Resource-Efficient Technologies 1 (2019) 20–26

TOMSK POLYTECHNIC UNIVERSITY

Resource-Efficient Technologies

journal homepage: http://ojs.tpu.ru/index.php/res-eff

Research Paper

THE USE OF INCORRECTLY POSED INVERSE PROBLEMS AND CATASTROPHE THEORY IN ACOUSTOPLASMIC STUDIES

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Abstract

If the discharge current into a plasma contains direct and variable components, the plasma develops wavelike acoustic instabilities and eventually becomes an acoustoplasma. Such instabilities lead to bistability, multistability, and hysteresis phenomena of the current–voltage characteristics, causing abrupt changes in the state of the plasma medium. These changes can be imagined as phase transitions and described using catastrophe theory. In the present study, the experimental plasma data are approximated by the equations of catastrophes. After reducing the catastrophe equation to canonical form, the points of possible phase transitions are determined. The phase transition coordinates are then converted to coordinates in the experimental system by inverse transformations. In this way, we determine the points of possible phase transitions in a real experiment. Finally, the parameter changes in an acoustoplasma discharge are obtained by solving incorrectly posed inverse problems. The inverse problem of the experimental data is solved at each current time. Within the neighborhoods of singular points, the incorrectly posed inverse problems are solved by the theory of catastrophes. The proposed methods are applicable to various fields of science and technology.

Key words: Incorrectly posed inverse problems, theory of catastrophes, phase transitions and jumps.

1. Introduction

As a nonlinear medium in a self-consistent system, plasma establishes both stable and unstable states [1, 2]. Under an external action (such as a current modulation), wavelike acoustic instabilities appear in the plasma, which then becomes an acoustoplasma [1, 3, 4]. Such instabilities cause abrupt changes in the plasma environment, hysteresis, bistability, and multistability and other bifurcations [1–10]. Jumps and instabilities in acoustoplasmas have been experimentally investigated [3, 4]. Other researchers have studied multistable states of the discharge parameters [3, 5, 6], hysteresis in gas-discharge plasmas [3, 7, 8], jumps in a magnetized gas-discharge plasma, and jumps in a plasma excited with radio-frequency waves [10, 11].

Recently, phase transitions have been interpreted not only in the conventional thermodynamic sense, but also as any transition from one metastable state to another in the system of interest [1, 3, 5, 9, 12]. In this view, each metastable state is considered as a

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☆ Peer review under responsibility of Tomsk Polytechnic University.
https://doi.org/10.18799/24056537/2019/1/228

separate phase. In 2016, Kosterlitz, Thouless, and Haldane were awarded the Nobel Prize in Physics for their discovery of topological phase transitions and topological phases of matter.

The bifurcations, jumps and hysteresis obtained in our experiments can also be represented as phase transitions. Expressing the measured dependencies of the parameters as potential functions, such jumps and phase transitions can be described by the mathematical theory of catastrophes (hereafter simply called the theory of catastrophes (TC)) [1, 3, 9, 13–17]. An early TC description of discharge jumps can be found in [18].

After the jump, the plasma can either transit to a new state or relax to its initial state.

A mathematical model of acoustoplasma interactions can be constructed from the experimental results. Some parameters not obtained from directly solvable problems can be calculated using inverse mathematical problems. The most intractable inverse problems are incorrectly posed inverse problems (IPIP).

The data processing methods discussed in the present work detect the presence of jumps and hysteresis in the plasma parameters that are not seen during experiments. For instance, if the experimental parameters are insufficiently discretized, or the jumps in one parameter are compensated by a change in other parameters, they will be absent in the experimental data. TC theory can purportedly identify the points of parameter jumps in acoustoplasmas [1, 3, 9]. This study also proposes a new method for solving IPIP, especially at the jump points (singular points). The equation of a canonical mathematical catastrophe is then built from the experimental results. Both direct and inverse problems are solved, even when a priori information about the parameter interdependences is lacking. In this paper, the proposed procedures are applied to acoustoplasmas, but they can also determine phase transitions, parameter jumps, and emergency situations in other areas of science and technology. The only condition is the potentiality (or at least quasipotentiality) of the studied functions. Sudden jumps in the parameters of real devices can lead to real (rather than mathematical) catastrophes. Therefore, determining the points of possible parameter jumps is important.

