

Mathematic modeling of the method of measurement relative dielectric permeability

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Abstract. The method of measuring relative permittivity's and the position of the interface between layers of a liquid medium is considered in the article. An electric capacitor is a system consisting of two conductors that are separated by a dielectric layer. It is mathematically proven that at any given time it is possible to obtain the values of the relative permittivity in the layers of the liquid medium and to determine the level of the interface between the layers of the two-layer liquid. The estimation of measurement errors is made.

1. Introduction

The big variety of the measuring tasks connected with the necessity of carrying out the measurements of various liquid environments level [1] is characteristic for the various fields of modern science and technology [2, 3]. So, for example, the necessity of measurement of the level arises in the majority of intellectual systems of environmental monitoring and safety, in modern productions and also at control of weight and an expense of liquid environments at a stage of their transportation or in the process of storage [4, 5].

Now there are more than twenty various ways of measurement of liquid environments level [6–9]. A part of them was widely used in the modern industry, and the others have the highly specialized field of application because of various shortcomings inherent in them. As a rule, not only the level of the liquid environment [10], but also position of the section limit between layers [11, 12] is required not only to measure in the majority practical applications if the liquid environment is multicomponent. For example, oil / commercial water in settling tanks can be an example of such multicomponent liquid environment. It should be noted that the majority of modern practical approaches for measurement of the section limit of two-layer liquid are unsuitable. So, for example, a number of methods of measurements can work only if the controlled liquid environment is transparent [10]. Therefore, there is a problem of consideration of other technical solutions to the measurement of level and limit of the section of the two-layer liquid environment.

Now most for measurement of liquid environments level the capacitor sensors of level measurement are widely used [13–17]. The dependence of value of electric capacity of the measuring converter in the form of the condenser educated, for example, by means of two plates which are partially entered on the measured environment from liquid level height is the basis for the principle of operation of the capacitor level gage. In this article the measurement method which is based on the usage of three capacitor sensors of level is considered.

2. Research object and method

The considered method of measurement of relative dielectric permeability and position of the section limit between layers of the liquid environment describes the measuring system, consisting of three



capacitor measuring converters in the form of flat condensers. It is necessary to notice that the electric condenser represents the system consisting of two conductors, which are divided by a dielectric layer. The value of electric capacity of the condenser C for a case of a plane-parallel arrangement of measuring electrodes can be determined by the following mathematical expression (at neglect by regional effect):

$$C = \varepsilon_0 \varepsilon \frac{s}{d}, \quad (1)$$

where ε is relative dielectric permeability of the interelectrode environment; ε_0 is a dielectric constant; s is area of measuring electrodes of the condenser; d is distance between measuring electrodes of the condenser.

The possible relative positioning and the geometrical sizes of measuring electrodes of three capacitor D1–D3 sensors are shown in the Figure 1. We will consider that the distance between measuring electrodes of sensors is equal d , and width and height measuring electrodes of capacitor sensors of level are equal a and b .

We will consider the tank with the two-layer liquid environment. We will consider at the same time that two-layer liquid occupies completely all volume of the tank. The relative dielectric permeability of the top layer of the liquid environment is equal ε_1 , and relative dielectric permeability of the lower layer of the liquid environment ε_2 (Figure 2).

In this case the problem of measurement of position of the section limit of liquid environments comes down to use of three capacitor sensors of level. We will assume that the limit of the section of two liquid environments passes through measuring electrodes of the D2 sensor (Figure 2). We will consider that the beginning of coordinates is in the top boundary surface of the tank. We will consider at the same time that the D1 sensor is shipped in the controlled two-layer liquid environment on the H_1 level, the D2 sensor on the H_2 level, and the D3 sensor on the H_3 level, and the limit of the section of two liquid environments is at the level H_L (Figure 2).

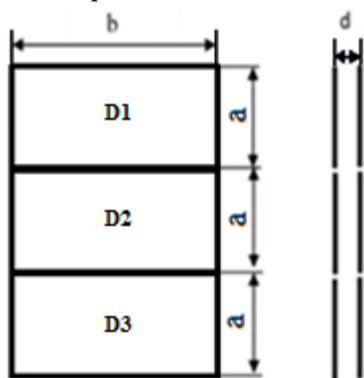


Figure 1. Arrangement and sizes sensors of level.

