IOP Conf. Series: Materials Science and Engineering **289** (2017) 012033 doi:10.1088/1757-899X/289/1/012033

Phenomenological model of mechanoelectric transformations in rocks

P I Fedotov[1], E A Kargina[2]
[1]Reseacher, National Research Tomsk Polytechnic University, Tomsk, Russia
[2]Graduate student, National Research Tomsk Polytechnic University, Tomsk, Russia
E-mail: pif@tpu.ru

Abstract. A phenomenological model is proposed on the example of the rock destruction development in underground mines. The characteristics of the electromagnetic signal generated due to the appearance and change of the dipole moment of cracks, whose beads are charged when the discontinuity is disturbed, are analytically investigated. The model is constructed using the theory of reliability and percolation theory, which allows to take into account the non-synchronism of the mechanical converters.

1. Introduction
The complexity of the qualitative explanation of the electric emission signals responses, which are an indicator of destruction, to mechanical or acoustic effects is due to the fact that the mechanisms of the phenomenon of mechanoelectric transformations are caused by a wide range of effects occurring at various levels from micro to macro-level. This may be manifestations of the piezoelectric effect, electrical polarization, electromagnetic-elasticity in the propagation of waves, etc [1, 2]. In rocks having a complex texture, porosity and fracturing, there are double electrical layers. Mechanoelectric transformations take place in them under mechanical or acoustic action on such rocks.

The main task of all the studies is practicing of the developed monitoring method of the change in the rock massive stress-strain state under conditions of an operating mine and a forecast of geodynamic manifestations.

During the mining of solid minerals by an underground method, the rock massive stress-strain state changes, that results to geodynamic manifestations activation, such as rock burst, rock- tectonic burst, shakes, and rock falls. This changes the physical fields. The influence of physical fields on the defects accumulation rate in a loaded rock is proved by numerous geophysical observations and in laboratory experiments. In addition, as a result of the mining of ore bodies, seismic effects are manifested by explosions. All these geodynamic manifestations are accompanied by mechanoelectric transformations, the consequence of which is electromagnetic emission [3–7]. The interrelation of these phenomena was proved in experiments on rock samples, which revealed a high sensitivity of electromagnetic emission to the action of acoustic waves [8–10]. The effect of electromagnetic emission activation as a response to the effect of physical fields is established for samples of rocks with different petrophysical properties [11, 12]. The regularities of the electromagnetic response of rocks of different sizes are revealed and this is due to the fact that in all cases electromagnetic emission arises from destruction [13].

A phenomenological model is proposed on the example of the rock destruction development in underground mines. The characteristics of the electromagnetic signal generated due to the appearance and change of the dipole moment of cracks, whose beads are charged when the discontinuity is disturbed, are analytically investigated. The model is constructed using the theory of reliability and percolation theory [14], which allows to take into account the non-synchronism of the mechanical converters.

2. Phenomenological model

The equation of electrodynamics, provided that the wavelength λ is large in comparison with the size of the radiator system L ($L < \lambda$ or $v < c$), allows to obtain the electric field strength [15, 16]:

$$E = (E_1 + E_2 + E_3) \exp(-r/\delta), \quad (1)$$

where $E_1 = \frac{\left[(3+r/\delta)(rD)r - Dr^2\right]}{4\pi\varepsilon r^5}$ characterizes the field strength in the near zone ($r < \lambda$), E_2 - field strength in the far field ($r > \lambda$), E_3 - field strength in the intermediate zone ($r \approx \lambda$). If the "correction for the delay" [17] is partially taken into account, neglecting it in calculating the potential $\phi = -div\frac{D}{r} \cong \frac{(rD)}{r^3}$, but taking it into account in calculating field strength $E = -grad\,\phi - \frac{1}{c}\frac{\partial}{\partial t}A$, *than*

$$E_2 = \frac{\left[n\times\left[n\times\ddot{D}\right]\right]}{4\pi\varepsilon c^2 r} - \frac{n\left(n\ddot{D}\right)}{4\pi\varepsilon c^2 r};\ E_3 = n(n\dot{D})/4\pi\varepsilon c r^2. \quad (2)$$

In the general case, in accordance with $\phi = \frac{(rD)}{r^3} + \frac{(r\dot{D})}{cr^2}$.

