

# Kinematical analysis of the nutation speed reducer

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**Abstract.** This paper discusses the development of a Nutating Speed Reducer (NSR) which is characterized by high reduction ratio, high tooth contact ratio, very high torque to weight/volume ratio, quiet and smooth operation under load and very high efficiency. All of these advantages are due to the presence of conjugate face-gear pairs, which incorporate each other, which called nutating/rotating gear mechanism. Details of the NSR, its kinematics, gear tooth load capacity, and mesh efficiency are explained. The NSR component speeds and speed reduction ratios of the NSR are calculated. Effect of the varying nutation angles on the geometry of the NSR is discussed and compared.

## 1. A brief history of the nutating gear

Robert Davison discovered the concept of nutation almost by accident [1]. While he was throwing a coin on the table, he noticed that as a speed of spinning coin begins to decrease, a peculiar wobble takes place. This wobble is nutation. This concept has been adapted to design high gear speed reductions, which is known as "Nutating Gear Device".

Nutating gears are high shaft angle bevel gears where one bevel gear is wobbling instead of rotating against a fixed gear. The rotation of the nutating gear depends on a difference in tooth numbers between the nutating gear and fixed gear instead of a difference in circumferences between them.

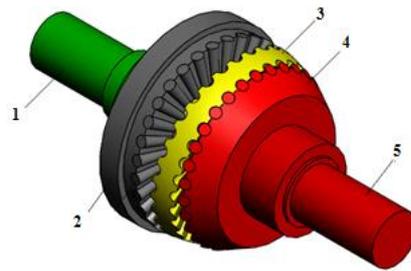
## 2. Introduction

Nutation speed reducer is a totally innovative gear transmission developed in recent years [2, 3], which bases its functioning on the mathematical concept of nutation motion, that is the main feature that characterizes nutation gear from conventional gear. It has unique features which include high reduction ratio in one stage, very high profile contact ratio and an overall higher power density transmission. In addition, the contact is theoretically pure rolling in the gear, the heat generated should be lower and the efficiency higher.

## 3. Structure of NSR

The Nutation Speed Reducer is illustrated in Figure 1. The component names illustrated in Figure 1 are (1) an input shaft; (2) the stator gear; (3) the nutator gear; (4) the rotor gear and (5) the output shaft.





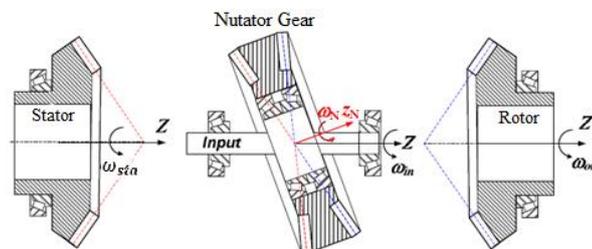
**Figure 1.** The Nutation Speed Reducer

The input shaft is driven by an engine and drives the nutator gear, which is fixed to the input shaft at a specified nutation angle (Typical range of the nutation angle  $\beta$  is  $2\text{-}5^\circ$  from vertical direction) [4]. The nutator has gear faces on either side of the body that are in mesh with both the stator and the rotor gears. The motion and torque transfer from nutator to rotor gear and output shaft.

The numbers of teeth on the stator gear 2 and rotor gear 4 is  $Z_1, Z_4$  respectively. The numbers of teeth on the nutator gear 3 meshing with stator gear is  $Z_2$ . The numbers of teeth on the nutator gear 3 meshing with rotor gear is  $Z_3$ . The gears of NSR simultaneously share the tooth loads with high contact ratios as a result of its unique assembly [5, 6].

**4. Kinematics of NSR**

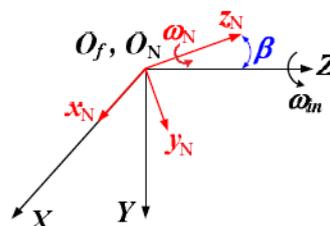
As illustrated in Figure 2, input shaft, stator gear and rotor gear rotate about the Z-axis of the global fixed-coordinate frame  $S_f$ . The nutator gear converts the purely rotational motion of the input shaft into both rotational motion about the Z axis and nutational motion (or precession motion), which is a rotation about the  $z_N$  axis. This second rotation is caused by the meshing engagement with the stator gear. The  $z_N$  axis is fixed to the input shaft; so, it rotates about the Z axis.



