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Mathematical model of solar radiation based on climatological data from NASA SSE

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Abstract. An original model of solar radiation arriving at the arbitrarily oriented surface has been developed. The peculiarity of the model is that it uses numerical values of the atmospheric transparency index and the surface albedo from the NASA SSE database as initial data. The model is developed in the MatLab/Simulink environment to predict the main characteristics of solar radiation for any geographical point in Russia, including those for territories with no regular actinometric observations.

1 Introduction

A reliable prediction of solar radiation arriving at the proposed plant site is crucial to design a solar power plant. For many years, the USSR climate reference books, which contain data on hourly, monthly and annual values of direct, scattered and total solar radiation, and sunshine duration in all regions of Russia for observation periods from 5 to 30 years, were the main source of actinometric data required for solar energy calculations [1, 2]. However, a limited edition and inconvenient form of data representation significantly limit the possibilities of the data practical application. A small number of meteorological stations that perform regular actinometric observations (no more than 130 [3]) considerably complicates obtaining reliable data on the intensity of solar radiation for most territories of Russia.

The list of the world and European electronic databases represented in a more convenient form and their main characteristics are provided in [4, 5]. The results of satellite measurements and the models of solar radiation propagation in the atmosphere are used for creating up-to-date databases to enable interpolation of the data on the intensity of solar radiation over large areas. Disadvantages of most foreign databases are commercial access, limited coverage of the territory, and as a result, the lack of data for most regions of Russia. Russian experience in creating electronic climate databases is not great. At present, data on actinometric observations for a limited number of stations included in the World Meteorological Organization (WMO) are available in the archive [6] of the World Radiation Data Center and on the official website of All-Russian Scientific Research Institute of Hydrometeorological Information [7].

Atlas of solar energy resources in Russia [3] developed at Joint Institute for High Temperatures RAS is of particular interest. The Atlas provides interactive maps of the distribution of the daily total and direct solar radiation arriving at a fixed surface tilted at different angles to the horizon and at the surface tracking the sun across the territory of Russia. The Atlas is available on the website of the project Geoinformation System, Renewable Energy Sources of Russia [8], and provides the designers of solar power plants with the required initial data to assess the potential of solar energy in various geographical regions of Russia.

However, in spite of the fact that a generalized technical and economic analysis of the efficiency of solar power plants can be performed using electronic databases, the data provided in databases are not enough to solve the majority of problems in technical design. The problems include the determination of the optimal ratio of capacities between the main and reserve generating sources, the required capacity for storage devices, the efficient tracking system and its parameters, the analysis of the energy balance in the designed system, etc. These problems are particularly relevant to hybrid power plants with renewable energy sources designed for autonomous consumer. The commensurability of generating power and load capacities requires the analysis of the plant operating conditions and the concurrence between the production and power consumption modes for both long (month, season) and relatively short (hours, days) time intervals. The stochastic nature of renewable energy sources significantly complicates the solution of the above problems and causes the necessity to use imitative mathematical modeling methods in designing similar power systems. Important components of the imitative models of power systems with renewable energy sources are the models of primary energy resources that contribute to their modeling with high time sampling.

The present study aims to develop a mathematical model of solar radiation arriving at an arbitrarily oriented surface located in any region of Russia.

2 Review of mathematical models of solar radiation

The value of solar radiation arriving at the receiving surface of solar panels depends on a number of factors, some of which are deterministic, and some are stochastic in nature. The deterministic factors include geographical latitude of the solar power plant location, day of year and time of day, and orientation angles of the receiving surface of the solar panel relative to the Sun. Stochastic factors that largely determine the intensity of solar radiation include the concentration of atmospheric gases, dust, aerosols and water vapor suspended in the air, and the nature of cloud cover and underlying surface.

A detailed review, classification and comparative analysis of the solar radiation models are presented in [9–13]. In most models, deterministic factors are determined using the known analytical equations established by solar geometry, and stochastic factors are determined on the basis of statistical processing of the results of long actinometric observations.

