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Modified Born–Infeld-Dilaton-Axion Coupling in Supersymmetry

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Abstract: We propose the supersymmetric extension of the modified Born–Infeld-axion-dilaton non-linear electrodynamics that has confined static abelian solutions used for describing the electromagnetic confinement in the presence of axion and dilaton fields, as well as charged matter. The supersymmetric extension also has the non-trivial scalar potential that implies the upper bounds on the matter fields.

Keywords: born–infeld model; supersymmetry; dilaton; axion; electromagnetic; confinement; abelian

1. Introduction

Born–Infeld (BI) theory [1] is the profound extension of Maxwell electrodynamics, distinguished by its several theoretical features: (i) the Lagrangian as a density; (ii) Lorentz invariance; (iii) the resolution of the Coulomb singularity of the static electric field of a point-like charge; (iv) the upper limits on the values of electric and magnetic fields; (v) causal propagation of waves (no shock waves); (vi) the electric-magnetic self-duality; (vii) the origin in string theory as the low-energy effective theory of open strings and D-branes (see [2–9] for all these known aspects of the BI theory).

The supersymmetric extensions of the BI theory exhibit the additional highly-non-trivial and unexpected features, such as (viii) the extra hidden (non-linearly) realized supersymmetries allowing (ix) the interpretation of the supersymmetric BI actions as the Maxwell–Goldstone actions associated with partial spontaneous supersymmetry breaking [10–14] and (x) adding Fayet–Iliopoulos (FI) terms [15].

The BI theory and its supersymmetric extensions are the very special and remarkable examples of the non-linear electrodynamics, whose applicability is not limited to string theory. Moreover, for the sake of phenomenological applications, one may sacrifice some of the distinguished features (i–x) mentioned above. The phenomenological applications may also include the coupling to axion and dilaton fields to the non-linear electrodynamics, beyond the standard (minimal) coupling to the charged fields.

One such interesting application was proposed in [16], where the natural extension of the Born–Infeld action was used, which preserves the properties (i) and (ii) above. The authors of [16]

discovered the existence of confined solutions to the modified BI equations of motion, i.e., the fields with finite support in spacetime, in the presence of axion and dilaton fields. This feature is not shared by the original BI theory and represents the electromagnetic confinement that may be of great interest to astrophysics and cosmology because it may imply the existence of new forms of matter (in the form of charged or uncharged stars invisible to us, like black holes).

It is, therefore, of interest, to construct the supersymmetric extension of the modified BI theory proposed in [16] and couple it to charged matter and a dilaton-axion superfield. Our paper is devoted to this purpose.

The paper is organized as follows. In Section 2, we briefly review the Born–Infeld theory and its $N = 1$ supersymmetric extension in four spacetime dimensions. In Section 3, we also review the standard coupling of the BI theory to dilaton and axion, together with its supersymmetric extension. Sections 2 and 3 provide the foundation for our new results given in Sections 4 and 5. Our conclusion is Section 6.

2. Supersymmetric Born–Infeld Theory

The standard Born–Infeld (BI) Lagrangian reads [1]:

$$\mathcal{L}_{\text{BI}} = \frac{1}{b^2} \left(1 - \sqrt{-\det(\eta_{mn} + bF_{mn})} \right), \quad F_{mn} = \partial_m A_n - \partial_n A_m, \quad (1)$$

with the BI coupling constant b . In four spacetime dimensions, it takes the form:

$$\mathcal{L}_{\text{BI}} = \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F\tilde{F})^2} \right) = -\frac{F^2}{4} + \frac{\frac{b^2}{16} [(F^2)^2 + (F\tilde{F})^2]}{1 + \frac{b^2}{4} F^2 + \sqrt{1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F\tilde{F})^2}}, \quad (2)$$

where we have used the notation $\tilde{F}^{mn} = \frac{1}{2}\epsilon^{mnpq}F_{pq}$, $F^2 \equiv F_{mn}F^{mn}$, and $F\tilde{F} \equiv \frac{1}{2}\epsilon^{mnpq}F_{mn}F_{pq}$ with Levi–Civita ϵ .

