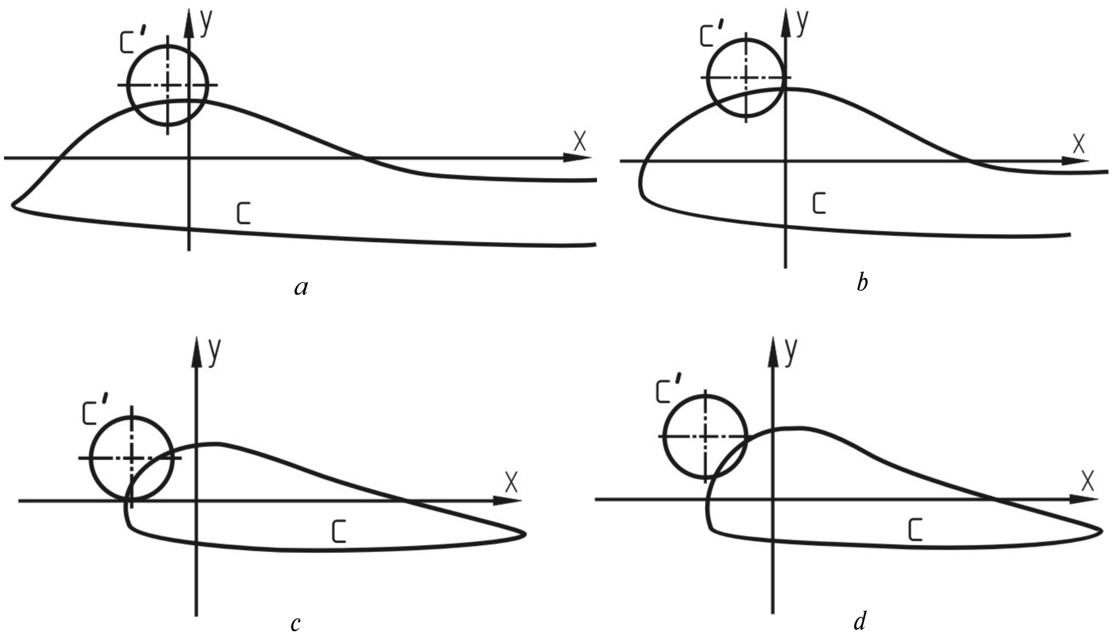

Fig. 6. *Different forms of curves for the parabolic pencil of circles*

Such pencil is suitable to be set up by zero circle $N(m,0)$ and a radical axis, for which the axis Oy is set. Solving the set of equations

$$\left\{\begin{matrix}(x-m)^2+y^2=0, \\ x=0\end{matrix}\right\},$$

we obtain the coordinates of imaginary base points $U(0,mi)$, $V(0,-mi)$. The point $A(x_A,y_A)$ distinguishes the circle k from the pencil described by the equation:

$$\left(x-\frac{x_A^2+y_A^2+m^2}{2x_A}\right)^2+y^2=\frac{(x_A^2+y_A^2+m^2)^2}{4x_A^2}-m^2.$$

The point $A'(x'_A,y'_A)$, diametrically opposite to the point A, let us consider the image of the point A in nonlinear involution, induced on the plane. Using the transformation operators for the elliptical pencil at $a=mi$, we obtain the transformation operators for the case involved:

$$x'=\frac{y^2+m^2}{x},$$
$$y'=-y.$$

The preimage of circle with the equation $(x'-x_0)^2+(y'-y_0)^2=R^2$ is the curve of the fourth order:

$$y^4+y^2x^2-2x_0xy^2+2y_0yx^2+2m^2y^2+$$
$$+(y_0^2-R^2+x_0^2)x^2+2m^2x_0x-m^4=0.$$

In homogeneous form the equation has the view:

$$x_2^4+x_1^2x_2^2-2x_0x_1x_2^2x_3+2y_0x_2x_1^2x_3+2m^2x_2^2x_3^2+$$
$$+(y_0^2x_3^2-R^2x_3^2+x_0^2x_3^2)x_1^2+2m^2x_0x_1x_3^3-m^4x_3^4=0.$$

This is a rational circular curve, different forms of which are presented in fig. 6.

The suggested method makes possible to design curves in the wide range of form and parameter transformation. Even at the stage of preimage one can have an idea of the form of constructed curve. Thus, for example, multiplicity of the curve points in the fundamental F-points is determined by the number of cross points of the preimage with p-lines, the presence of improper points is done by location of preimage with regard to limiting line. For designed curve to be closed it is necessary for the circle-preimage not to cross the limiting line.

To apply this method to practice of real designing the program which permits us to construct curves meeting the specified requirements has been developed in Turbo Pascal algorithmic language.

REFERENCES

1. Ivanov G.S. Construction of engineering surfaces (mathematical simulation on the bases of nonlinear transformations). – Moscow: Machinostroyeniye, 1987. – 192 p.
2. Sturm R. Die Lehre von den geometrischen Verwandtschaften. – Leipzig und Berlin: Druck und Verlag von B.G. Teubner, 1908. – Bd. 4. – 484 s.

Arrived on 02.06.2006