A numerical solution of the problem of crown forest fire initiation and spread

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Abstract. Mathematical model of forest fire was based on an analysis of known experimental data and using concept and methods from reactive media mechanics. The study takes in to account the mutual interaction of the forest fires and three-dimensional atmosphere flows. The research is done by means of mathematical modeling of physical processes. It is based on numerical solution of Reynolds equations for chemical components and equations of energy conservation for gaseous and condensed phases. It is assumed that the forest during a forest fire can be modeled as a two-temperature multiphase non-deformable porous reactive medium. A discrete analog for the system of equations was obtained by means of the control volume method. The developed model of forest fire initiation and spreading would make it possible to obtain a detailed picture of the variation in the velocity, temperature and chemical species concentration fields with time. Mathematical model and the result of the calculation give an opportunity to evaluate critical conditions of the forest fire initiation and spread which allows applying the given model for of means for preventing fires.

1. Introduction
The characteristics of forest fires reveals their dependence on the specific conditions under which the experiments were conducted, and their insufficient accuracy, which results from the fact that under actual conditions it is impossible to control the meteorological situation and the homogeneity of forest fuel layer. For this reason, the physical modeling of forest fires must be combined with mathematical experiments using numerical methods and computers. Considering that, natural investigations of these problems are merely impossible. It was used the methods of mathematical modeling. Mathematical model of forest fire was based on an analysis of known experimental data and using concept and methods from reactive media mechanics. Usually crown fires are initiated by surface fires. The one of the first explanation of this process was given by Van Wagner [1, 2]. The theory proposed there depends on three simple crown properties: crown base height, bulk density and moisture content of forest fuel. Also crown fire initiation and hazard have been studied and modeled in detail [3–11]. The more complete mathematical model of forest fires was provided by Grishin [12–14]. In particular, this model which was obtained by Grishin [12] based on an analysis of known and original experimental data [2, 14], and using concepts and methods from reactive media mechanics. The physical multiphase model used in [15] may be considered as a continuation and extension of the formulation proposed by Grishin [12]. However, the investigation of crown fires has been limited mainly to cases studied of forest fires initiation without take into account the mutual interaction of the forest fires and three-dimensional atmosphere flows. This restriction has been removed in present setting of problem.

Boundary layer of the atmosphere, forest canopy and the lower tier of the forest is considered as a single region.

2. Physical and mathematical model

The basic assumptions using in this setting are: 1) the forest represents a multi-phase, multistoried, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non - deformed medium (trunks, large branches, small twigs and needles), affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the coordinate reference point x_1, x_2 , $x_3= 0$ be situated at the centre of the surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the horizontal surface to the right in the direction of the unperturbed wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (Figure 1).
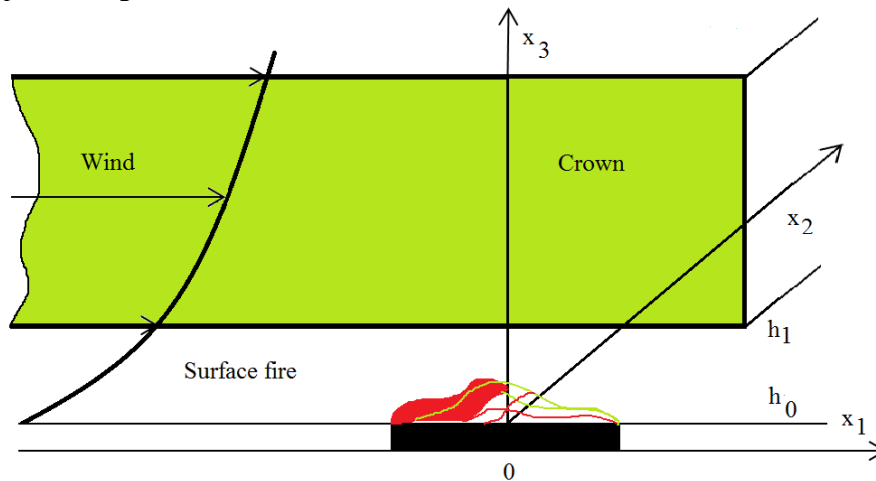

Figure 1. Computational domain.

