

## ROUTING OF RETAIL PRODUCT DELIVERY IN URBAN ROAD SYSTEM ON THE BASIS OF GENETIC ALGORITHM

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*A model simulating dynamic processes in a urban road system has been developed. On the basis of genetic search the methods of service route construction was created. System of automated transport routing delivering different goods was designed.*

### Introduction

In modern conditions of market economy the competitive advantage over the other participants of a certain market sector is a symbol of stable and successful development of any enterprise. Beneficial terms of manufacturing may be achieved due to wide spectrum of action. Qualitative organization of production distribution process is the competitive advantage for the enterprise. The main optimization directions are: operative preparation of applications and stable execution of orders in full volume and range. For this purpose the optimized traffic plan is required. The routs should meet a number of specific requirements:

- automobiles used in delivery have limited carrying capacity;
- limited number of automobiles defines the possibility of their repeated use;
- each route starts and finishes at finished-products storage area;
- clients automobile distribution and sequence of their route should minimize delivery costs;
- routs should reflect road situation changing during the day and conform to existing rule of the road.

Vehicle Routing Problem (VRP) is widely discussed in literature. The first investigations on this problem had already appeared at the beginning of the 70's of the last century [1, 2]. At present, not unsuccessful attempts are realized for solving this problem and its expansions both by exact and approximate methods. The majority of techniques existing up today solve the statistic variant of VRP. This situation is simplified and not always adequate to real conditions. For urban road systems (URS) the inconstancy of carrying capacity at many movement sections during the day are typical that influences inevitably on quality of work of various delivery services. Besides, almost each real URS abounds in one-way movement streets with constantly or occasionally illegal turns. The route overcoming time may also significantly depend on its configuration, as broken bypass schemes decrease the average rate of motion. The problem is especially urgent for retail food stores, where regular deliveries of relatively small consignments of goods are required. To construct storage rooms in such conditions is usually meaningless: the sold production has mostly very limited use-by date; additional areas involve additional costs for their maintenance.

VRP peculiarity at any graph is in the fact that the total rout, traversed by all used traffic, the time of all orders execution are influenced both by quality of many clients rout distribution and the quality of found route cycles of all clients by traffic units. This peculiarity is rather important at modeling networks with high inconstancy of carrying capacity that is typical for modern cities.

In the given paper the stated peculiarity of VRP is taken into consideration. The suggested technique for solution of routing problem consists of several stages: conversion of initial URS graph to the graph of a certain type, applying a specially modified Deicstra algorithm; routs construction by means of developed metaheuristics on the basis of genetic algorithm (GA). The package permitting for efficient traffic plans, minimizing delivery costs is suggested as a part of GA. The choice of the method is justified by the fact that routing problem is NP complex, i. e. the size of search space has an exponential dependence on variables amount. In suggested extension it has very large and complex decision space even at relatively small quantity of clients. There is a number of sources, for example, [3], where the effectiveness of metaheuristics, especially GA, application for graph problems is shown in tests. And, the quality of solutions obtained in GA increases at growth of problem dimension in comparison with other methods.

### Problem on construction of delivery plan for retail client network

Dynamic graph which simulates changes of streets carrying capacity, existence of one-way movement sections, constantly and occasionally illegal turns, increase of rout overcoming time, using more gradient turns connected with necessity to brake and to race again is used as the URS model. In the conditions of the dynamic graph the target criterion is an extent and duration balance of required routes.

Let us represent the urban road system in the form of directed bound graph  $G = \langle D, U \rangle$ , where  $D = \{1, 2, \dots, n_D\}$  is the set of graph nodes with  $n_D$  power,  $U$  is the set of arcs connecting nodes. Graph junctions are considered to be both various points of production selling, for example, stores, dining halls, hospitals and kindergartens, and common crossings of highways. Let us distinguish a target subset of nodes  $\hat{P} = \{1, 2, \dots, \hat{n}\}$ ,  $\hat{P} \subset D$ , which consists only of service customers.

In Fig. 1 vertex set  $D$  is all graph nodes,  $\hat{P}$  is the black nodes,  $u_{ij} \in U$ ,  $i, j \in D$  are the arcs between any adjacent nodes, and  $u_{ij} \neq u_{ji}$ . The finished-product storage area is labeled in left upper corner.