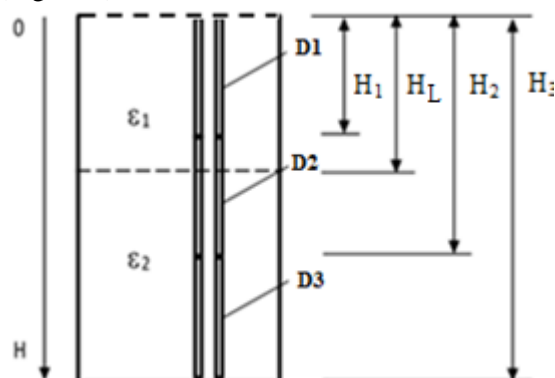


Figure 2. The tank with three capacitor sensors of level.

According to (1) we will find the value of capacity for each of three sensors of level. We will write down value of capacity of the D1 sensor as:

$$C_1 = \frac{\varepsilon_0 \varepsilon b H_1}{d}. \quad (2)$$

We will write down value of capacity of the D2 sensor as:

$$C_2 = \frac{\varepsilon_0 \varepsilon_1 b (H_L - H_1)}{d} + \frac{\varepsilon_0 \varepsilon_2 b (H_2 - H_L)}{d}. \quad (3)$$

We will write down value of capacity of the D3 sensor as:

$$C_3 = \frac{\varepsilon_0 \varepsilon_2 b (H_3 - H_2)}{d}. \quad (4)$$

From expressions (2) and (4) we will write down values of relative dielectric permeability as ε_1 and ε_2 :

$$\varepsilon_1 = \frac{dC_1}{\varepsilon_0 b H_1}, \quad \varepsilon_2 = \frac{dC_2}{\varepsilon_0 b (H_3 - H_2)}. \quad (5)$$

We will consider that $H_3 = 3H_1$, $H_2 = 2H_1$.

Having substituted the received expressions for ε_1 and ε_2 in expression (3), we receive the expression for value of section limit level of two liquid environments:

$$H = H_1 \frac{c_2 - 2c_3 + c_1}{c_1 - c_3}. \quad (6)$$

Thus, values of relative dielectric permeability of both layers of the liquid environment and the level of limit of the section between layers of two-layer liquid are known at any moment.

3. Assessment of errors of measurement

We will assume that in a system which is represented in the Figure 2 the measurement of capacity value happens with a margin error c_{ci} that is

$$c_1 = c_{1t} + \xi_{c1}, \quad c_2 = c_{2t} + \xi_{c2}, \quad c_3 = c_{3t} + \xi_{c3},$$

where c_{1t} , c_{2t} , c_{3t} are true values of capacity of sensors of level, and ξ_{c1} , ξ_{c2} , ξ_{c3} are full errors of measurement of the corresponding capacities.

We will define errors of measurement of relative dielectric permeability ε_1 , ε_2 and position of limit of sections H_L through errors of measurement of capacitor sensors of the level ξ_{c1} , ξ_{c2} и ξ_{c3} .

At nonlinear dependence of position of limit of section H_L on the measured values of parameters according to (6) mean square deviation of result of indirect measurement is defined as follows:

$$\sigma_{\varepsilon 1} = \frac{d}{\varepsilon_0 b H_1} \sigma_{c1}, \quad \sigma_{\varepsilon 2} = \frac{d}{\varepsilon_0 b (H_1 - H_2)} \sigma_{c3}, \quad (8)$$

where σ_{c1} , σ_{c3} mean square deviations of capacitor D1 and D3 sensors.

At nonlinear dependence of position of limit of section H_L on the measured values of parameters according to (6) mean square deviation of result of indirect measurement is defined as follows:

$$\sigma_{H_L} = \sqrt{\left(\frac{\partial H_L}{\partial c_1}\right)^2 \cdot (\sigma_{c1})^2 + \left(\frac{\partial H_L}{\partial c_2}\right)^2 \cdot (\sigma_{c2})^2 + \left(\frac{\partial H_L}{\partial c_3}\right)^2 \cdot (\sigma_{c3})^2} \quad (9)$$