**Figure 2.** Motions of the NSR gears

The nutation speed reducer velocities are defined by three major characteristics, the nutation angle of the nutator gear, the number of teeth in the two respective meshes between the nutator and the stator, and the nutator and the rotor, and the rotational velocity applied to the stator gear (when driving the stator gear via an external power input as in Elmoznino’s work [7]). In this work the stator gear is fixed to body of reducer, so the gear tooth numbers and nutation angle will be the major factors in the velocity terms [8].

Coordinate relations of the  $S_f$  and  $S_N$  are illustrated in Figure 3. The  $S_f$  and  $S_N$  coordinate frames are coincident at the common origins  $O_f$  and  $O_N$ . The directions of the frames are set according to the right hand rule.



**Figure 3.** Coordinate relations of the nutator gear and global frame

Assuming an input speed for the input carrier being  $\omega_{in}$ , we can formulate the rotational speed of each body. The input shaft rotational velocity can be defined as:

$$\vec{\omega}_{in} = \omega_{in} \cdot \vec{K} = \begin{bmatrix} 0 \\ 0 \\ \omega_{in} \end{bmatrix} \quad (1)$$

Where the symbol  $\vec{K}$  presents unit vector in the Z direction of the  $S_f$  coordinate. The rotor angular velocity is:

$$\vec{\omega}_{out} = \omega_{out} \cdot \vec{K} = \begin{bmatrix} 0 \\ 0 \\ \omega_{out} \end{bmatrix} \quad (2)$$

The motion of the nutator gear is a combination of the input speed ( $\omega_{in}$ ) and the speed component from the meshing engagement between the stator gear and the nutator gear ( $\omega_N$ ). It is:

$$\vec{\omega}_{Nut} = \omega_{in} \cdot \vec{K} + \omega_N \cdot \vec{k} \quad (3)$$

Where the symbol  $\vec{k}$  is the unit vector in the z direction of the  $S_N$  coordinates. The  $\omega_N$  gives as:

$$\vec{\omega}_N = (\omega_{Sta} - \omega_{in}) \frac{Z_1}{Z_2} \vec{k} \quad (4)$$

A coordinate transformation is necessary to present nutator gear angular velocity in the  $S_N$  coordinate. As illustrated in Figure 4 to form  $S_N$  frame the  $S_f$  coordinate is rotated about the X axis. The coordinate transformation matrix defining a rotation of  $\beta$  degrees about the X axis is:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \quad (5)$$

and the transformation of the coordinate is established by:

$$\begin{bmatrix} I \\ J \\ K \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \cdot \begin{bmatrix} i \\ j \\ k \end{bmatrix} \quad (6)$$

The input speed component of the nutator gear angular velocity is transformed as:

$$\vec{\omega}_{Nut} = \omega_{in} \cdot (R_x \cdot \vec{k}) + \omega_N \cdot \vec{k} \quad (7)$$

with expanding:

$$\vec{\omega}_{Nut} = \begin{bmatrix} 0 \\ -\omega_{in} \sin \beta \\ \omega_{in} \cos \beta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_N \end{bmatrix} \quad (8)$$

and the sum of the angular velocity component vectors results in :

$$\vec{\omega}_{Nut} = \begin{bmatrix} 0 \\ -\omega_{in} \sin \beta \\ \omega_{in} \cos \beta + \omega_N \end{bmatrix} \quad (9)$$

with a more compact form:

$$\vec{\omega}_{PMC} = -\omega_{in} \sin \beta \vec{j} + (\omega_{in} \cos \beta + \omega_N) \vec{k} \quad (10)$$

The nutator gear angular velocity with the number of teeth relation is then becomes:

$$\vec{\omega}_{Nut} = \begin{bmatrix} 0 \\ -\omega_{in} \sin \beta \\ \omega_{in} \cos \beta + -\omega_{in} \frac{Z_1}{Z_2} \end{bmatrix} \quad (11)$$

### 5. Speed Reduction Ratio

A simple schematic of the NSR is illustrated in Figure 4. Here, the symbols  $Z_1$ ,  $Z_2$ ,  $Z_3$  and  $Z_4$  present the numbers of teeth on the stator, nutator and rotor gears, respectively.