The factor exp ($-r$ / δ) takes into account the weakening of the field in the medium and $\delta = c(2\pi\sigma f)^{-1/2}$, σ - is the conductivity, f – is the frequency of the signal, c – is the light speed of (at f = 10 kHz, $\sigma = 10^{-4}$ (Ohm·m)$^{-1}$, $\delta \approx 1$ km). If we use, for the prediction of rock disruption, already existing equipment and the operating frequency range is 1–10^2 kHz, then for distances smaller than the thickness of the skin layer $r < \delta$, the ratio $r / \delta < 1$ for $\sigma = 10^{-2}$-10^{-5} (Ohm· m)$^{-1}$. Therefore, electromagnetic effects in the mines can be described by the first term E_1. In this case, after averaging over the angles for the isotropic distribution of dipoles, we obtain that for a set of cracks

$$\left\langle E^2\right\rangle = \frac{1}{4\pi}\int E_1^2 d\Omega = 2N\left\langle D^2\right\rangle / r^6 \quad (3)$$

where $N = nV$ is the number of cracks in the volume V, n - is the volume concentration of cracks, $\left\langle D\right\rangle = D\sqrt{N}$, D - is the dipole moment of a single crack. We get the integral when integrating expression (3) over the volume occupied by elementary sources of the signal,

$$I = \int\frac{dU'}{r^6} = 2\pi\int_0^{\pi}\int_0^{R_{max}}\frac{r'^2 dr'\sin v dv}{\left|r-r'\right|^6} = \frac{2\pi}{r^3}\int\int\frac{x^2 dx d\cos v}{(1-x\cos v)^6},$$

where $x = r/r'$, r - is the radius vector of the observation point, r' - is the radius vector of the elementary source of the signal. As

$$\int_0^{\pi}\frac{\sin v dv}{(1-x\cos v)^6} = \frac{1}{5x}\left\{\frac{1}{(1+x)^5} - \frac{1}{(1-x)^5}\right\},$$

then finally we get that

$$I = 2\pi\frac{40x^3 + 8x^5}{60\left(1-x^2\right)^4 r^3} \cong \frac{4\pi}{3}R_{max}^3\frac{1}{r^6}.$$

Thus,

$$|E| = \sqrt{\left\langle E^2\right\rangle} = \sqrt{2n_* P_0 P_1 P_2 U}\,D/|\varepsilon|h^3, \quad (4)$$

Where, h – is the distance from the center of the investigated region to the antenna; n_* - is the critical crack concentration and $k = 2 \div 5$ [18]; $n_*^{-1/3} = k \cdot l$; l - is the linear dimension of the crack; P_0 - is the scalar function characterizing the damage to the medium, $U = \frac{4\pi R_{\max}^3}{3}$; $R_{\max}$ – is the radius of the investigated region; $D = \sigma_0 U_{src}$ – is the crack dipole moment, U_{src} – volume of a single crack, σ_0 – surface charge density on the cracks sides; P_1 – is the probability that at a time t the system of elementary sources will be able to radiate; P_2 – is the probability that the system of signal sources will work reliably on the time interval $(t, t + \tau)$ - the time interval during which the measurement is made.

From (4) we obtain for the magnitude of the electric field strength change rate

$$dE / dt = E_* \dot{P}_0 / \sqrt{P_0} \tag{5}$$

where $E_* = \sqrt{2n_* P_1 P_2 U} \rangle D \langle / 2|\varepsilon| h^3$.

Investigation of the destruction process of a geomaterial in time within the framework of reliability theory makes [16] it possible to obtain the probability of the formation of a system of elementary sources at time t

$$P_0 = \left\{ 1 - \exp\left(-\int_0^t \lambda(t) dt \right) \right\}, \tag{6}$$

where $\lambda(t)$ is the rate of formation of elementary sources (ES).

The probability $P_2 = \exp(-\tau/T_1)$, and

$$P_1 = 1 - F(t) + \int_0^t [1 - F(1-x) h(x) dx],$$

$$h(x) = \sum_{n=1}^{\infty} \phi(n), \phi_n(x) = \Phi_n'(x), \Phi_n = \int_0^t F_n(t-x) dG_n(x),$$

$$F_n(t) = \int_0^t F_{n-1}(t-x) dF(x), G_n(t) = \int_0^t G_{n-1}(t-x) dG(x);$$

$$G_1(t) \equiv G(t); F_1(t) \equiv F(t);$$

$F(t)$ – is the distribution function of the working time duration (the system radiation time);

$G(t)$ – is the distribution function of the system pause duration.

