Modelling of depth stabilization and submerging of tethered underwater garage in conditions of sea oscillating motion

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Abstract. The paper is dedicated to examining dynamics of a submersible underwater garage in conditions of significant sea oscillation. During the considered research, the mathematical model of the electromechanical depth control system, considering interval parametric uncertainty of the system and distribution of tether mass, was developed. An influence of sea oscillation on submerging underwater garages and their depth stabilization processes was analyzed.

1. Introduction

Nowadays, the World ocean is being actively developed with the help of remotely operated and autonomous unmanned underwater vehicles (UUV). In order to prevent any damage of expensive underwater equipment, while ascending-descending a UUV during sea oscillation, and to save a resource of UUV batteries; UUVs are submerged to an operating depth in submersible underwater garages (SUGs). Such SUG can be submerged and stabilized with the help of a hoist with a tether, mounted on a carrier vessel.

Sea oscillation may cause a rapid tension of the SUG tether. Consequently, a joint between a tether and a SUG or a SUG itself may be broken [1–4]. Considering this, a problem of mathematical modeling of dynamics in a mechanical system, including a hoist, a tether and a SUG is highly relevant. To solve this problems, a proper mathematical model, considering interval parametric uncertainty of the system, added masses of water and distribution of a tether mass, must be derived.

2. Mathematical modelling of a tether with a distributed mass

Let us consider a process of manipulating a tension force of a tether with one fixed end by applying a controllable force to another end. A flexible vertical tether with length \( l \) is influenced by two tensing forces, applied to an upper (\( F_{tT} \)) and a lower (\( F_{bT} \)) end of the tether. In a stationary condition, force \( F_{tT} \), applied to an upper end of a tether, is equal to a sum of a tether weight and a force \( F_{bT} \), applied to a lower end of a tether. Static tension force in each point of a vertical flexible tether, which mass is measured, is fully determined by three parameters: distances between a considered point and ends of a tether and a value of a tension force in a lower point of a tether \( F_{bT} \).

Let us make the following designations: \( F_{tT} \) – increment of a tension force, applied to a lower end of the tether; \( F_{bT} \) – increment of tension force, applied to an upper end of the tether; \( l_{tT} \) – increment of a length of a tether upper end; \( l_{bT} \) – increment of a length of a tether lower end, \( m_r \) – mass of the tether.
According to the theory of oscillating systems with distributed parameters, a transfer function between an increment of a tether end length and a tension force increment can be written as follows:

\[ W_t(s) = \frac{\Delta F^{\alpha T}}{\Delta l^{\alpha T}} = \frac{\Delta F^{\beta T}}{\Delta l^{\beta T}} = \frac{C_{\text{ort}} \sqrt{b} \cdot \text{ch}(\sqrt{b})}{\text{sh}(\sqrt{b})}, \]

where \( b = \frac{s}{a^2} (s + 2h) \), \( 2h = \frac{\chi_{\text{ort}}}{m_r}, a^2 = \frac{C_{\text{ort}}}{m_r} \) – accordingly, internal damping coefficients and specific stiffness of the tether.

One can notice that the transfer function between tension forces of tether ends can be written as follows:

\[ W_f(s) = \frac{\Delta F^{\alpha T}}{\Delta F^{\beta T}} = \frac{\Delta F^{\beta T}}{\Delta F^{\alpha T}} = \frac{1}{\chi(\sqrt{b})}. \]