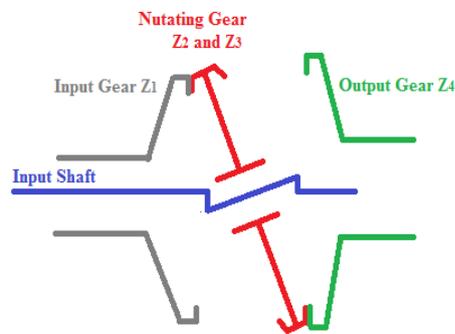


Figure 4. Simple schematic of the NSR

The reduction ratio ( $i$ ) is presented as [9]:

$$i = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{1 - \frac{Z_1 Z_3}{Z_2 Z_4}} \quad (12)$$

### 6. Nutator Gear Geometry

The nutator gear geometry is highly dependent upon its nutation angle, which defines the internal bevel gear pitch cones size and shape [9]. To calculate the size and shape of pitch cones, the number of gear teeth, the diametral pitch, and nutation angle has to be given. All of the pitch cone vertexes within the NSR align at the point, that the pitch cone vertexes of the two gear meshes within the PMC are coincident of one another and back to back. This is the point about which the nutator both rotates and nutates during its meshing cycle. As illustrated in Figure 5 can be seen this point with the pitch cones being shown and coming together.

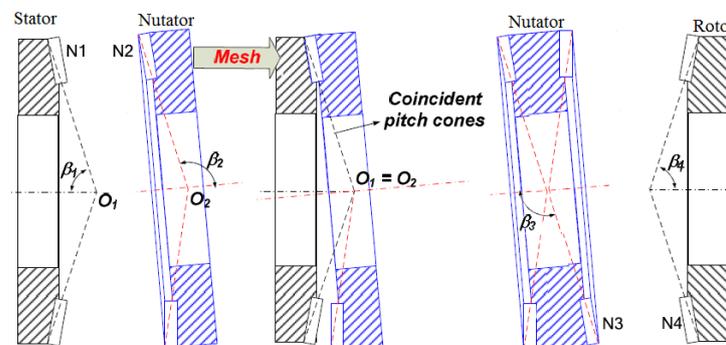


Figure 5. Meshing between the stator and nutator

The pitch cone angles of these three bodies are mathematically defined by five parameters, the teeth numbers of each gear pair ( $Z_1, Z_2, Z_3, Z_4$ ) and the nutation angle  $\beta$ . To find the pitch cone angles of the four gear pitch cones using trigonometric relations:

$$\beta_1 = \tan^{-1} \left( \frac{\sin(\pi - \beta)}{\frac{Z_2}{Z_1} + \cos(\pi - \beta)} \right) \quad (13) \quad , \quad \beta_2 = \pi - \beta - \beta_1 \quad (14)$$

$$\beta_3 = \tan^{-1} \left( \frac{\sin(\pi - \beta)}{\frac{Z_4}{Z_3} + \cos(\pi - \beta)} \right) \quad (15) \quad , \quad \beta_4 = \pi - \beta - \beta_3 \quad (16)$$

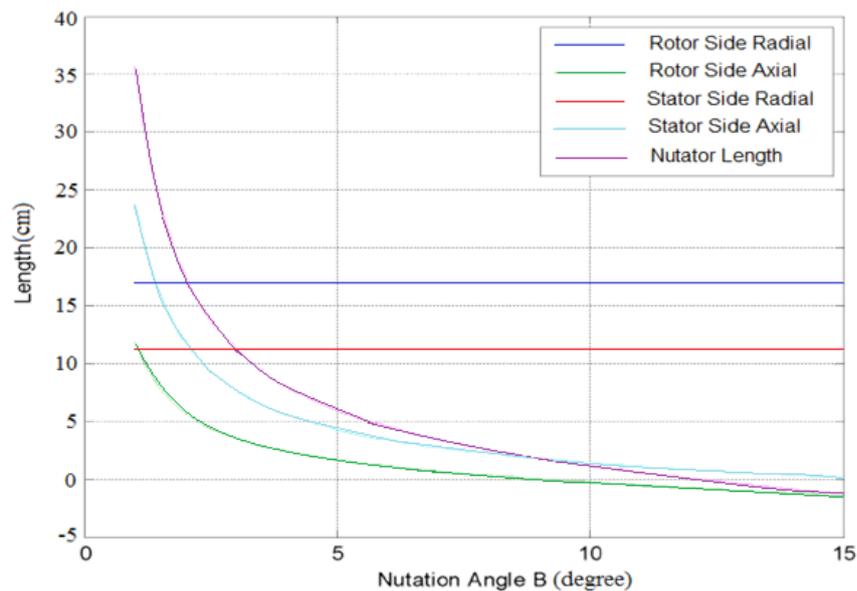
**7. Effect of Altering Nutation Angle on NSR Geometry**