The equations of motion (EOM) and Bianchi identities (BIs) of the BI theory can be written as follows:

$$\partial^n G_{mn} = 0, \quad (3)$$

$$\partial^n \tilde{F}_{mn} = 0, \quad (4)$$

respectively, where we have used the notation:

$$G_{mn} = -2 \frac{\partial \mathcal{L}}{\partial F^{mn}}. \quad (5)$$

The EOM and BIs are invariant under the $SO(2)$ electric-magnetic duality rotations:

$$\begin{aligned} F_{mn} &\rightarrow \cos \gamma F_{mn} + \sin \gamma \tilde{G}_{mn}, \\ G_{mn} &\rightarrow \cos \gamma G_{mn} + \sin \gamma \tilde{F}_{mn}, \end{aligned} \quad (6)$$

where we have used the notation $\tilde{G}^{mn} = \frac{1}{2}\epsilon^{mnpq}G_{pq}$. As regards a generic non-linear electrodynamics with the Lagrangian $\mathcal{L}(F)$, the condition of the electric-magnetic duality reads [5]:

$$G\tilde{G} = F\tilde{F}. \quad (7)$$

The Lagrangian (2) can be expanded as:

$$\mathcal{L}_{\text{BI}} = -\frac{1}{4}F^2 + \frac{b^2}{32} [(F^2)^2 + (F\tilde{F})^2] + \frac{b^4}{128} [(F^2)^3 + F^2(F\tilde{F})^2] + \mathcal{O}(b^6), \quad (8)$$

whose leading term is the standard Lagrangian of Maxwell electrodynamics.

The form (2) of the BI theory is most convenient for its (rigid) supersymmetrization in superspace. In $N = 1$ superspace, the supersymmetric BI theory is described by the Lagrangian [10,17] (for a review of the Born–Infeld theory and its supersymmetric extensions, see, e.g., [9,14]):

$$\mathcal{L}_{\text{sBI}} = \frac{1}{4} \left(\int d^2\theta W^2 + \text{h.c.} \right) + \frac{b^2}{4} \int d^4\theta \frac{W^2 \bar{W}^2}{1 + \frac{b^2}{2}(\omega + \bar{\omega}) + \sqrt{1 + b^2(\omega + \bar{\omega}) + \frac{b^4}{4}(\omega - \bar{\omega})^2}}, \quad (9)$$

in terms of the superfield strength $W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$ of the real gauge superfield V with $W^2 \equiv W^\alpha W_\alpha$ and:

$$\omega \equiv \frac{1}{8} D^2 W^2 = \frac{1}{4} (F^2 - 2D^2 - iF\tilde{F}) + \dots, \quad (10)$$

where the dots stand for the fermionic and the higher order (in θ) terms.

The Lagrangian (9) is invariant under the $U(1)$ gauge transformations:

$$V \rightarrow V - \Lambda - \bar{\Lambda} \quad (11)$$

with the chiral superfield gauge parameter Λ . In addition to the manifest $N = 1$ supersymmetry (SUSY), the SUSY BI action is also known to be invariant under the second (non-linearly-realized) supersymmetry [11,12].

The bosonic part of the Lagrangian (9) reads:

$$\mathcal{L}_{\text{sBI}} = -\frac{A}{4} + \frac{\frac{b^2}{16}(A^2 + B^2)}{1 + \frac{b^2}{4}A + \sqrt{1 + \frac{b^2}{2}A - \frac{b^4}{16}B^2}} = \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2}A - \frac{b^4}{16}B^2} \right), \quad (12)$$

where $A \equiv F^2 - 2D^2$ and $B \equiv F\tilde{F}$. The real auxiliary field D can be eliminated by its equation of motion, $D = 0$, then the resulting Lagrangian coincides with Equations (1) and (2).

The SUSY BI theory (9) can be further generalized by adding matter chiral superfields Φ_i (charged under the $U(1)$ gauge symmetry with charges q_i), together with a Fayet–Iliopoulos (FI) term:

$$\mathcal{L} = \mathcal{L}_{\text{sBI}} + \int d^4\theta \left[K(\bar{\Phi}_i e^{q_i V} \Phi_i) + \zeta V \right], \quad \Phi_i \rightarrow e^{q_i \Lambda} \Phi_i, \quad (13)$$

where we have introduced the arbitrary function K (we do not demand renormalizability) and the real constant FI parameter ζ . In terms of the field components, we find:

$$\mathcal{L} = \mathcal{L}_{\text{sBI}} - XD + \dots, \quad (14)$$

where we have introduced the notation:

$$X \equiv -\frac{1}{2}(K_V + \zeta) \quad \text{and} \quad K_V \equiv \frac{\partial K}{\partial V} \Big|_{\theta=0}, \quad (15)$$

and the dots stand for the terms that are irrelevant in the D equation of motion.