The process of crack formation in geomaterials is random [14]. Such processes are well described by an exponential law [16]: $F(t)$=1-exp($-\lambda t$); $G(t)$ = 1-exp ($-\mu t$).

Wherein

$$P_1(t) = \frac{\mu + \lambda \exp -(\lambda + \mu)t}{\mu + \lambda} = \frac{T_1 + T_2 \exp(-t/(T_1 + T_2)}{T_1 + T_2} \tag{7}$$

and $\underset{t \to \infty}{P_1}(t) = T_1 / (T_1 + T_2)$; here $T_1 = \left[\sum_{m=1}^{n} T_m^{-1} \right]^{-1}$ – average duration of the system operating period; T_m – average duration of the working period of the m-th element (ES); T_2 - average system pause time.

For the destruction process in mines [19]: $\lambda(\tau) = \omega/(1-\omega)$, $P_0 = \omega$,

where the damage

$$\omega = (2-m)\, D(t)/[r/a)^{m+1} - 3n/(m+1)], \tag{8}$$

here $D(t) = D_1' \exp(-vt) = 3nx^{-(m+1)} / (m+1)(2-m)$,

$$m = \left(2 + \sqrt{\frac{2}{3}\frac{a}{\beta}} \right) / \left(1 - \sqrt{\frac{2}{3}\frac{a}{\beta}} \right);$$

$$n = \gamma / \left(\beta - a\sqrt{\frac{2}{3}} \right);$$

α, β, μ_0, μ_∞ - medium constants; $v = \tau^{-1}$, τ - shear relaxation time; $x(t) = a/b(t)$; a – passage linear dimension; $b(t)$ - thickness of the fractured layer near the working surface; breakdown time t_* is found from condition $\omega(a, t_*) = 0$ and

$$t_* = -\frac{\tau\mu_\infty}{\mu_0}\ln\left[1 - \frac{\mu_\infty}{\mu_0}\left(\sqrt{\frac{2}{3}}\frac{P_*}{\Delta P} - 1\right)\right] \cong \tau\left(\sqrt{\frac{2}{3}}\frac{P_*}{\Delta P} - 1\right) \cong 0.63\tau; \quad (9)$$

$$P_* = 2\mu_0\gamma / a;$$

$$\Delta P = P_\infty - P_0$$

ΔP – pressure drop, P_∞– hydrostatic pressure; α^2/β- $2\mu_0 = M$ – modulus of decline.

The value of $x(t)$ is found from Equation

$$\dot{x} = \frac{n(v-\bar{v})x}{(2-m)} + \frac{9n\zeta(x - x^{-m})}{(2-m)(m+1)^2} + x\left(\frac{\Delta\dot{P} + v\Delta P}{4\mu_0} + \frac{3\mu_\infty}{2\mu_0}v\ln x\right) / \left(\frac{9n\zeta x^{-(m+1)}}{(2-m)(m+1)} - \frac{3}{2}\varepsilon_*\right), \quad (10)$$

where,

$$\bar{v} = v\left(1 - \frac{\mu_\infty}{\mu_0}\right),\ \varepsilon_* = \frac{\alpha\gamma}{2\mu_0\beta}\sqrt{\frac{2}{3}},$$

$$\zeta = \left(1 - \frac{\alpha^2}{2\mu_0\beta}\right) / \left(1 - \frac{\alpha}{\beta}\sqrt{\frac{2}{3}}\right).$$

Since μ_o/μ_∞<<1, the main role in the Equation (10) is played by the last term

$$(\dot{x}/x) \cong \frac{\Delta P v}{4\mu_0} / \left(\frac{9n\zeta x^{-(m+1)}}{(2-m)(m+1)} - \frac{3}{2}\varepsilon_*\right). \quad (11)$$

Equation (11) leads to

$$b(t) = b_c^{(0)}\left[1 - \sqrt{\frac{(m+1)\Delta P v}{3\mu_0\varepsilon_*}\Delta t}\right]^{\frac{1}{m+1}}, \quad (12)$$

$\Delta t = t_{\max} - t$, $t_{\max} \cong \tau$.