The transparency coefficient K_T and the diffusion coefficient K_D that show the intensity of solar radiation arriving at the horizontal surface are most often used as the integral characteristics of the state of the atmosphere:

$$K_T = \frac{H}{H_0}, \quad K_D = \frac{H_D}{H} \tag{1}$$

where H and H_D are the average daily intensity of total and scattered solar radiation arriving at the horizontal surface, respectively; H_0 is the intensity of solar radiation at the upper boundary of the atmosphere.

Numerous studies have shown a close correlation between the diffuse coefficient K_D and the transparency coefficient K_T, which can be represented as an empirical equation:

$$K_D = f(K_T) \tag{2}$$

Polynomial equations are often used to describe the dependence (2), the most common of which are Erbs's model [14], Orgilland Hollands's model [15], Reindl's model [16], etc. A detailed review of the models of this type is reported in [11].

This study employs the NASA methodology [17] to determine the value of the diffuse coefficient K_D . According to the methodology, the fraction of scattered radiation is determined from the system of polynomial equations as a function of the latitude of the locality φ, the transparency coefficient K_T, the sunset hour angle ω_{ss}, and the midpoint elevation angle of the Sun above the horizon h.

There are two main groups of models to convert the scattered radiation from a horizontal surface to a tilted one: isotropic and anisotropic. The isotropic models (Liu and Jordan's [18], Koronakis' [19] et al.) assume a homogeneous distribution of scattered radiation in the sky. The anisotropic models

(Klucher's [20], Hay's [21] et al.) consider the relationship between the direction of scattered radiation propagation and the zenith angle of the Sun.

In the present study, anisotropic Hay's model is used to determine the values of scattered radiation [21]. According to the results of the studies presented in [22], the model provides higher accuracy in calculating the values of scattered radiation in high northern latitudes.

3 Theoretical description of the proposed model

The study uses a combined model of the solar radiation arrival, in which some of the parameters are calculated from analytical equations, and some are determined using empirical coefficients from the NASA SSE database [23] for the specified solar power plant location. The total solar radiation arriving at the tilted surface is determined by the equation:

$$G = (G_H - G_{DH}) \cdot \frac{\cos\theta}{\cos\theta_z} + G_{DH} \cdot \left[A_i \cdot \frac{\cos\theta}{\cos\theta_z} + (1 - A_i) \cdot \left(\frac{1+\cos\beta}{2} \right) \right] + G_H \cdot \rho \cdot \frac{1-\cos\beta}{2}, \qquad (3)$$

where G_H and G_{DH} are the values of total and scattered solar radiation arriving at the horizontal surface, respectively; θ is the angle between the incident radiation direction to the surface and the normal to the surface; θ_z is the solar zenith angle; ρ is the Earth's surface albedo; β is the inclination angle of the receiving surface to the horizontal surface; A_i is the anisotropy index determined by the equation:

$$A_i = \frac{G_H - G_{DH}}{G_0}, \qquad (4)$$

where G_0 is extra-atmospheric solar radiation on the horizontal surface.

The value of the angle θ is determined by formula:

$$\begin{aligned} \cos\theta = {} & \sin\delta \cdot \sin\varphi \cdot \cos\beta - \sin\delta \cdot \cos\varphi \cdot \sin\beta \cdot \cos\gamma + \cos\delta \cdot \cos\varphi \cdot \cos\beta \cdot \cos\omega + \\ & + \cos\delta \cdot \sin\varphi \cdot \sin\beta \cdot \cos\gamma \cdot \cos\omega + \cos\delta \cdot \sin\beta \cdot \sin\gamma \cdot \sin\omega, \end{aligned} \qquad (5)$$

where φ is the latitude of the receiving surface locality; δ is the solar declination angle; γ is the azimuth angle of the receiving surface (with a south-facing direction γ=0°, with a deviation to the east γ is considered negative, with a deviation to the west γ is positive); ω is the solar hour angle.
The declination angle is calculated by the formula:

$$\delta = 23.45 \cdot \sin\left[\frac{360}{365} \cdot (N + 284) \right], \text{ deg.} \qquad (6)$$

where N is the day number, such that N=1 on the 1st January.
The solar zenithal angle is calculated by the formula:

$$\theta_z = \arccos\left[\sin\varphi \cdot \sin\delta + \cos\varphi \cdot \cos\delta \cdot \cos\omega \right], \text{ deg.} \qquad (7)$$