When the matter fields are absent, $K = 0$, the BI theory with the FI term (13) retains its hidden $N = 2$ SUSY [15], although the FI term spontaneously breaks the linear $N = 1$ SUSY. When the charged matter fields are present, the second non-linear SUSY is explicitly broken.

Given the Lagrangian (14), the EOM of D reads:

$$D = X \sqrt{1 + \frac{b^2}{2}(F^2 - 2D^2) - \frac{b^4}{16}(F\tilde{F})^2}, \quad (16)$$

and its solution is given by:

$$D = \frac{X}{\sqrt{1+b^2X^2}} \sqrt{1 + \frac{b^2}{2}F^2 - \frac{b^4}{16}(F\tilde{F})^2}. \quad (17)$$

Substituting the solution back into the Lagrangian yields:

$$\mathcal{L} = \frac{1}{b^2} \left(1 - \sqrt{1+b^2X^2} \sqrt{1 + \frac{b^2}{2}F^2 - \frac{b^4}{16}(F\tilde{F})^2} \right). \quad (18)$$

Therefore, we obtain the scalar potential:

$$V_D = \frac{1}{b^2} \left(\sqrt{1+b^2X^2} - 1 \right) = \frac{1}{2}X^2 - \frac{b^2}{8}X^4 + \frac{b^4}{16}X^6 + \mathcal{O}(b^6). \quad (19)$$

3. BI Coupling to Dilaton-Axion and Its Supersymmetrization

The standard coupling of Born-Infeld theory to dilaton field ϕ and axion field C reads (the full theory also includes the dilaton and axion kinetic terms):

$$\begin{aligned} \tilde{\mathcal{L}}_{\text{BI}} &= \frac{1}{b^2} \left(1 - \sqrt{-\det(\eta_{mn} + be^{\phi/2}F_{mn})} \right) + \frac{C}{4}F\tilde{F} \\ &= \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2}e^{-\phi}F^2 - \frac{b^4}{16}e^{-2\phi}(F\tilde{F})^2} \right) + \frac{C}{4}F\tilde{F}. \end{aligned} \quad (20)$$

In this case, the $SO(2)$ electromagnetic self-duality can be extended to the $SL(2, \mathbb{R})$ self-duality [5] (in quantum theory (superstrings), the $SL(2, \mathbb{R})$ is broken into its discrete subgroup $SL(2, \mathbb{Z})$). After introducing the complex dilaton-axion field and its vacuum expectation value (VEV),

$$\tau \equiv C + ie^{-\phi}, \quad \tau_0 = \frac{\Theta}{2\pi} + \frac{4\pi i}{e^2}, \quad (21)$$

respectively, where Θ is the vacuum theta-angle and e is the $U(1)$ electric charge, the $SL(2, \mathbb{R})$ transformations read:

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad \text{with } a, b, c, d \in \mathbb{R}, \quad ad - bc = 1. \quad (22)$$

The SUSY extension of the action (20) takes the form [18]:

$$\tilde{\mathcal{L}}_{\text{sBI}} = \frac{1}{4i} \left(\int d^2\theta \tau W^2 - \text{h.c.} \right) + \frac{b^2}{16} \int d^4\theta \frac{|\tau - \bar{\tau}|^2 W^2 \bar{W}^2}{1 + \frac{b^2}{2}(\omega' + \bar{\omega}') + \sqrt{1 + b^2(\omega' + \bar{\omega}') + \frac{b^4}{4}(\omega' - \bar{\omega}')^2}}, \quad (23)$$

where we have used the notation:

$$\omega' \equiv \frac{1}{16i}(\tau - \bar{\tau})D^2W^2 = \frac{1}{4}e^{-\phi}(A - iB) + \dots, \quad (24)$$

and have promoted τ to the chiral superfield. The bosonic part of the Lagrangian (23) reads:

$$\tilde{\mathcal{L}}_{\text{sBI}} = \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2}e^{-\phi}(F^2 - 2D^2) - \frac{b^4}{16}e^{-2\phi}(F\tilde{F})^2} \right) + \frac{C}{4}F\tilde{F}. \quad (25)$$