Using the results of [12] and known experimental data [2, 12] governing equations are written as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = \dot{m}, \ j = 1,2, \ i = 1,2; \tag{1}$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j}(-\rho \overline{v'_i v'_j}) - \\ - \rho s c_d v_i |v| - \rho g_i - \dot{m} v_i; \tag{2}$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p \overline{v'_j T'}) + q_5 R_5 - \alpha_v (T - T_s) + k_g (cU_R - 4\sigma T^4); \tag{3}$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho \overline{v'_j c'_\alpha}) + R_{5\alpha} - \overset{\bullet}{m} c_\alpha \quad , \alpha = 1,5; \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right) - kcU_R + 4k_s\sigma T_S^4 + 4k_g\sigma T^4 = 0, k = k_g + k_s; \tag{5}$$

$$\sum_{i=1}^{4} \rho_i c_{pi} \varphi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k_s(cU_R - 4\sigma T_S^4) + \alpha_V(T - T_S); \tag{6}$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_C R_1 - \frac{M_C}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \tag{7}$$

$$\sum_{\alpha=1}^{5} c_\alpha = 1, \; p_e = \rho RT \sum_{\alpha=1}^{5} \frac{c_\alpha}{M_\alpha}, \vec{v} = (v_1, v_2, v_3), \vec{g} = (0,0,g); \; \overset{\bullet}{m} = (1-\alpha_c)R_1 + R_2 + \frac{M_c}{M_1}R_3 + R_{54} + R_{55};$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2}R_5, R_{52} = v_g(1-\alpha_c)R_1 - R_5, R_{53} = 0, R_{54} = \alpha_4 R_1, R_{55} = \frac{\alpha_5 v_3}{v_3 + v_{3*}}R_3.$$

The system of equation defines the state of the medium in the forest fire zone. Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models [12]. The boundary conditions for the system of equations (1)–(7) are specified as follows:

$$t = 0 : v_1 = 0, v_2 = 0, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e}, T_s = T_e, \varphi_1 = \varphi_{ie}; \tag{8}$$

$$x_1 = -x_{1e} : v_1 = V_e, v_2 = 0, \frac{\partial v_3}{\partial x_1} = 0, T = T_e, c_\alpha = c_{\alpha e}, -\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + cU_R/2 = 0; \tag{9}$$

$$x_1 = x_{1e} : \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_1} + \frac{c}{2}U_R = 0; \tag{10}$$

$$x_2 = x_{20} : \frac{\partial v_1}{\partial x_2} = 0, v_2 = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{\partial U_R}{\partial x_2} = 0; \tag{11}$$

$$x_2 = x_{2e} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2}U_R = 0; \tag{12}$$

$$x_3 = 0 : v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2}U_R = 0, v_3 = v_{30}, T = T_g, |x_1| \le \Delta, |x_2| \le \Delta,$$
$$v_3 = 0, \; T = T_e, |x_1| > \Delta, |x_2| > \Delta; \tag{13}$$

$$x_3 = x_{3e} : \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2}U_R = 0. \tag{14}$$

Here and above $\dfrac{\mathrm{d}}{\mathrm{d}t}$ is the symbol of the total (substantial) derivative; α_V is the coefficient of phase exchange; ρ is density of gas – dispersed phase, t is time; v_i is the velocity components; T, T_S, are temperatures of gas and solid phases, U_R is density of radiation energy, k *is* coefficient of radiation attenuation, P is pressure; c_p is constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i are specific heat, density and volume of fraction of condensed phase (1 is dry organic substance, 2 is moisture, 3 are condensed pyrolysis products, 4 is mineral part of forest fuel), R_i are the mass rates of chemical reactions, q_i are thermal effects of chemical reactions; k_g , k_S are radiation absorption coefficients for gas and condensed phases; T_e is the ambient temperature; c_α are mass concentrations of α - component of gas - dispersed medium, index α=1,2,...,5, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide *CO*, 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R is universal gas constant; M_α , M_C, and M are molecular mass of α -components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum, g is mass fraction of gas combustible products of pyrolysis, α_4 and α_5 are empirical constants. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase were used the following formulae for the rate of formulation of the gas-dispersed mixture $\dot{m}$, outflow of oxygen R_{51}, changing carbon monoxide R_{52}, generation of black R_{54} and smoke particles R_{55} .

Coefficients of multiphase (gas and solid phase) heat and mass exchange are defined $\alpha_v = \alpha S - \gamma C_P \dot{m}, S = 4\phi_S / d_S$. Here $\alpha = Nu\lambda/d_S$ is coefficient of heat exchange for sample of forest combustible material (for example needle), Nu is Nusselt number for cylinder, λ is coefficient of heat conductivity for pine needle; γ is parameter, which characterize relation between molecular masses of ambient and inflow gases. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is "grey"), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. The components of the tensor of turbulent stresses, as well as the turbulent fluxes of heat and mass are written in terms of the gradients of the average flow properties [12]. It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different type of forest (for example, pine forest [12-16]).

3. Numerical solution and results
The boundary-value problem (1)–(14) was solved numerically using control volume method and the SIMPLE like algorithm [16]. Also numerical calculation carried out using software PHOENICS [17, 18]. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained for different instant of time. Figures 2*a-g* present the distribution for temperature and velocities.