An enterprise manufactures production cyclically. Therefore, the whole set of clients  $\hat{P}$ , are distributed per production cycles when applications being received. Then, our task is to plan delivery for clients of a certain production cycle.  $P \subset \hat{P}$  is taken as clients subset of a certain production cycle. Then,  $n$  is the number of clients, corresponding to a certain cycle, which should be served.

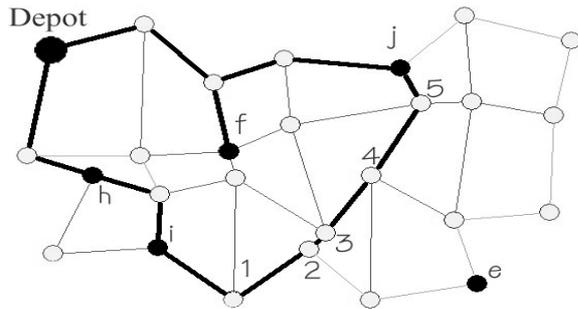


Fig. 1. Graph  $G$  is the model of urban road system

Streets carrying capacity is the time function and changes almost cyclically every day. Therefore, let us divide the whole daily range into  $T$  segments, inside of which the index is constant. Then, each arc  $u_{ij} \in U$  of graph  $G$  connecting nodes  $i, j \in D$  and representing a certain section of a street possesses the extent  $l(u_{ij})$ , km and average rate  $w(u_{ij}, \tau)$ ,  $\tau = 1 \dots T$ , km/h at the moment  $\tau$ , where  $\tau$  is the discrete time determining a number of daily range. The number of range  $t$  is determined as time function  $\tau = \tau(t)$ , where  $t$  is the current simulated time.

$q$  vehicles are at the disposal of shipping department. Each vehicle is characterized by lift capacity  $K$ , kg (m cube). The order of  $I$  client has the form:  $d_i = (d_{iv})$ ,  $i \in P$ ,  $v \in S$ , where  $d_{iv}$  is the volume  $d$  of product  $v$ , which should be delivered to the client  $i$ ,  $S$  is the set of production manufactured at the enterprise.

The set of clients  $P$  should be broken down per vehicles so that total orders volume of the subset of clients assigned to a concrete vehicle for transportation does not increase this vehicle lift capacity. Let us denote set partition  $P$  into disjoint subsets by  $p_1 \cup \dots \cup p_k \cup \dots \cup p_\alpha$ , so that  $p_k \cap p_s = \emptyset$ , where  $\alpha$  is the total amount of runs,  $Y$  is the set of all admissible set partitions  $P$ . For example, there is  $p_1 = \{h, f, j, i\}$ ,  $p_2 = \{e\}$  in Fig. 1. Then, loading of vehicles for products transportation to assigned customers is determined by inequality

$$\sum_{i \in p_k} d_i \leq K_k, \quad k = 1 \dots \alpha. \quad (1)$$

It is necessary to calculate an optimal route of nodes of the assigned subset  $p_k$  of partition  $y$  for every used vehicle. Let us denote by permutation  $z_k = (i_1 \dots i_m \dots i_{g_k-1} i_{g_k})$  the alternative order of nodes  $i_m \in p_k$ ,  $m = 1 \dots g_k$  route by vehicle  $k$  from set of all possible permutations  $Z_k$ . Here  $i_1$  is the starting node, denoting route beginning, i. e. the finished-products storage area;  $g_k - 1 = |p_k|$  is the amount of target nodes at the route. It is  $z_k = (i, h, i, j, f, i)$  in Fig. 1. The completion of route is in star-

ting node, i. e.  $i_{g_k} = i_1$ . The routes should cover the nodes of sets  $p_k$  in order, providing minimal total delivery cost.

The main factors determining its cost are duration and length of routes. Therefore, their linear combination per each traffic unit is considered as target criteria that results in optimal use of traffic resources and, as a result, costs optimization. The target function is of the following form:

$$\sum_{k=1, \alpha} [\lambda l(z_k) + (1 - \lambda)\theta(z_k)] \rightarrow \min, \quad (2)$$

where  $l(z_k)$  and  $\theta(z_k)$  are the length and duration of the route  $z_k$  respectively,  $\lambda$  is the weighting coefficient.