By replacing hyperbolic functions in (1) and (2) with first two terms of their Maclaurin series \( \text{ch}(\sqrt{b}) = 1 + \frac{b l_{y}^2}{2} \); \( \text{sh}(\sqrt{b}) = \sqrt{l_{y}^2 b (1 + \frac{l_{y}^2 b}{6})} \), transfer functions (1) and (2) can be written as follows:

\[ W_t(s) = \frac{\Delta F^{\alpha T}}{\Delta l^{\alpha T}} = \frac{\Delta F^{\beta T}}{\Delta l^{\beta T}} = \frac{3C_{\text{ort}}}{l_{y}^3 m_r s^2 + \chi_{\text{ort}} l_{y}^3 s + 2C_{\text{ort}}} \]

\[ W_f(s) = \frac{\Delta F^{\alpha T}}{\Delta F^{\beta T}} = \frac{\Delta F^{\beta T}}{\Delta F^{\alpha T}} = \frac{2C_{\text{ort}}}{l_{y}^3 m_r s + \chi_{\text{ort}} l_{y}^3 s + 2C_{\text{ort}}}. \]

Considering this, a mathematical model of a heavy tether can be considered as an object with two inputs \( l^{\alpha T} \) and two outputs \( F^{\beta T} \). Let us derive an expression for increments of tension forces applied to tether ends:

\[ \Delta F^{\alpha T} = \Delta F_{1}^{\alpha T} - \Delta F_{2}^{\alpha T} = l^{\alpha T} \cdot W_{f} - l^{\beta T} \cdot W_{i} \cdot W_{f}. \]

\[ \Delta F^{\beta T} = \Delta F_{1}^{\beta T} - \Delta F_{2}^{\beta T} = l^{\beta T} \cdot W_{i} - l^{\alpha T} \cdot W_{i} \cdot W_{f}. \]

The previous expression can be written in a matrix form as follows:

\[ \begin{bmatrix} \Delta F^{\alpha T} \\ \Delta F^{\beta T} \end{bmatrix} = \begin{bmatrix} W_{f} & W_{i} \end{bmatrix} \begin{bmatrix} l^{\alpha T} \\ l^{\beta T} \end{bmatrix}; \]

where a transfer function matrix can be written as:

\[ \begin{bmatrix} W_{f} & W_{i} \\ W_{i} & W_{f} \end{bmatrix}. \]

On the base of transfer functions (3) and (4), a structural diagram of a heavy tether mathematical model, shown in figure 1, was developed.

![Figure 1. Structural diagram of heavy tether](image)

3. **Considering an added mass of water and interval parametric uncertainty**
If a SUG with a mass equal to \( m_{SUG}^* \) moves vertically under the water surface under influence of \( F \) force, then a force of water resistance is proportional to SUG acceleration \( a \), according to \( F = m_{SUG}^* a \), and:

\[ m_{SUG}^* = \left( m_{SUG} + \mu \right), \]

where \( \mu \) – an added mass of water. Added mass of water depends on SUG geometry, motion direction and water density. According to [5, 6], an added mass of water of a parallelepiped shaped SUG can be calculated with the help of following expression:

\[ \mu = \frac{\pi r l^2}{4r^2 + l_{SUG}^2} \left( 1 - 0.425 \frac{r_{SUG}^2}{r^2 + l_{SUG}^2} \right) \]

where \( r \) – SUG width; \( l_{SUG} \) – SUG length.

Some parameters of the system are considered as interval ones: \([l_T], [C_{aT}], [X_{aT}], [m_{SUG}], \rho\) .

Considering interval uncertainty of these parameters, (3) – (6) can be rewritten as follows:

\[ W_j(s) = \frac{\Delta F_{SUG}^T}{\Delta l_T} = \frac{\Delta F_{SUG}^T}{\Delta X_{aT}} = [3[C_{aT}], \frac{[l_T]^3[m_T]s^2 + 2[C_{aT}]}{[l_T]^3[m_T]s^2 + 2[C_{aT}]} \], \]

\[ W_j(s) = \frac{\Delta F_{SUG}^T}{\Delta l_T} = \frac{2[C_{aT}]}{[l_T][m_T]s^2 + 2[C_{aT}]} \frac{\pi \rho r_{SUG}^2}{4r^2 + l_{SUG}^2} \left( 1 - 0.425 \frac{r_{SUG}^2}{r^2 + l_{SUG}^2} \right), \]

\[ m_{SUG}^* = \left( m_{SUG} + \mu \right). \]

