Mathematical modeling of heating of a dimetallic plate by a high-energy concentrated radiation flux

Mikhail Purin[1,*], and *Ekaterina* Zakharova[1]

[1]National Research Tomsk Polytechnic University, 634050 Tomsk, Russia

Abstract. Mathematical modeling of heat transfer processes proceeding together under conditions of intense phase transformations (melting and evaporation of metals) under laser action on a dimetallic target has been carried out. Based on the results of numerical simulation, it is established that the laser power exerts a significant influence on the dynamics of the melting process. It is shown that evaporation of metal in the boundary layer of the plate can also have a significant effect on the heat transfer characteristics. It is established that metal vapors forming a gas mixture in the near-wall region can absorb laser radiation and cause the phenomenon of optical resonance with increasing their critical concentration, in other words, when the metal melts, optical breakdown is possible.

1 Introduction

Laser treatment of metals in industry is becoming increasingly important due to its advantages such as high processing accuracy, the absence of mechanical damage on the processed products, and as a result, a lower consumption of metal for tolerances and allowances of blanks, as well as the ability to work with complex geometric shapes [1,2]. It should be noted that industrial laser metal processing plants are less harmful for the environment, since they do not generate waste industrial oils that are used to cool and lubricate moving parts of mechanical metalworking machines.

In connection with the growing interest in laser processing of metal, studies of the processes that occur during laser processing become topical, however, field experiments are quite complex and expensive. In these cases, the most suitable is the apparatus of mathematical modeling, which makes it possible to carry out studies of the processes under consideration with high accuracy and efficiency without any material costs.

The melting process is accompanied by a change in the structure of the material with a change in the thermophysical characteristics: heat capacity, density, thermal conductivity. It should be noted that under the conditions of laser action, intense heating of the metal takes place in a limited region. Accordingly, in the plate structure it is possible to form an essentially nonlinear and spatially-distributed fusion front, the shape of which does not yield to polynomial approximation.

* Corresponding author: mvp21@tpu.ru

A significant contribution to the solution of the multidimensional formulation of Stefan problem was made by V.I. Mazhukin [3-5], A.A. Samarskii [6,7], and L.I. Rubinshtein [8]. However, the methods of [3-8] are based on the explicit separation of the fusion front, and this requires the use of a very complex mathematical apparatus based on a combination of the experimental results of the melting process and the apparatus of differential topology that takes into account the dynamic changes in the calculated difference grids.

The aim of the work is mathematical modeling of the processes of heat and mass transfer proceeding together under conditions of intense phase transformations (melting and evaporation of a metal) under the action of a laser.

2 Physical statement of the problem

At the initial instant of time (t = 0), a laser beam is incident on the bounded region of the dimetallic target, as a result, the metal of both layers begins to heat up. When the phase transition temperatures are reached, the process of metal melting is initiated, resulting in a heterogeneous structure consisting of liquid and solid phases of metals with different thermophysical characteristics. With continued heating, the process of evaporation of the metal into the gas region is initiated. The metal vapor forms a vapor-gas mixture in the external environment, which has a significant effect on the radiation transfer characteristics.

When solving the problem, it was assumed that the thermophysical characteristics of metals depend on the position of the fusion front. The thermophysical characteristics of metals and gas, the heat of melting and evaporation of metals are given in Table 1. The laser beam is directed perpendicular to the target surface.

Table 1. Basic physical characteristics in the system “Dimetallic plate-gas”.