The auxiliary field D is eliminated by its EOM, $D = 0$, and the resulting Lagrangian coincides with (20). In the parametrization (21), the Lagrangian should be rescaled as $\mathcal{L} \rightarrow \frac{e^2}{4\pi}\mathcal{L}$, in order to obtain the canonical kinetic term of F_{mn} .

4. Modified BI Theory with Dilaton-Axion-Like Couplings

As was argued in the Introduction, let us consider the modified coupling of the dilaton-axion field to BI theory, which was proposed in [16]:

$$\begin{aligned} \mathcal{L}_{BI'} &= \frac{1}{b^2} \left(1 - \sqrt{-\det(\eta_{mn} + b\alpha F_{mn} + b\beta \tilde{F}_{mn})} \right) \\ &= \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2} (e^{-\phi} F^2 - CF\tilde{F}) - \frac{b^4}{16} (e^{-\phi} F\tilde{F} + CF^2)^2} \right), \end{aligned} \tag{26}$$

where α and β are related to ϕ and C as:

$$\alpha^2 - \beta^2 = e^{-\phi}, \quad -2\alpha\beta = C. \tag{27}$$

The Lagrangian (26) is obtained from the original BI theory (1) by the substitution $F_{mn} \rightarrow \alpha F_{mn} + \beta \tilde{F}_{mn}$. As is clear from a comparison of Equations (20) and (26) (see also Equation (28) below), the two theories differ in their C -dependence, but their leading (b -independent) terms coincide.

We refer to the modified theory (26) as the BI' theory and still associate the fields ϕ and C with the dilaton and axion, respectively, as in [16], because they regain their original meaning in the weak coupling limit $b \rightarrow 0$. Indeed, in the BI' theory, the shift symmetry $C \rightarrow C + const.$, defining the axion, is lost (together with the electromagnetic self-duality, since $G\tilde{G} \neq F\tilde{F}$), while C directly affects the equations of motion for the electromagnetic field A_m . However, after expanding the action (26),

$$\begin{aligned} \mathcal{L}_{BI'} &= -\frac{1}{4}e^{-\phi}F^2 + \frac{1}{4}CF\tilde{F} + \frac{b^2}{32}(e^{-2\phi} + C^2) [(F^2)^2 + (F\tilde{F})^2] - \\ &\quad - \frac{b^4}{128}(e^{-2\phi} + C^2)(e^{-\phi}F^2 - CF\tilde{F}) [(F^2)^2 + (F\tilde{F})^2] + \mathcal{O}(b^6), \end{aligned} \tag{28}$$

in the leading order with respect to b , the BI' theory coincides with the Maxwell theory coupled to the dilaton and axion. Hence, for the weak F_{mn} field, the axionic shift symmetry approximately holds (as well as the approximate $SL(2, \mathbb{R})$ duality).

A supersymmetrization of the BI' theory is straightforward and results in:

$$\mathcal{L}_{sBI'} = \frac{1}{4i} (\int d^2\theta \tau W^2 - \text{h.c.}) + \frac{b^2}{4} \int d^4\theta \frac{|\tau|^2 W^2 \bar{W}^2}{1 + \frac{b^2}{2}(\omega' + \bar{\omega}') + \sqrt{1 + b^2(\omega' + \bar{\omega}') + \frac{b^4}{4}(\omega' - \bar{\omega}')^2}}, \tag{29}$$

where we have used the notation:

$$\omega' \equiv \frac{\tau}{8i} D^2 W^2 = -\frac{\tau}{4} (B + iA) + \dots \tag{30}$$

together with $A \equiv F^2 - 2D^2$, $B \equiv F\tilde{F}$, and $\tau = C + ie^{-\phi} + \mathcal{O}(\theta)$.