The length  $l(z_k)$  of the route  $z_k$  is made up of paths lengths  $\tilde{u}_{i_m i_{m+1}}(\tau) \in U(\tau)$  connecting the nodes following one after another  $i_m, i_{m+1} \in p_k$ ,  $m = 1 \dots (g_k - 1)$ . There is  $\tilde{u}_{i_m i_{m+1}}(\tau) = (i, 1, 2, 3, 4, 5, j)$  in Fig. 1. The path  $\tilde{u}_{i_m i_{m+1}}(\tau)$  is the function of discrete time  $\tau$ , as the optimal route, connecting nodes  $i_m, i_{m+1}$  may considerably change in the course of the day,  $\tau = \tau(t_{i_{m-1}})$ , where  $t_{i_{m-1}}$  is the departure time from the node  $i_{m-1}$ . The algorithm of set construction is described more detailed below.

The duration  $\theta(z_k)$  of the route  $z_k$  is determined in the following way.

Let the time from current node  $i_m$  be calculated by the formula

$$t_{i_m} = \begin{cases} t_{i_m}^{\text{do}} + t_{i_m}^{\text{pazc}} & | m = 2 \dots g_k \\ t_k^{\text{cmappm}} & | m = 1 \end{cases}, \quad (3)$$

where  $t_{i_m}^{\text{do}}$  is the arrival moment into the node  $i_m$  from the node  $i_{m-1}$ ;  $t_k^{\text{cmappm}}$  is the vehicle  $k$  departure time from the storage are;  $t_{i_m}^{\text{pazc}}$  is the time interval, necessary for unloading client order  $i_m \in p_k$ ,  $k = 1 \dots \alpha$ .

The magnitude  $t_{i_m}^{\text{do}}$  is made up of departure time from the previous node  $t_{i_{m-1}}$  and the time for covering the distance  $w(\tilde{u}_{i_{m-1} i_m}(\tau))$  at average rate  $l(\tilde{u}_{i_{m-1} i_m}(\tau))$ ,  $\tau = \tau(t_{i_{m-1}})$ , returns the number of daily interval, corresponding to the current value  $t_{i_{m-1}}$ .

$$t_{i_m}^{\text{do}} = t_{i_{m-1}} + \frac{l(\tilde{u}_{i_{m-1} i_m}(\tau))}{w(\tilde{u}_{i_{m-1} i_m}(\tau))}, \quad i_m \in p_k. \quad (4)$$

The time of unloading order  $t_{i_m}^{\text{pazc}}$  is determined by the formula

$$t_{i_m}^{\text{pazc}} = \tilde{t} \cdot \sum_{v \in S} d_{i_m, v}, \quad i_m \in p_k, \quad (5)$$

where  $\tilde{t}$  is the average quantity of time for unloading 1 unit of product.

Then the time of the whole route of a single vehicle  $\theta(z_k)$  equals the magnitude  $t_{i_{g_k}}^{\text{do}}$ , defining the recurrence time of the vehicle  $k$  to the storage – node  $i_{g_k}$

$$t_{i_{g_k}}^{\text{do}} = t_{i_{g_k-1}} + \frac{l(\tilde{u}_{i_{g_k-1} i_{g_k}}(\tau))}{w(\tilde{u}_{i_{g_k-1} i_{g_k}}(\tau))}, \quad \tau = \tau(t_{i_{g_k-1}}), \quad i_{g_k} \in p_k, \quad k = 1 \dots \alpha.$$

### Method of solution

URS together with retail chain may number more than a thousand points. Therefore, a large quantity of va-



described estimation algorithm, using efficiently the dedicated storage having simplified as much as possible the structure of coded solutions (6) and excepting a single monitor unit of acceptability of alternatives obtained during evolution. The infeasible solutions, from the point of view of traffic overloading, are excluded.

At every estimation the capacities of available vehicles are imposed to the alternative sequence of clients in strictly fixed order for the whole evolution, that is the strictly consequence dynamic traffic loading by clients is performed. Therefore, within the evolutionary process, those alternatives where comparatively close clients are grouped according to the vehicles capacities and in the sequence, providing minimum of target function, become the best solutions (1).

The combination of solution representation system and estimation technique in the give GA possesses the key properties for the problem: in evolutionary process the parallel search of nodes optimal partition by the routes and optimal order of clients' route inside each set occur.