Thermophysical characteristics of metals							
	Solid phase			Fluid phase			
	c, J/(kg·K)	λ, W/(m·K)	ρ, kg/m^3	c, J/(kg·K)	λ, W/(m·K)	ρ, kg/m^3	
Aluminum	951.3	240	2712	1090	90.7	2368	[9]
Plumbum	127.5	32.9	11340	148	15.12	10592	[9]
Thermophysical characteristics of gas							
	c, J/(kg·K)		λ, W/(m·K)		ρ, kg/m^3		
Argon	519		0.0394		1.661		[10]
Heat of fusion, heat of vaporization and melting point of metals							
	Q_{melt}, kJ/kg		Q_{evp}, kJ/kg		T_{melt}, K		
Aluminum	393		10900		933.3		[9]
Plumbum	24.3		860		600.4		[9]

3 Mathematical formulation of the problem

The mathematical model of the process under consideration includes the energy equations for metals in dimensionless form, taking into account the dependence of thermophysical characteristics on temperature:

The energy equation for the upper plate:

$$\frac{C_1(\theta)P_1(\theta)}{Fo_1}\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial X}\left(\Lambda_1\frac{\partial\theta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\Lambda_1\frac{\partial\theta}{\partial Y}\right)+Pom_1, Pom_1=\frac{2\cdot Q_1\cdot W_1\cdot\delta\cdot l^2}{h\cdot\lambda_0\cdot T_0}, \quad (1)$$

The energy equation for the lower plate:

$$\frac{C_2(\theta)P_2(\theta)}{Fo_2}\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial X}\left(\Lambda_2\frac{\partial\theta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\Lambda_2\frac{\partial\theta}{\partial Y}\right)+Pom_2, Pom_2=\frac{2\cdot Q_2\cdot W_2\cdot\delta\cdot l^2}{h\cdot\lambda_0\cdot T_0}, \quad (2)$$

here $C = c / c_0$, $P = \rho / \rho_0$, $\Lambda = \lambda / \lambda_0$, $\theta = T / T_0$ are the dimensionless heat capacity, density, thermal conductivity, and temperature, respectively; c_0, ρ_0, λ_0, T_0 are the dimensionless parameters $X = x / l$, $Y = y / l$ are dimensionless coordinates; Pom_1, Pom_2 - Pomerantsev criteria for the melting process for the upper and lower plates, respectively; Fo_1, Fo_2 - Fourier numbers for the upper and lower plates, respectively; W_1, W_2 - melting rates of plates, kg / ($m^2 \cdot s$); Q_1, Q_2 - heat of melting of metals, kJ / kg; δ is the Dirac delta function; h, l - linear dimensions of plates, m.

The energy equation for the described process in dimensionless quantities for a gas:

$$\frac{1}{Fo_3}\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}+PomI\cdot divE_{isl}, PomI=\frac{I_0\cdot l}{\lambda\cdot T}, \quad (3)$$

here I_0 is the intensity of the laser radiation, W/m^2; $E_{isl} = I/I_0$ is the dimensionless radiation intensity.

The diffusion equation in the gas region:

$$\frac{1}{Fo_d}\frac{\partial C}{\partial t}=\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right), Fo_d=\frac{t\cdot D_{12}}{l^2}, \quad (4)$$

here C is the mass concentration of metal vapor in the gas, kg/kg; Fo_d is the Fourier diffusion number.

The diffusion coefficient was determined according to [11]:

$$D_{12}=2{,}66\cdot 10^{-2}\cdot\frac{\sqrt{T^3\cdot(M_1+M_2)/(2\cdot M_1\cdot M_2)}}{P\cdot\sigma_{12}^2\cdot\Omega^{(1,1)*}(T^*)}, \frac{m^2}{s}, \quad (5)$$

here σ_{12} is the parameter of the function expressing the potential interaction energy of the molecules of the first and second components; $\Omega^{(1,1)*}(T^*)$ is the reduced interaction integral of the molecules of the first and second components; M_1, M_2 are molar masses, kg / kmol; P is the pressure, Pa.

The reduced interaction integral of the molecules of the first and second components [11]:

$$Q_{12}^{(1,1)}\cdot\left(\frac{k\cdot T}{J_2}\right)^m\cdot\alpha_2^{-\frac{2}{3}}=a\cdot\exp\left(b\cdot\frac{J_2}{J_1}\right), \quad (6)$$

here J_1, J_2 is the ionization energy of the metal atom and the gas molecule, eV; α is the gas polarizability coefficient; $k = 0.863\cdot 10^{-4}$ eV / K is the Boltzmann constant; m is the temperature dependence of $D_{12}\sim T^n$; a, b are coefficients that depend on the values of J_1.