After expanding the Lagrangian in components, we find:

$$\begin{aligned} \mathcal{L}_{sBI'} &= -\frac{1}{4}(e^{-\phi}A - CB) + \frac{b^2}{16} \cdot \frac{|\tau|^2(A^2 + B^2)}{1 + \frac{b^2}{4}(e^{-\phi}A - CB) + \sqrt{1 + \frac{b^2}{2}(e^{-\phi}A - CB) - \frac{b^4}{16}(e^{-\phi}B + CA)^2}} \\ &= \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2}(e^{-\phi}A - CB) - \frac{b^4}{16}(e^{-\phi}B + CA)^2} \right) \\ &= \frac{1}{b^2} \left(1 - \sqrt{1 + \frac{b^2}{2}(e^{-\phi}F^2 - 2e^{-\phi}D^2 - CF\tilde{F}) - \frac{b^4}{16}(e^{-\phi}F\tilde{F} + CF^2 - 2CD^2)^2} \right). \end{aligned} \tag{31}$$

In the absence of matter fields and FI terms, the auxiliary field D is eliminated by its EOM, $D = 0$, and the resulting Lagrangian coincides with (26).

4.1. Adding Supersymmetric Matter and the FI Term

Adding to the BI' theory the charged matter and the FI term along the lines of Equations (13) and (14), with:

$$\mathcal{L}' = \mathcal{L}_{\text{sBI}'} - XD + \dots, \quad (32)$$

yields the EOM for D in the form:

$$2b^2\psi D - \frac{b^4}{2}CD \left(\psi F\tilde{F} + CF^2 - 2CD^2 \right) = \\ = 2b^2X \sqrt{1 + \frac{b^2}{2}(\psi F^2 - 2\psi D^2 - CF\tilde{F}) - \frac{b^4}{16}(\psi F\tilde{F} + CF^2 - 2CD^2)^2}, \quad (33)$$

where we have introduced the notation $\psi \equiv e^{-\phi}$. Since $D = 0$ is no longer a valid solution for its EOM, this generates a scalar potential.

Let us study solutions to EOM of D . After setting $F_{mn} = 0$, there are three of them (in terms of D^2),

$$D_1^2 = \frac{1}{3b^2C^2} \left[-4\psi - b^2X^2 + \frac{(2\psi - b^2X^2)^2}{\lambda} + \lambda \right], \quad (34)$$

$$D_{2,3}^2 = \frac{1}{3b^2C^2} \left[-4\psi - b^2X^2 - \frac{\frac{1}{2}(1 \pm i\sqrt{3})(2\psi - b^2X^2)^2}{\lambda} - \frac{1}{2}(1 \mp i\sqrt{3})\lambda \right], \quad (35)$$

where we have used the notation:

$$\lambda \equiv \left(6b|X| \sqrt{3\sigma(\psi^2 + C^2)} + \sigma + 27(\psi^2 + C^2)b^2X^2 \right)^{\frac{1}{3}}, \quad (36)$$

$$\sigma \equiv 8\psi^3 + (15\psi^2 + 27C^2)(bX)^2 + 6\psi(bX)^4 - (bX)^6. \quad (37)$$

The two real solutions are $D_1 = \pm\sqrt{D_1^2}$. For consistency, the sign of the square roots should be correlated with that of X , i.e., $\pm\sqrt{D_1^2}$ should correspond to $\pm|X|$, respectively.

Since σ appears under the square root in (36), when $\sigma < 0$, the D_1^2 becomes complex, while $D_{2,3}^2$ generically stays complex as well (however, for certain values of X , ψ , and C , the imaginary parts of $D_{1,2,3}$ may vanish). Furthermore, D_1^2 has to be positive, i.e., the condition:

$$\frac{(2\psi - b^2X^2)^2}{\lambda} + \lambda - 4\psi - b^2X^2 \geq 0 \quad (38)$$

must hold. It can be easily checked that (38) is always satisfied for $\sigma \geq 0$.

As is clear from the Lagrangian (31) or EOM (33), the solution (34) does not guarantee that the expression under the square root is positive. This means we have to require that (for $F_{mn} = 0$):

$$1 - b^2\psi D^2 - \frac{b^4}{4}C^2D^4 \geq 0, \quad (39)$$

which yields (also demanding the D^2 to be positive):

$$D^2 \leq 2 \frac{-\psi + \sqrt{\psi^2 + C^2}}{b^2C^2}. \quad (40)$$

Substituting the solution (34) for D^2 leads to a complicated inequality involving X , ψ , and C .