The variation in nutation angle impacts all pitch cone angles while not changing the reduction ratio of the NSR, therefore it is a good candidate to examine. Using the geometric parameters given in Table 1 and altering the nutation angle, we will examine how the nutator gear size is affected.

**Table 1.** Basic parameters of Nutation Speed Reducer

Parameters	Z1	Z2	Z3	Z4	i	Module	Face Width
Value	42	44	81	80	30	4.23 mm	20 mm

The nutator gear geometry has two main dimensions, the radial dimension from the nutator center, and the axial dimension from the nutator axis. Since the nutator has two sides, so four dimensions with varying nutation angle from 1 degree to 15 degrees were created, the resulting calculated values are illustrated in Figure 6 below.



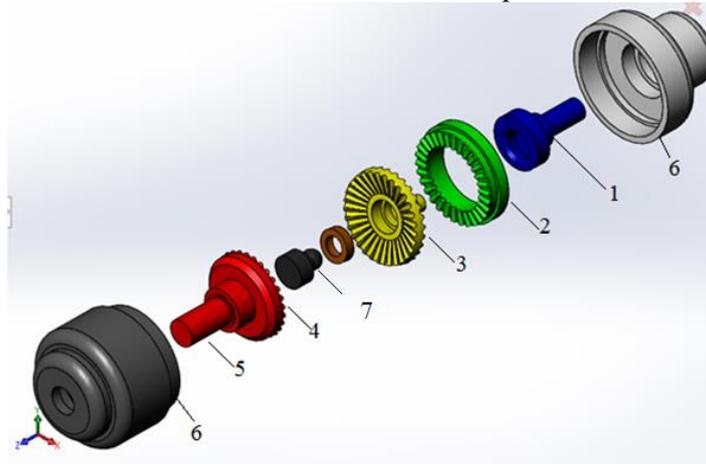
**Figure 6.** Nutator gear size against varying nutation angles

We can observe that there are no change in radial lengths, and this is due to their value being tied to the diametral pitch and number of teeth on each gear, which is held constant in this case. While increasing nutation angle causes a decrease in axial dimensions as stator and rotor side axial and nutator length. This exponential decrease in axial length immediately translate to an exponential decrease in mass of the nutator body. But the axial decrease is limited, due to it decreases the conformity of the gear meshes as well as the number of teeth simultaneously in contact, and limiting the area available for the nutator bearings. At approximately twelve degrees nutation the nutator length turns negative. Because

of an inversion of the pitch cones where the NSR transfers from the speed reducer regime to a humpage drive regime [10]

### 8. Kinematic Simulation

The virtual assembling for the 3D models of NSR is carried out in a virtual environment as shown in figure 7. To verify the transmission theory and the interference detection of the nutation speed reducer, the kinematic simulation were verified. It should be noted that, the motion constraint is 3D contact pair[5] between the stator and nutator ,as well as the rotor and nutator. It can be proven from the simulation that there are no interferences between the movable parts.



**Figure 7.** Assembly exploded views of NSR

### 9. Conclusion

In this work, a novel and advanced transmission which called Nutation Speed Reducer has been studied. The kinematical analysis of the nutation speed reducer was performed, which showed that the motion of the nutator gear is combination of two motions produced by the input shaft and stator gear mesh. These coupled motions produce the kinematics to the NSR system.

The general formulae for calculating the reduction ratio and main design parameters of the novel gear drive were developed. The geometry of the nutator gear was examined along with the impact of nutation angle on overall size and shape. The kinematic simulation was carried out in a virtual simulation environment to verify the transmission theory and the interference detection of the nutation speed reducer.

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