The radiation transfer equation [12]:

$$\Omega\cdot grad\, I+\chi\cdot I=\frac{\chi\cdot\sigma\cdot T^4}{\pi}, \quad (7)$$

here Ω is the direction of the ray; I - radiation intensity, W / m^2; χ is the absorption coefficient of the medium, m^{-1}; σ is the Stefan-Boltzmann constant, $W\cdot m^{-2}\cdot K^{-4}$.

The physical constants for equations (5-7) are given in [8,10,13]. The system of equations (1-7) was solved under the following boundary conditions (Fig. 1):

$$t=0;\ 0<x<l;\ 0<y<H_1;\ T(x,y)=T_s;$$

$$t=0;\ 0<x<l;\ H_1<y<H_2;\ T(x,y)=T_s;$$

$$t=0;\ 0<x<l;\ H_2<y<H_{gas};\ T(x,y)=T_{gas}$$

$$\left.\frac{\partial\theta}{\partial X}\right|_{x=0}^{0<y<H_{gas}} = \left.\frac{\partial\theta}{\partial X}\right|_{x=l}^{0<y<H_{gas}} = \left.\frac{\partial\theta}{\partial Y}\right|_{y=0}^{0<x<l} = \left.\frac{\partial\theta}{\partial Y}\right|_{y=H_g}^{0<x<l} = 0, t>0;$$

$$\Lambda_2(T)\left.\frac{\partial T}{\partial Y}\right|_{y=H_2}^{0<x<x_1} = \Lambda_{gas}\left.\frac{\partial T}{\partial Y}\right|_{y=H_2}^{0<x<x_1}, t>0;$$

$$\Lambda_2(T)\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{x_1<x<x_2} = \Lambda_{gas}\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{x_1<x<x_2} - (Ki_{EV}\text{- }Ki_L)\cdot\Psi(X_L;Y_L), t>0,$$

here $Ki_{EV} = \frac{W_{EV}\cdot Q_{EV}}{T\cdot\lambda}$ and $Ki_L = \frac{I_0\cdot l\cdot(1-k_{ref})}{T\cdot\lambda}$ – Kirpichev criteria; k_{ref} – reflection coefficient.

$$\Lambda_2(T)\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{x_2<x<l} = \Lambda_{gas}\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{x_2<x<l}, t>0;$$

$$\Lambda_1(T)\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{0<x<l} = \Lambda_2(T)\left.\frac{\partial\theta}{\partial Y}\right|_{y=H_2}^{0<x<l}, t>0.$$

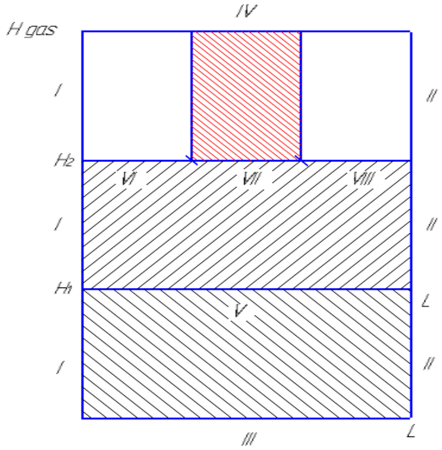


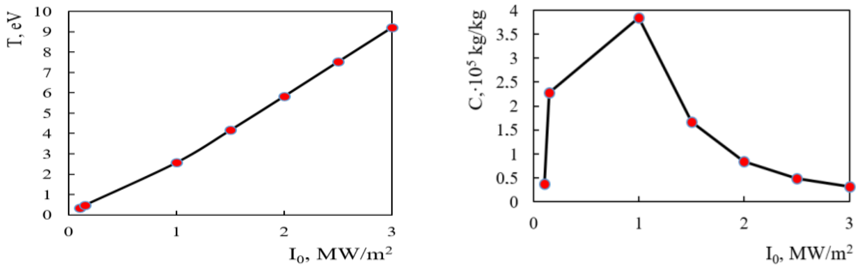
Fig. 1. Scope of the problem.