Here **the selection policy** of parents by the principle of roulette wheel and elite strategy is used. Thus, the loss of good solutions at application of genetic operators is excluded. Using **genetic operators** the creation of new individuals not presenting in population before is performed. Scanning search space is performed by operators of **crossing, mutation, inversion**. The corresponding operator of crossing is partially applied. The choice is made due to such problem situation that any client may be served only once by one vehicle:  $p_k \cap p_s = \emptyset$ ,  $k \neq s$ ,  $k, s = 1 \dots \alpha$ . That is, the descendant should have the same gene structure that the initial individuals. The modifications appear only in the order of nodes sequence. Application of mean mutation – **local searching strategies** modifying the original solution on the basis of target function analysis or some of its parameters allows increasing significantly the quality of solutions. The description of local searching methods and various genetic operators is widely represented in literature and Internet, for example, in [5, 6].

#### Conditions of search stop:

- algorithm performed the given quantity of iterations  $AbsN$ ;
- during  $LocN$  iterations the best solution is not changed.

#### Comparative results and approach efficiency

Trail calculations for static test problems published in scientific literature and available in Internet show good results comparable with best achievements published

in literature [8]. In many problems deviations achieve less than 2 %. There are no standard problems on the dynamic graphs. It should be mentioned that the quality of solutions including their stability depends essentially on the time set for searching. For example, for the problem with 200 clients, at  $LocN=200$ , the average search time is 40 s, and scatter of readings may reach 5...7 % from the best solution searched by algorithm. At  $LocN=2000$  the average search time already equals 7 min, and the scatter of readings is 2...3 %.

The approach suggested above implies such search strategy that the routes found on the dynamic graph provide the best results at their use in real conditions in comparison with the results found on the static graph. The following estimation technique of searching efficiency in the dynamic graph was suggested. Let  $P_1$  be the solution of the problem found on the static graph,  $P_2$  is the same on the dynamic graph. Let us denote the quality of the solution determined by the weights of the static graph by  $L(P)$ , where  $P$  is a certain solution. The estimation of solution  $P$  in the dynamic graph is  $H(P)$ . Let us take  $L(P_1)$  as a lower estimate, as the solution is found in easy road conditions. Probably,  $H(P_1)$  is the worst estimation, since the solution  $P_1$  does not include the peculiarities of the dynamic graph. The simulation error is denoted by  $\Delta_1 = H(P_1) - L(P_1)$ . Then the efficiency of solution technique is determined by

the formula  $\Delta_2 = \frac{H(P_1) - H(P_2)}{\Delta_1}$  with the assumption

that  $L(P_1) \leq H(P_2) \leq H(P_1)$ . The closer  $\Delta_2$  to the unit, the higher is the effect from simulating at the dynamic graph. The calculations showed that  $\Delta_2 \approx 0,2$ , i. e. simulating on the dynamic graph, allows reducing delivery time or both indices approximately by 20 %. Besides, the parameter depends significantly on the degree and structure of graph loading, amount and spread of clients.

#### Conclusion

The model of urban road system simulating complex characteristics, changing within the time and inherent to real road networks of modern cities has been created. The technique of routes searching of service traffic movement taking into account the peculiarities of real road network was realized on the basis of genetic search architecture. The software product representing the system of vehicles routing was developed. The system collects, stores and prepares information about clients, current orders, URS state, available vehicles as well as direct VRP solution by one of realized technique.

#### REFERENCES

1. Christofides N., Eilon S. An algorithm for the vehicle-dispatching problem // Operational Research Quarterly. – 1969. – № 20. – P. 309–318.
2. Christofides N., Mingozzi A., Toth P. The vehicle routing problem. In: Christofides N., Mingozzi A., Toth P. Sandi C., editors. Combinatorial optimization. – London: Wiley, 1979.
3. Minakov I.A. Comparative analysis of some methods of random search and optimization // The Bulletin of RAS SC, Samara. – 1999. – № 2. – P. 286–293.
4. Kristofides N. Graph theory. Algorithm approach. – Moscow: Mir, 1978.
5. Emelyanov V.V., Kureichik V.M., Kureichik V.V. Theory and practice of evolutionary modeling. – Moscow: Phisimatlit, 2003.
6. Materials of the site <http://qai.narod.ru/GA/index.htm>
7. Materials of the site [www.citforum.ru](http://www.citforum.ru)
8. Materials of the site <http://neo.lcc.uma.es/radi-aeb/WebVRP/main.html>

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