We conclude that the supersymmetric BI' theory leads to restrictions on the values of $\psi = e^{-\phi}$, C , and $X \equiv -\frac{1}{2}(K_V + \zeta)$, in contrast to the standard supersymmetric BI theory, namely Equation (40) together with the condition:

$$\sigma \geq 0. \tag{41}$$

4.2. The Case of a Constant τ

To gain more insight, let us consider the particular case of $\tau = \tau_0$, i.e.:

$$\tau = \frac{\Theta}{2\pi} + \frac{4\pi i}{e^2}. \tag{42}$$

We find that the scalar potential of the supersymmetric BI' theory has a restricted domain in terms of the values of X . In order to get numerical results, as an example, let us set $\Theta = 2\pi$ and $e^2 = 4\pi$, so that $\tau = 1 + i$. We first derive the upper limit on X , using the inequality $\sigma \geq 0$ (with the definition (37)):

$$-(bX)^6 + 6(bX)^4 + 42(bX)^2 + 8 \geq 0. \tag{43}$$

The inequality (43) is respected by:

$$|X| \equiv \frac{1}{2}|K_V + \zeta| \leq \sqrt{5 + 3\sqrt{3}}b^{-1} \approx 3.19 b^{-1}. \tag{44}$$

Note that the BI parameter b has the dimension $(\text{mass})^{-2}$ and can be written in terms of the BI mass scale as $b = M_{\text{BI}}^{-2}$, whereas the FI parameter has the dimension of $(\text{mass})^2$.

Figure 1 illustrates the solution D_1 acquiring the non-vanishing imaginary part for $X > X_{\text{max}} \equiv \sqrt{5 + 3\sqrt{3}} b^{-1}$.

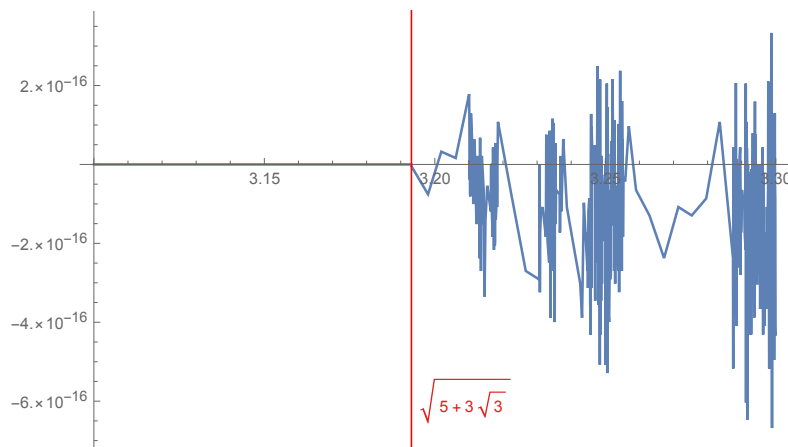


Figure 1. $\text{Im}D_1$ (vertical) versus X (horizontal). For $|X| \leq \sqrt{5 + 3\sqrt{3}}$ (with $b = 1$), the imaginary part of D_1 is zero, so the solution is valid in this region.

If we ignore the matter fields, $K = 0$, we arrive at the upper limit on the absolute value of the FI parameter:

$$|\zeta| \leq 2\sqrt{5 + 3\sqrt{3}} b^{-1}. \tag{45}$$

On the other hand, when the charged matter is present, the condition (44) gives the upper limit on K_V for a given ζ . For instance, with the canonical choice $K = \Phi e^{2V} \bar{\Phi}$ and $K_V = 2|\varphi|^2$, where φ is the leading component of Φ , the charged scalar φ is bounded from above. Setting $\zeta = 0$ for simplicity, we find:

$$|\varphi|^2 \leq \sqrt{5 + 3\sqrt{3}} b^{-1}. \tag{46}$$

Note that the upper limit of $|X|$ is of the same order ($1/b$) as that for F_{mn} .