The problem was solved by the method of finite differences using the implicit four-point difference template with the use of discontinuous coefficients [13, 14]. The nonstationary multidimensional Stefan problem was solved by the method of implicit separation of the fusion front [14].

4 Results

When solving the problem, a dimetallic plate was considered, the upper layer of which was made of aluminum, and the lower one was made of lead. The choice of these materials is due to their extensive use in industry.

Fig. 2 (a, b) shows the temperature and concentration of metal vapors as a function of the intensity of the incident radiation.

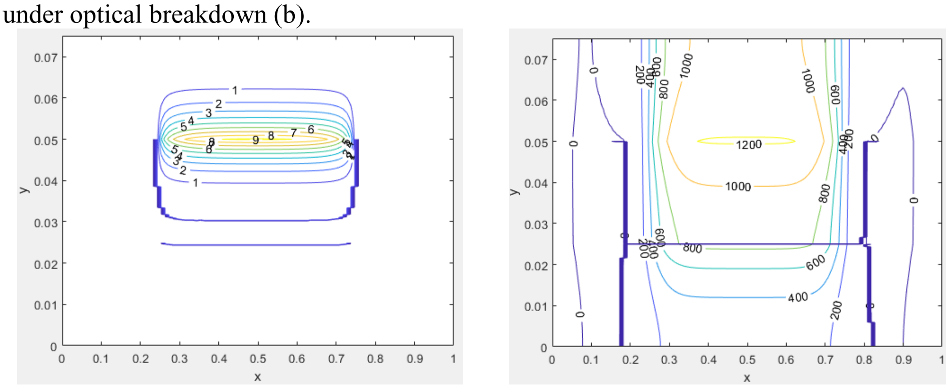


a b

Fig. 2. Dependence of temperature (a) and metal vapor concentration (b) in the gas on the intensity of the incident radiation at the upper plate-gas interface in the middle of the upper plate.

With increasing intensity of the incident radiation, the concentration of metal vapor in the gas region increases to a critical value, and then begins to decrease. This can be explained by the intensive outflow of metal vapors from the region under consideration, owing to a greatly increased diffusion coefficient. According to (5), the diffusion coefficient depends nonlinearly on the temperature. With a continuous increase in the radiation intensity, starting from a certain instant of time, the diffusion coefficient becomes so large that an intensive outflow of metal vapors occurs and the concentration decreases.

Fig. 3 shows the temperature field in the dimetallic plate (a) and in the dimetallic plate under optical breakdown (b).



a b

Fig. 3. Temperature field (eV) (a) and fusion fronts (b) in a dimetallic plate.

According to the results of mathematical modeling, it is established that with sufficiently intense laser action (> 900 MW/m^2), the effect of optical breakdown in the near-wall gas region is possible. This is due to the fact that metal vapors, forming a gas mixture in the near-wall region, can substantially absorb laser radiation, as a result, an intense, even exponential temperature increase is possible. The latter leads to an increase in the optical thickness of the wall gas layer, and also forms zones with intense self-heating in the gas region as a result of the phenomenon of reradiation. In other words, in some cases of metal melting, there are risks of destruction of the metal preform and damage to the laser.

As can be seen from Fig. 3,a, melting of the lower plate can begin even before the moment of contact with it of the melting front of the upper plate.

5 Conclusion

It is established that the laser power exerts a significant influence on the dynamics of the melting process. It is shown that the evaporation of metal into the near-surface gas layer can also have a significant effect on the heat transfer characteristics. It is established that metal vapors, forming a gas mixture in the near-wall region, can substantially absorb laser radiation and cause the phenomenon of optical resonance.

In conclusion, we can say that in order to ensure a fairly stable process of melting by a laser beam, it is necessary to organize a constant flow of inert gas in the melting and welding region, which ensures the entrainment of metal vapors from the melting region.

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