2. Background Information

2.1. Generality of IPIP and the theory of catastrophes

This subsection briefly discusses solutions to inverse problems [9, 19–24] and the TC [9, 13–17]. Defining a function obtained from experimental data as $u(x)$, an experimental result can be written as [19, 22]

$$u(x) = A[x, z(s)], \tag{1}$$

where x is an independent variable, A is an operator, and $z(s)$ is a dependent variable.

To solve the inverse problem, we must find $z(s)$ given the experimentally determined function $u(x)$ [20].

Direct problems (in which the reasons are known and the consequences must be found) can be solved when a single causal relationship is known. In contrast, inverse problems (in which the consequences are known but the reasons must be found) can be solved only when many causal relationships are known, and are sourced from several reasons. Therefore, inverse problems require a large experimental database [23].

Inverse problems are usually nonlinear and have several roots, which also complicates their solution [21, 22].

Let us compare the formulations of the TC and IPIP.

The theory of catastrophes was formulated by Arnold [13]. In this context, a *catastrophe* is any abrupt response of the system to a smooth change in the external conditions.

IPIP as they are discussed in [19]. A task is posed correctly if 1) the solution exists, 2) the solution is unique, and 3) the solution is stable to small data perturbations. If at least one of these conditions is violated, the task is posed incorrectly.

Conditions 2 and 3 are more often violated than Condition 1. The uniqueness of the solution is violated by nonlinearity in the inverse problem [1, 3, 9]. Meanwhile, the stability of the solution is violated when a small change in an independent parameter induces a jump in the dependent parameters. Thus, under a violation of Condition 3, the task becomes incorrectly posed and coincides with the formulation of catastrophes. Such problems can be solved by TC [9, 17].

2.2. Theory of catastrophes and the definition of jump points (singular points).

TC is a mathematical tool that approximates a function by a Taylor series, considering the Morse Lemma and Thom's Theorem [13, 15].

Morse Lemma: In the vicinity of a non-degenerate point, a function of a changing variable can be reduced to a simple standard form.

Thom's theorem: When the number of independent parameters is four or fewer, almost any physically realizable function can be reduced to seven polynomials defining seven elementary catastrophic responses [13–16] to local changes of variables. Among these is the butterfly catastrophe

$$V_{abcd}(x) = (x^6/6) + (dx^4/4) + \\ +(cx^3/3) + (bx^2/2) + (ax) \quad (2)$$

or

$$y(x) = \sum_{k=0}^{n} a_k x^k \quad (3)$$

In Eq. (3), n is the degree of the polynomial (in catastrophe equations, n = 3, 4, 5, 6).

Let us consider the detection of phase transitions and parameter jumps in experimental data. As mentioned above, these jumps may be unseen due to insufficient discretization of the measurements, or when jumps occur between some coefficients a_k but do not appear directly in the curve formed by the measurements.

For this reason, TC [13–17] proposes the use of separatrices, defined as curves that divide the function areas into segments with sufficiently many and different numbers of jumps, or with no jumps at all. When a line connecting the values of the coefficients for different parameters of the experiment intersects separatrix, there will be a jump of some parameter, or some coefficient a_k in Eq. (2). As shown in Fig. 1, regions bounded by separatrices can contain different numbers of minima and maxima, implying different numbers of unstable states when jumps are possible. Inside the triangle formed by the separatrices in Fig. 1 there are two maxima. Above the triangle there are no maxima, whereas the remaining areas each contain one maximum. The number of maxima corresponds to the number of unstable points, where jumps are possible. These jumps can be interpreted as phase transitions in the studied system.