The zenithal angle is the conjugate angle of the solar elevation above the horizon h, which can be calculated from:

$$h = 90° - \theta_z \qquad (8)$$

The solar azimuth angle Az can be calculated from the following equation:

$$\cos Az = \frac{\sinh \cdot \sin\varphi - \sin\delta}{\cosh \cdot \cos\varphi} \qquad (9)$$

The azimuth angles at sunrise ω_{sr} and sunset ω_{ss} can be calculated from:

$$\begin{aligned} \omega_{sr} &= 0 - \arccos\left[-\mathrm{tg}\varphi \cdot \mathrm{tg}\delta \right] \\ \omega_{ss} &= 0 + \arccos\left[-\mathrm{tg}\varphi \cdot \mathrm{tg}\delta \right] \end{aligned} \qquad (10)$$

The average daily values of extra-atmospheric radiation arriving at the horizontal surface are determined by the equation:

$$H_0 = \frac{24}{\pi} \cdot G_{sc} \cdot \left(1 + 0.033 \cdot \cos\frac{360 \cdot N}{365} \right) \cdot \left(\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \sin\varphi \cdot \sin\delta \right), \qquad (11)$$

where G_{sc}=1 367 Wt/m^2 is the solar constant.

The current daily values of total and scattered solar radiation on the horizontal surface are calculated by the formulas:

$$G_H = r_t \cdot H, \quad G_{DH} = r_d \cdot H_D \tag{12}$$

To calculate the coefficients from equation (12), we use the formulas proposed by Collares-Pereira [24]

$$r_t = \frac{\pi}{24} \cdot \left(a + b \cdot \cos \omega\right) \cdot \frac{\cos \varphi - \cos \omega_s}{\sin \omega_s - \omega_s \cdot \cos \omega_s}, \tag{13}$$

with coefficients

$$\begin{aligned} a &= 0.409 + 0.5016 \cdot \sin\left(\omega_s - 60\right) \\ b &= 0.6609 - 0.4767 \cdot \sin\left(\omega_s - 60\right), \end{aligned} \tag{14}$$

and by Liu and Jordan [17]:

$$r_d = \frac{\pi}{24} \cdot \frac{\cos \varphi - \cos \omega_s}{\sin \omega_s - \omega_s \cdot \cos \omega_s} \tag{15}$$

To determine the real characteristics of solar radiation arriving at a certain territory, the model uses the numerical values of the transparency coefficient K_T and the surface albedo ρ from the NASA SSE database [23]. Advantages of this database are open access, convenient service and up-to-date models of the solar radiation propagation in the atmosphere that take into account the features of the climate zones, the nature of radiation reflected from the Earth's surface, cloud amount, and atmospheric pollution. The NASA database is based on the ground and satellite observations of solar radiation in the period from 1983 to 2005, and the calculated values of solar radiation are interpolated to a 1°×1° grid covering the Earth. The comparative analysis of the NASA database data with the results of direct long-term measurements of solar radiation from 50 Russian meteorological stations located in different regions of the country showed that the average radiation error for most regions of Russia does not exceed 15% [5].

Based on equations (1)–(15), a mathematical model was developed in the MatLab/Simulink environment to determine the amount of solar radiation arriving at the arbitrarily oriented surface for any day of the year in any location of the solar power plant.

The model verification using the observation data from 10 meteorological stations located in the territory 43º to 62º N, 50º to 158º E showed that the model provides a satisfactory error in modeling both the annual and daily course of solar radiation. The average relative error for all the cities under consideration does not exceed 11.7% for total solar radiation and 24.5% for scattered radiation [25], which is quite acceptable for engineering calculations.

4 Results and discussion

Illustrate the possibilities of the proposed model through a specific example of determining the potential solar energy resources for the solar power plant located in Tomsk region (56.5º N, 85º E).