The value $b_c^{(0)}$ can be found from the condition $\omega = \omega_c$, where ω_c is the percolation limit [20], so

$$b_c^{(0)} = a\left[1 + \omega_c(m+1)/3n\right]^{1/(m+1)} \quad (13)$$

Thus, from the relations (8), (11), (12) we finally obtain that

$$\frac{\dot{E}}{E_*} = \frac{\dot{\omega}}{\sqrt{\omega}} = \frac{\Delta P v}{6\mu_0\varepsilon_*}\left(\frac{b_c^{(1)}}{b}\right)^{m+1}\left[\frac{b_c^{(1)}}{b} - 1\right]^{-1}\sqrt{\frac{3n(m+1)}{\left[\left(\frac{b}{a}\right)^{m+1} - 1\right]}} = , \quad (14)$$

$$= \sqrt{\frac{n\Delta p v}{4\mu_0\varepsilon_*}} / \sqrt{\Delta t}\left(-1\left(\frac{a}{b}\right)^{m+1}\right)^{1/2}$$

here $b_c^{(1)} = \left[\frac{(2-m)(m+1)\varepsilon_*}{6\zeta n}\right]^{1/(m+1)}$, a - is the critical value of $b(t)$ at which a sharp increase in the damage rate $\dot{\omega}$ occurs. The value $b_c^{(1)}$ does not depend on ΔP and the occurrence of failure can be determined only by the material parameters α, β, μ_o, μ_∞, γ. The pressure differential ΔP determines only the beginning of the collapse process in accordance with the expression (9).

Since $\gamma/\alpha = \varepsilon_c << 1$,

$$\frac{\alpha^2}{2\mu_0\beta} > \text{и } \frac{a}{\beta}\sqrt{\frac{2}{3}} < 1, \text{ than } \frac{b_c^{(1)}}{b_c^{(0)}} < 1.$$

Therefore, the moment of speed increase $\dot{\omega}$ precedes the moment of damage increase ω, so that fracture prediction can be based on the measurement of $\dot{\acute{\Gamma}}$ magnitude. In accordance with (14), we estimate the possibility of recording signals during the development of collapse:

$\dot{\acute{\Gamma}} = E_* \dot{\omega}/\sqrt{\omega}$, where $E_* = \frac{D\sqrt{2n_*U}}{2|\varepsilon|h^3} = E_0 \frac{l_{Ц}^3}{h^3}\sqrt{\frac{8\pi n_*}{3}R_*^3}$, ε - the dielectric constant, $l_{Ц}$ - linear dimension (ES), $R_* = l_{Ц}^3\sqrt{\frac{E_0}{E_n}}$ - radius of signal collection, $E_П$ - instrument sensitivity, $\acute{\Gamma}_0 = \sigma_0 / 2|\varepsilon|$; $\dot{\omega}/\sqrt{\omega}$ - is defined by (14).

Taking into account the concentration criterion $n_*^{-1/3} = kl_{Ц}$, we obtain

$$\frac{E_*}{E_n} = \frac{2\sqrt{\frac{2\pi}{3}}}{h^3 n_* k^{3/2}}\left(\frac{E_0}{E_n}\right)^{3/2}. \quad (15)$$

The right-hand side of (15) must be greater than one, for which it is necessary that $h \leq 10^{-1}$ m, $\sigma_o \geq 10^{-8}$ C/m^2, $n_* \leq 10^9$ 1/m^3. Since $\sigma_o = 10^{-2} \div 10^{-6}$ C/m^2 [21], the electromagnetic signal generated during the preparation of the destruction is available for registration by the currently used equipment.

3. Result

Thus, the developed phenomenological model of rock failure allows qualitative and quantitative prediction of it by the precursor of the electromagnetic nature. Within the framework of the reliability theory and the percolation theory, the problem of the non-synchronization of the elementary radiation sources is solved, which is important for the practice of predicting geodynamic phenomena based on precursors of electromagnetic nature. It is proved that fracture prediction can be based on a characteristic change in the electric field strength derivative.

Acknowledgments

The work was supported by the Ministry of Education and Science of the Russian Federation within the framework of the State task in the field of scientific activity, project No. 11.980.2017 / 4.6.

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