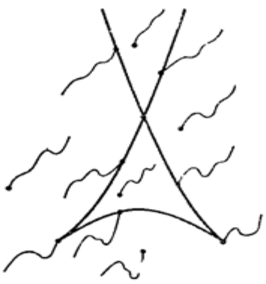

Fig. 1. Schematic of potentials in different areas [14]

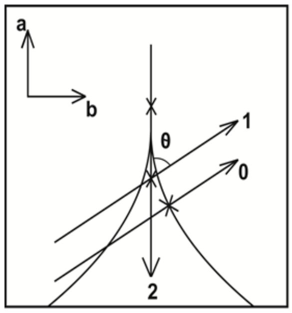

Fig. 2. Orders of phase transitions [15]

Figure 2 shows how the order of the phase transition can be determined by the method described in [15]. For two different states of an experiment, we obtain two catastrophe equations of the same form (e.g., Eq. (2)) but with different coefficients, (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2). Figure 2 shows the situation for the coefficients (a, b). If a straight line connecting two points with coefficients (a_1, b_1) and (a_2, b_2) in these same-form catastrophe equations intersects one separatrix, then a first-order phase transition occurs. If the coordinates of the parameters of one of the equations coincide (but do not intersect) with the separatrix, then a zero-order phase transition occurs. If a straight line intersects both separatrices simultaneously, then a second-order phase transition occurs. If the straight line never touches the separatrices, then no phase transition occurs. These operations are sufficient for detecting the presence or absence of phase transitions in the system.

2.3. Processing experimental results using catastrophe theory

As an example, let us consider the measurement of current and voltage. A two-beam oscilloscope acquires oscillograms of the discharge current in the discharge tube and the voltage across the discharge. The discharge current is modulated by a sinusoidal signal of a certain frequency. To study phase transitions in acoustoplasmas, this sinusoidal signal must necessarily contain a constant and a variable component. Owing to the processes occurring in the acoustoplasma, the oscillograms differ from the sinusoidal ones and exhibit a phase shift between the current and the voltage. We modulate the current (i.e., set the current as the independent parameter) and measure the voltage at discharge (i.e., the dependent parameter) [9].

In our case, the current and voltage are quantized over time during the modulation period (from the beginning to the end of the period). The start point of the quantization is determined by an independent parameter (in this case, the current). The quantization is separately performed for each of the independent parameters *z*, and for each of the dependent parameters. Note that the beginnings of the quantizations of all parameters must be synchronized with that of any one independent parameter.

To minimize the errors in calculating the fast Fourier transforms of the first six harmonics, we quantized the modulation period into 120 pixels (120 is the doubled lowest common multiple for the six harmonics). As only the modulation period is quantized, the quantization process is frequency-independent. The instantaneous voltages obtained by the quantization depend on the instantaneous value of the current. Here, "instant" denote a time interval that is 120 times shorter than the modulation period.

We thus obtain the current–voltage relationship at 120 points during the modulation period. The coefficients *a*, *b*, *c*, *d* in Eq. (2), denoted by a_i in Eq. (3), are determined by solving a system of 120 equations. The definition of the a_i is described in [25]. As each coefficient is determined from 120 values, Eq. (2) and Eq. (3) approximate experimental data with high accuracy.

The coefficients are then determined as [25]

$$a_0 N + \sum_{k=1}^{n-1} a_k \sum_{i=1}^{N} x_i^k = \sum_{i=1}^{N} y_i \qquad (4)$$

$$\sum_{k=1}^{n} a_{k-1} \sum_{i=1}^{N} x_i^k = \sum_{k=1}^{n} \sum_{i=1}^{N} x_i^k y_i \qquad (5)$$

Figure 3 shows the experimental curve of the normalized discharge power (i.e., the ratio of the instantaneous power to the average one) versus voltage during the modulation period (blue curve) and the experimental curve approximation by equation (2) (pink curve). From Fig. 3 one can see a good agreement between the approximating and experimental curves.

Let us consider the dynamic voltage–power characteristics, which reveal the presence of power surges and voltages under a smooth current change [9]. A *dynamic* characteristic describes the behavior of an instantaneous value of a measurable quantity during the monitoring period. The instantaneous powers are obtained by multiplying the corresponding instantaneous currents and voltages (recall that the modulation period is divided into 120 pixels). The discharge reactance can be ignored over the entire measurement area.