At the same time

$$\frac{\partial H_L}{\partial c_1} = \frac{H_1(c_2 - c_3)}{(c_1 - c_3)^2}, \quad \frac{\partial H_L}{\partial c_2} = \frac{H_1}{c_1 - c_3}, \quad \frac{\partial H_L}{\partial c_3} = \frac{H_1(c_1 - c_2)}{(c_1 - c_3)^2},$$

$$\text{Then } \sigma_{H_L} = \sqrt{\left(\frac{H_1(c_2 - c_3)}{(c_1 - c_3)^2}\right)^2 \cdot (\sigma_{c_1})^2 + \left(\frac{H_1}{c_1 - c_3}\right)^2 \cdot (\sigma_{c_2})^2 + \left(\frac{H_1(c_1 - c_2)}{(c_1 - c_3)^2}\right)^2 \cdot (\sigma_{c_3})^2}$$

4. Modeling of work of a method with three capacitor sensors of level

The considered method of measurement of relative dielectric permeability and position of section limit between two layers of the liquid environment has been simulated in the MathCAD package of the following entry conditions:

- height of the tank is 0.6 m;
- height of measuring electrodes of capacitor sensors of the level is 0.2 m;
- width of measuring electrodes of capacitor sensors of the level is 0.3 m;
- distance between measuring electrodes of capacitor sensors of the level is 0.01 m;
- relative dielectric permeability of the first layer of the liquid environment is 2.2;
- relative dielectric permeability of the second layer of the liquid environment is 81;
- time of modeling is 1000 sec.

It was also supposed that errors of measurement of capacitor sensors of level represent white noise with an amplitude of 1% from extreme value of the indication of sensors.

In the course of modeling on every second selection of 10 measurements of values of capacities for each of three sensors of level was formed. Then the mathematical expectations and mean square deviations for values of capacities of sensors of level were determined by the received selection. Besides, on each measurement of selection were calculated relative dielectric permeability of liquid layers, and position of section limit between layers for which later the mathematical expectations and mean square deviations were defined.

In Figures 3 and 4 schedules with the results of mathematical modeling are submitted.

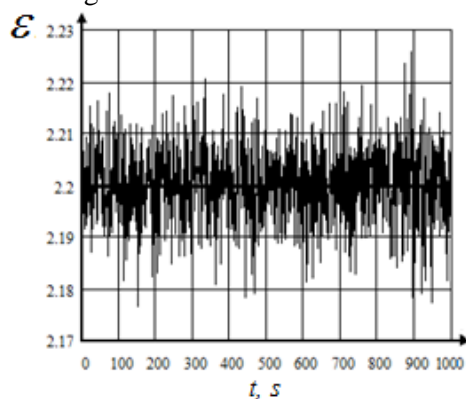


Figure 3. Relative dielectric permeability of the top layer.

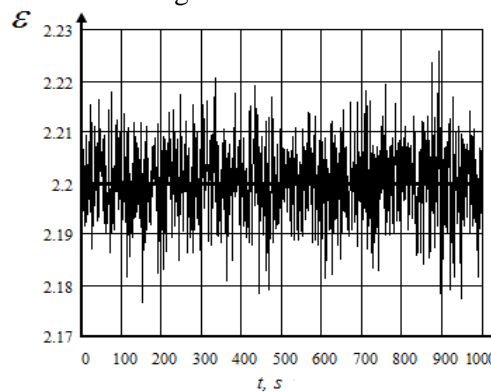


Figure 4. Relative dielectric permeability of the lower layer.

As the result of mathematical modeling we have received that for the capacitor D1 sensor the extreme value of capacity is 11.68 pF, and for the capacitor D3 sensor is 430.1 pF.

The value of amplitude of white noise for the D1 sensor will be $3\sigma_{c_1} = 0.12$ pF and the value of amplitude of white noise for the D3 sensor will be $3\sigma_{c_3} = 4.3$ pF.

Having substituted these found values in expressions (8), we will receive:

$$\sigma_{\varepsilon_1} = 2.26 \cdot 10^{-3}, \sigma_{\varepsilon_2} = 0.081.$$

As the expression (11) includes the current value of capacity of the D2 sensor, for determination of theoretical value σ_{H_s} will calculate it for timepoint of $t=1000$ sec.

In this time point $C1=11.69$ pF, $C2=428.3$ pF, $C3=430.5$ pF, $H_L=0.20$ m, and the mean square deviation for the position of section limit H_L will make $\sigma_{H_s} = 6.35 \cdot 10^{-4} m$.

5. Summary

In article the method of measurement of relative dielectric permeability of layers and position of section limit of the two-layer liquid environment based on the usage of three capacitor sensors of level is considered. The results of the mathematical modeling are given and assessment of errors of measurement is executed.

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