Table 1 summarizes the average daily values of total solar radiation on the horizontal surface under actual cloud conditions, according to the data from the USSR Scientific and Applied Reference Book on Climate [1], the NASA SSE database [23] and the results of modeling.

Table 1. The average daily total solar radiation on the horizontal surface under actual cloud conditions (kWh/m^2) in Tomsk

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	year
Reference book	0.67	1.54	3.15	4.27	4.99	5.70	5.48	4.20	2.87	1.32	0.74	0.45	35.39
NASA SSE	0.69	1.59	2.94	4.29	5.48	5.79	5.80	4.55	2.83	1.58	0.83	0.46	36.83
Model	0.66	1.53	2.90	4.17	5.43	5.79	5.84	4.48	2.72	1.44	0.77	0.45	36.18

While modeling, the "average day of the month" method proposed by Klein [26] was used to reduce the amount of calculation. According to the method, the daily average intensity of solar radiation for the considered month of the year is equal to the daily radiation intensity for the corresponding day. The analysis of the results presented in Table 1 shows that the model provides satisfactory accuracy in calculating the total values of solar radiation arriving at the horizontal surface: the relative error for all months of the year is no more than 10%.

For the comparative evaluation of the efficiency of solar tracking systems in Tomsk, a series of computational experiments was performed to determine the values of solar radiation arriving at the receiving surface of satellites oriented at different angles. As an example, Figure 1 illustrates the calculated dependences of solar radiation arriving at the surface of the solar panel on the day of the summer solstice.

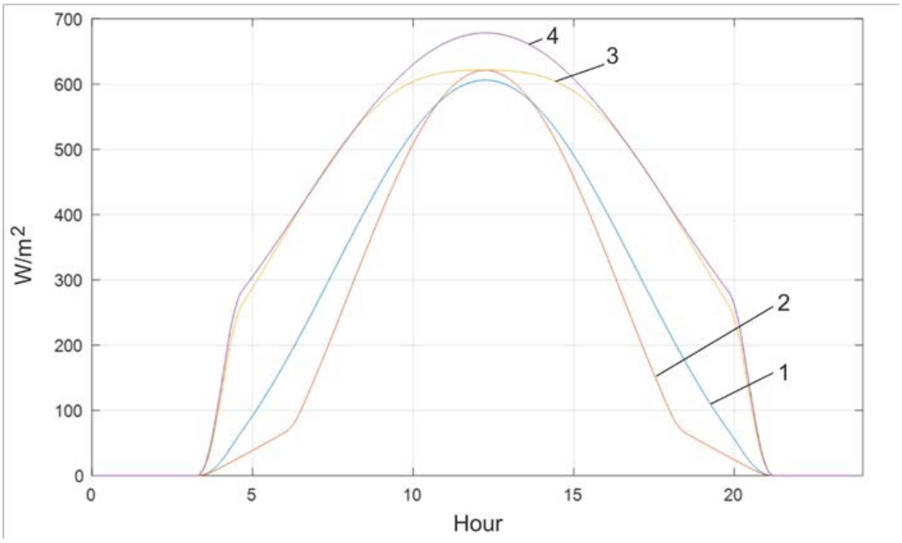

Figure 1. Dependencies of total solar radiation arriving at the surface of the solar panel on the day of the summer solstice. The solar panel is installed horizontally (1), at the angle of 56.5º to the horizon (2), with a single-axis (3) and two-axis tracking systems (4).

The analysis of modeling results shows that the proposed model adequately reflects the daily course of solar radiation and can be used as a convenient tool for solving many practical problems associated with the design and optimization of the power systems, which employ solar radiation as a primary energy source.

5 Conclusions

The original model of solar radiation arriving at the arbitrarily oriented surface has been developed. The peculiarity of the model is that it uses numerical values of the atmospheric transparency index and the surface albedo from the NASA SSE database as the initial data. This enables using the model to predict the main characteristics of solar radiation for any geographical point in Russia, including those for territories with no regular actinometric observations.

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