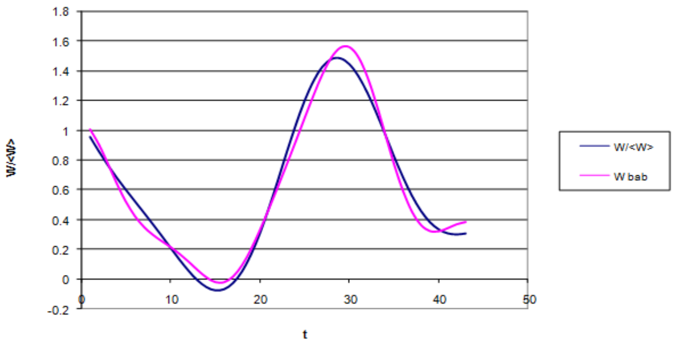

Fig. 3. Approximation of the experimental curve (blue line) by Eq. (2) describing a butterfly catastrophe (pink line)

Note that both voltage and power are dependent parameters, meaning that this method is valid not only for connecting independent and dependent parameters, but also for analyzing two parameters that depend on a common variable.

The presence of jumps is detected by the following five-step procedure:

Step 1. Normalize the experimental data to their average values and transform the normalized data into dimensionless values.

The dynamic power–voltage characteristics are plotted in Fig. 4. Here the power and voltage scales are normalized to the average power (<*W*>) and average voltage (<*U*>), respectively. Whether jumps and phase transitions occur in this dynamic characteristic is not clarified in Fig. 4.

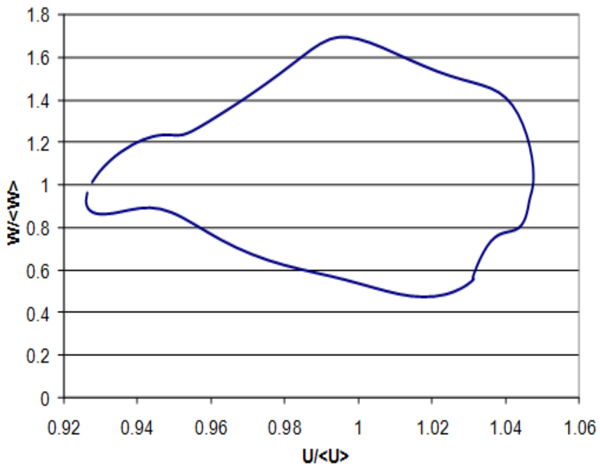

Fig. 4. Normalized dynamic power–voltage characteristics

Step 2. Construct an approximate catastrophe equation, and reduce it to canonical form [13–15].

Separate power–voltage characteristics are constructed in distinguishable sections: in the first section, the voltage increases from its minimum to its maximum; in the second section, it decreases from its maximum to its minimum.

For example, in the section of increasing the voltage the butterfly catastrophe Eq. (2) is constructed as follows:

$$\begin{aligned} W(t) &= AU^6(t)+BU^4(t)+ \\ &+CU^3(t)+DU^2(t)+EU+F= \\ &=(1711)U^6(t)+(-8044)U^4(t)+(1016)U^3(t)+ \\ &=(19682)U^2(t)+(-20508)U(t)+6143 \end{aligned} \tag{6}$$

Reducing (6) to the canonical form of (2), we get

$$\begin{aligned} W_{cannon} &= \frac{1}{6}U^6(t)+\frac{1}{4}dU^4(t)+ \\ &+\frac{1}{3}cU^3(t)+\frac{1}{2}bU^2(t)+aU(t)= \\ &=\frac{1}{6}U^6+\frac{1}{4}(-3.13)U^4+\frac{1}{3}(0.3)U^3+ \\ &+\frac{1}{2}(3.83)U^2+(-2)U(t) \end{aligned} \tag{7}$$

where $a = -2$, $b = 3.83$, $c = 0.3$, and $d = -3.13$. Note that W_{cannon} is not (W/<W>).

Step 3. Construct the separatrices of the canonical equation.