### 4. Mathematical model of a SUG depth control system

An electric drive of a carrier vessel hoist is described with equation

\[ J \frac{d \omega}{dt} = M_d + M_{q}, \]

where \( M_d \) – an actuating moment of a hoist drive, \( J \) – hoist moment of inertia, \( \omega \) – an angular velocity of hoist drum rotation, \( M_{q} = F_{q}R \) – a moment of tether tension force, \( R \) – a radius of hoist drum. Actuating moment of a hoist drive can be calculated with the help of expression \( M_d = k_m(U_{c} - U_{e}) \), where \( U_{c} \) is an output voltage of a hoist controller, \( k_m \) – a moment transfer coefficient of a hoist, \( U_{e} = k_c \omega \) – a voltage of counter electromotive force of a hoist drive, \( k_c \) – a coefficient of counter electromotive force of a ship hoist. Voltage \( U_{c} \) depends on a signal from linear setpoint adjuster of a hoist rotation velocity. Input voltage of hoist \( U_{i} \) is determined by an amplifier with a \( k_a \) coefficient on the base of \( U_{i} = k_a (U_{m} - U_{f}) \), where \( U_{f} = k_f \omega \).

Considering this, a structural diagram of a mathematical model of a SUG depth control system is shown in figure 2.

**Figure 2.** A structural diagram of a SUG depth control system
5. Modeling a control signal and a disturbance signal

In order to provide a smooth acceleration and deceleration, a signal of motion velocity setpoint adjuster must have a form, shown in figure 3. In figure 3, T is a desired time of SUG submerging on an operating depth.

![Figure 3. Setpoint adjuster signal of a SUG motion velocity](image)

Figure 3 shows that a SUG accelerates in a time interval \([t_1, t_2]\), moves with a constant velocity in a time interval of \([t_2, t_3]\) and decelerates in a time interval \([t_3, t_4]\). From the \(t_4\) moment, SUG switches to stabilization mode. To model an irregular oscillation of the water, a wave specter was used which mathematical description is given in [7]–[10]. According to [7, 8], to model a random process with desired spectral density, a “white noise” signal must be processed with the help of filter with a rational transfer function. One of such filters, modeling a sea oscillation, was derived in [7, 8] and has the following transfer function:

\[
W_{as}(s) = \frac{10.966s^4}{(s^2 + 1.497s + 1.361)(s^2 + 0.483s + 0.664)(s^2 + 0.854s + 0.941)(s^2 + 2.466s + 6.288)}.
\]

According to [7,8], in order to obtain a “white noise” signal, it is proposed to use a standard block White Noise from a Simulink-Band-Limited library. Considering this, a structural diagram of an irregular sea oscillation was developed (figure 4).

![Figure 4. Model of an irregular sea oscillation](image)

Output signal of the model is an ordinate of sea oscillation \(\zeta\), which determines a vertical motion of a vessel. Its plot is shown in figure 5.
6. Modeling a submerging and a stabilization of SUG

In order to analyze an influence of a sea oscillation on a submerging and stabilization process of SUG, an simulation modeling of a SUG depth control system was performed with the help of Matlab software. Results of simulation modeling of SUG submerging on a depth of 1000 meters in conditions of 4 grade sea oscillation are shown in figure 6.

Figure 6 shows, that mean square error of SUG velocity is 0.8 mps.

7. Conclusion

The considered system, which includes a hoist, a tether and SUG, is a mass-elastic system with distributed parameters. Such system may be influenced by resonance oscillations, caused by significant sea oscillations. These oscillations may cause tether breaking, SUG ground impact and UUV failure. In order to compensate sea oscillation, the SUG depth control system must be synthesized, which would damp sea oscillations during SUG submerging and stabilizing it on a desired depth. The research resulted in a mathematical model, which allows one to synthesize such
system, considering its interval parametric uncertainty, distribution of parameters and inner interaction between manipulated variables of the system.

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References