Figures 2*a-g* present the distribution for temperature (*a-t=7s, b-t=9s, c – t=14s, d- 16s, e – t=17s, f – t=21s, g – t=27 s.*) and velocities. The isothermes are moved in the forest canopy and deformed by the action of wind. The results of calculation give an opportunity to evaluate critical condition of the forest fire spread, which allows applying the given model for preventing fires. It overestimates the crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel and etc. The model proposed there gives a detailed picture of the distribution in the velocity, temperature and component

concentration fields with time, and determine as well as the influence of different conditions on the crown forest fire spread.
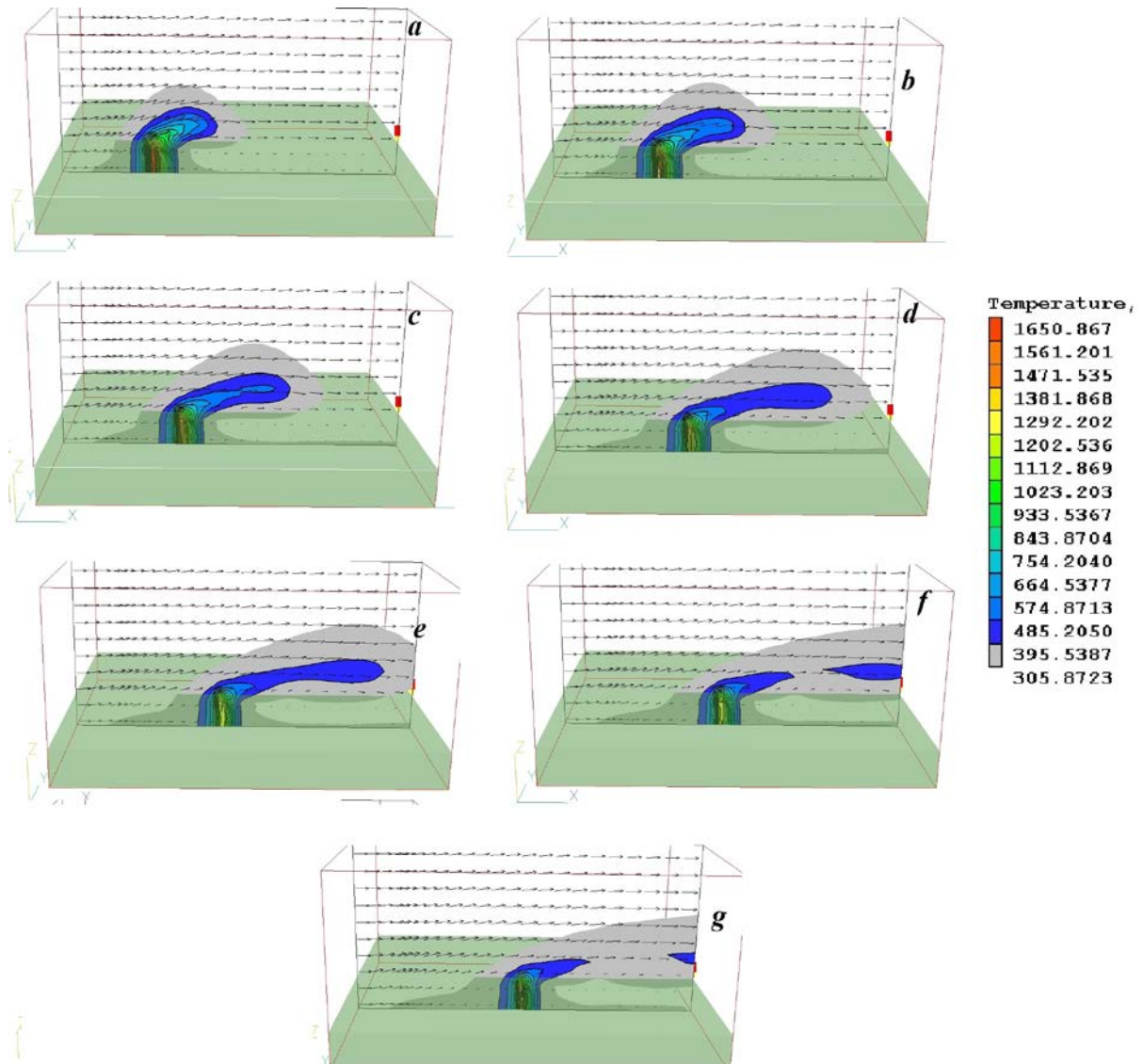

Figure 2. The distribution of temperature and field of velocity.

4. Conclusion

A multiphase mathematical model of crown fire initiation and spread was developed using the multi-purpose software, PHOENICS [17, 18]. The model accounts for all the important physical and physicochemical processes: drying, pyrolysis, char combustion, turbulent combustion of gaseous products of pyrolysis, exchange of mass, momentum and energy between gas and solid phase, turbulent gas flow and convective, conductive and radiative heat transfer.

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