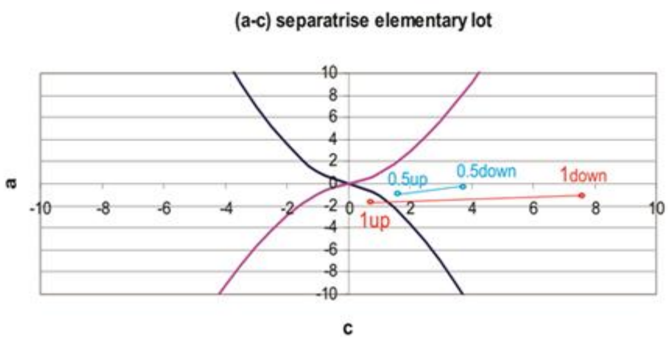

Fig. 5. Separatrix for the coefficients a and c of Eq. (7)

Step 4. Determine the points of possible phase transitions.

The point with coordinates (a, c), denoted as (0.5, up) in Fig. 5, corresponds to a voltage increase from minimum to maximum during the modulation period at 0.5 kHz. Meanwhile, the point (0.5, down) corresponds to a voltage decrease from maximum to minimum at the same frequency. Similar points (1, up) and (1, down) indicate the voltage increase (from minimum to maximum) and decrease (from maximum to minimum), respectively, at 1.0 kHz. As evidenced by the lack of intersections between the curve and the separatrices, no phase transition occurred during the modulation period at 0.5 kHz. However, at 1.0 kHz, the plot intersects one separatrix, indicating a first-order phase transition. Thus, at a modulation frequency of 1.0 kHz, the system undergoes a phase transition period in each modulation period of the discharge current.

Step 5. If necessary, restore the measured values by inverse transformation of the normalized values.

By processing the experimental data as described above, one can find new dependencies between pairs of parameters. Moreover, these dependencies can be expressed both by already known laws and as catastrophe polynomials.

2.4. Procedure for solving IPIP, including at singular points

To solve an incorrectly posed inverse problem, we determine or the coefficients of the equations, or the area over which the operator acts, or the initial conditions, or combinations of these factors. The most difficult task is determining the solutions in the neighborhoods of singular points, where the system parameters become discontinuous (note that at the singular points themselves, there is no solution). After the jump, the system either returns to its previous state or transitions to a new state [19, 24].

It should be noted that acoustoplasmas are characterized by parameter jumps and phase transitions. Accordingly, almost all acoustoplasma phenomena are nonlinear with non-unique solutions, and most of them are incorrectly stated [9].

For a system of linear algebraic equations, Eq. (1) can be expressed as [20, 22]

$$Az = U \tag{8}$$

where z is the vector to be found, U is a known vector, and $A = \{a_u\}$ is a square matrix with elements of a_u.

Calculating the determinant of a system of N equations requires N^3 operations. Error accumulation while calculating the measured values degrades the calculations, which eventually become meaningless. However, several methods for solving such systems have been proposed [20].

This paper proposes a new method for solving linear and nonlinear algebraic equations. The method is based on the processing of experimental data by TC, and uses non-square matrices [9, 26].

First, the (experimentally measured) oscillograms of the independent and dependent parameters are time-quantized as described above.

In the case of one independent (z) and one dependent (u) parameter, the matrices are simple and consist of one column and 120 rows.

$$A\begin{Bmatrix} z_1 \\ z_{120} \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_{120} \end{Bmatrix} \quad (9)$$

If there are several independent parameters, then each of the z_m independent parameters and u_m dependent experimental parameters are quantized and inserted into the following matrix:

$$A\begin{Bmatrix} z^*_{1.1} & \dots & z^*_{m.1} \\ \dots & \dots & \dots \\ z^*_{1.120} & \dots & z^*_{m.120} \end{Bmatrix} = \begin{Bmatrix} u^*_{1.1} & \dots & u^*_{m.1} \\ \dots & \dots & \dots \\ u^*_{1.120} & \dots & u^*_{m.120} \end{Bmatrix} \quad (10)$$

This matrix describes 120m equations $Az_{ij}* = u_{ij}*$; $\{i = 1–120\}$, $\{j = 1–m\}$. The index * denotes a proximity value.