Next, let us consider the condition (40). When $\psi = C = 1$, it takes the form:

$$|D| \leq \sqrt{2\sqrt{2} - 2} b^{-1}. \quad (47)$$

After a substitution of the solution (34) into (47), the resulting inequality can be numerically solved for X , yielding the conditions:

$$|X| \lesssim 2.02 b^{-1} \quad \text{and} \quad |X| \gtrsim 3.04 b^{-1}. \quad (48)$$

5. The Scalar Potential

The scalar potential of the supersymmetric matter-coupled BI' theory (32) can be obtained by using the solutions (34) for D and setting $F_{mn} = 0$. The exact potential is very complicated, and it seems impossible to obtain its perturbative expansion in terms of b , because the solution to D has the overall factor of b^{-1} .

Though the exact features of the potential depend on the choice of X , it is nevertheless possible to get some general results. For instance, the potential is always non-negative, and its minimum is at $X = 0 \rightarrow D = 0$, if it exists, with unbroken SUSY and Minkowski vacuum. This is similar to the ordinary D -term potentials proportional to D^2 . With the positive FI term and no matter, the minimum is of the de Sitter type with the D -term SUSY breaking.

The main difference against the ordinary D -term potentials is the existence of restrictions on the values of $X \equiv \frac{1}{2}(K_V + \xi)$ for given values of $\psi = e^{-\phi}$ and C . These restrictions arise from the conditions (41) and (47) for general ψ and C . If we set $\psi = C = 1$, we obtain the specific results:

$$|X| \lesssim 2.02, \quad 3.04 \lesssim |X| \lesssim 3.19, \quad (49)$$

in units of $b = 1$.

We provide the plot of the scalar potential as the function of X in Figure 2. For negative values of X , the negative root of D_1^2 should be used as the solution to D , so that the potential is symmetric under $X \rightarrow -X$.

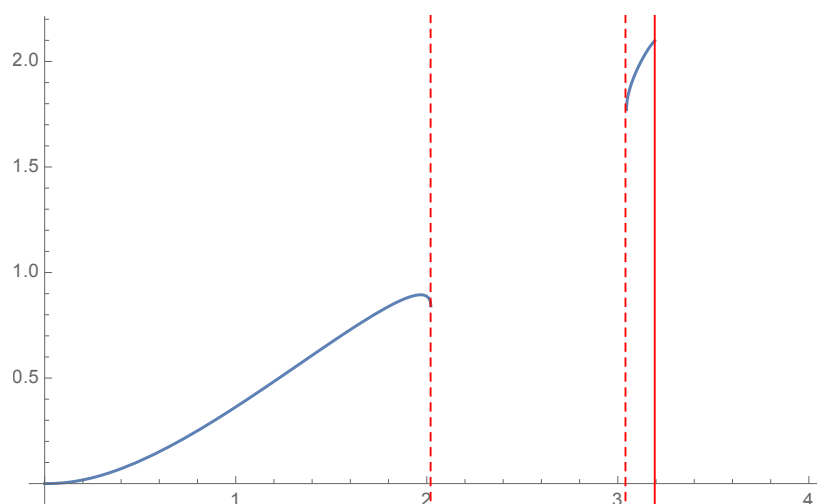


Figure 2. The scalar potential (vertical) versus X (horizontal). The dashed red lines mark the values $X \approx 2.02$ and $X \approx 3.04$, while the solid red line stands at $X \approx 3.19$, the upper limit on X .

6. Conclusions

Our main results are given in Sections 4 and 5.

It is of further interest to explore whether the confining mechanism of [16] also exists in non-abelian extensions of the (modified) Born–Infeld theory and/or their supersymmetric extensions, in the presence of the dilaton-axion superfield. For example, a non-abelian (and non-supersymmetric) Born–Infeld (NBI) theory with a Θ -term, having a similar structure to the BI' theory, was considered in [7], where it was found that this NBI theory (together with the Higgs sector) admits monopole solutions and has Witten's effect (i.e., a shift of the electric charge of the monopole induced by the presence of the Θ -term [19]), like that in the usual non-abelian gauge theory. Unfortunately, a non-abelian extension of the BI theory and, hence, a supersymmetric NBI theory, is not unique [9].

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