We thus obtain a set of (i = 1–120) instantaneous values of the studied functions $u_{ij}*$, which depend on $z_{ij}*$. The instantaneous values of each function $Az_j* = u_j*$ are approximated by TC. These approximations are true values of u_j functions.

Note that these approximating functions are expressed through the equations of TC, and not through the well-known mathematical formulations of well-known physical laws.

Recall that the beginnings of the quantizations of all parameters must be synchronized with any single independent parameter. In this case, we can further investigate the dependencies between different u_{ij}^*, thus solving inverse problems using the obtained database.

Those, the present study is divided into several stages.

In the first stage, based on the obtained experimental database, all the independent and dependent parameters are quantized to 120 pixels (and initially synchronized with one of the independent parameters).

In the second stage, the direct problems using the first group of operators A are solved and matrices of $u_{ij}*$ elements are obtained.

In the third stage, based on the obtained set of values, a catastrophe approximation is constructed using Eqs. (3)–(5).

In the fourth stage, it is determined whether or not there are singular points in the area under consideration. For this the TC is using.

In the fifth stage, the inverse problems are easily solved in the smoothly continuous regions. In regions containing discontinuities, the inverse problems cannot be solved at the singular points. Instead, the cross-linkages are analyzed using bifurcation theory [9, 27]. The procedure is as follows:

1. In the existence domain of the experimental parameters {A–D}, determine the set of singular points {B, C}.

2. Divide the solution area (A–D) of the inverse problems into several sections $\{U_A; U_B; U_C\}$, which exclude the singular points but are described by the same catastrophe equation. Solve the inverse problems separately in each of these areas. Formally, this step is represented as

$$\int_{x(A)}^{x(D)} U dx = \int_{x(A)}^{x(B)} U_A dx + \int_{x(B)}^{x(C)} U_B dx + \int_{x(C)}^{x(D)} U_C dx \quad (11)$$

3. In the neighborhood of singular points where there are bifurcations, the solutions of the inverse problems are stitched [9, 27]. Integral functions are expressed in terms of catastrophe equations.

4. Interpret the parameter jumps at the stitching points as phase transitions.

It should be noted that from a purely mathematical perspective, the TC equation is a reasonable approximation to potential functions [14]. However, for applied tasks, the class of functions can be extended. For example, in the formulations of [13, 14], the electric field is a potential function, whereas the voltage (potential difference) is not. However, the potential difference in physics is a potential function determining the energy gained by a charged particle. In addition, the voltage function meets all requirements of TC. Sociology and other fields with no developed mathematical apparatus can also be described by TC.

3. Conclusions

The main conclusions of the study are summarized below.

1. A method for solving direct and inverse problems (including incorrectly posed ones) based on experimentally obtained data is proposed. The method digitizes the experimental oscillograms into 120 discrete points (pixels) during each modulation period. The obtained instantaneous values of the processes are assembled into non-square matrices with 120 rows and m columns (where m is the number of independent parameters of the discharge current into a plasma). The resulting 120m equations link the instantaneous values of the dependent and independent parameters. The operator is not constructed as a matrix, but is assumed constant over the entire column. As the digitization is synchronized, the instantaneous relationships between any dependent and inde-

pendent pair of discharge parameters can be obtained during the modulation period.

2. A method of determining the presence of jumps and phase transitions of the dependent parameters (including implicit transitions that are not noticeable in the experimental data) is proposed. This method represents the obtained links of the parameters as mathematical catastrophe equations, and reduces them to canonical form, revealing the presence or absence of jumps and phase transitions in the considered area.

3. A method that solves incorrectly posed inverse problems at singular points is developed. This method adopts the theory of catastrophes and a bifurcation analysis method for cross-linked systems.

4. The described procedures can be used to determine jumps, phase transitions, and emergency situations in various fields of science and technology.

Acknowledgment

The authors thank the management of the Institute of Applied Problems of Physics and their loved ones for their patience and help.

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Received: 27.06.2018