Published for SISSA by Springer

Received: *August 6, 2018*
Revised: *September 10, 2018*
Accepted: *September 11, 2018*
Published: *September 17, 2018*

Massive vector multiplet with Dirac-Born-Infeld and new Fayet-Iliopoulos terms in supergravity

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Abstract: We propose a four-dimensional $N = 1$ supergravity-based Starobinsky-type inflationary model in terms of a single massive vector multiplet, whose action includes the Dirac-Born-Infeld-type kinetic terms and a generalized (new) Fayet-Iliopoulos-type term, without gauging the R-symmetry. The bosonic action and the scalar potential are computed. The inflaton is the superpartner of the Goldstino in our model, and supersymmetry is spontaneously broken after inflation by the D-type mechanism, whose scale is related to the value of the cosmological constant.

Keywords: Supergravity Models, Supersymmetry Breaking

ArXiv ePrint: 1808.00669

https://doi.org/10.1007/JHEP09(2018)094

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1 Introduction

Supergravity is the natural framework for unification of bosons and fermions, and for unification of elementary particles with gravity. On the one hand, it is possible (though non-trivial) to unify the dark matter (as the lightest supersymmetric particle), the dark energy (as the positive cosmological constant) and the cosmological inflation (with the inflaton scalar field having the proper scalar potential) in supergravity. On the other hand, supergravity emerges as the low-energy effective theory from (compactified) superstrings (quantum gravity), and can be connected to the Standard Model at the electro-weak scale. All phenomenological applications of supergravity require (spontaneous) supersymmetry breaking and a non-vanishing gravitino mass.

In supersymmetry (SUSY), the inflaton should belong to a supermultiplet. A spontaneous SUSY breaking implies the existence of the spin-1/2 Goldstino that should also belong to a supermultiplet. In the literature of the inflationary model building based on supergravity, one usually assumes that the inflaton belongs to a chiral multiplet and the

Goldstino belongs to another chiral multiplet [1–3], whose Kähler potential and superpotential can be appropriately chosen "by hand" [4].[1] This gives rise to *four* real physical scalars and the need to distinguish the inflaton among them, while stabilizing the remaining three scalars during a single-field inflation. It can be done in many ways, thus reducing the predictive power.

This freedom of choice can be reduced by minimizing the number of the physical degrees of freedom involved. The inflaton and Goldstino chiral multiplets can be identified, which leads to the viable and more economic inflationary models based on supergravity [6–9].

It is also possible to employ a massive *vector* multiplet [10] that has only *one* physical scalar to play the role of the inflaton, and then to identify its fermionic superpartner with the Goldstino, as the truly minimal option. This opportunity was investigated in [11, 12], where it was found that it is flexible enough to accommodate a cosmological inflation with any values of the Cosmic Microwave Background (CMB) radiation tilts n_s and r. However, it was also observed that SUSY is necessarily restored after inflation in this class of the supergravity-based inflationary models, which requires an extra mechanism of spontaneous SUSY breaking after inflation for reheating and viable phenomenology of particles. A solution to this problem was proposed in [13, 14], by adding a chiral (Polonyi) multiplet with a linear superpotential.

It is, therefore, the good question: is it possible to get rid of Polonyi multiplet, but still describe a viable cosmological inflation together with a spontaneous SUSY breaking after inflation, by using only a *single* (massive) vector multiplet? The affirmative answer apparently requires extra tools in supergravity theory, beyond the standard ones.

In this paper we employ the new supergravity construction that includes the following theoretical resources (tools):

- the manifest (*linearly* realized) local $N = 1$ supersymmetry,
- the inflaton and the Goldstino in a single (massive) $N = 1$ vector multiplet,
- the kinetic terms of the vector multiplet have the Dirac-Born-Infeld (DBI) structure inspired by superstrings and D-branes [15–18],
- the new Fayet-Iliopoulos (FI) terms in supergravity, that do not require gauging the R-symmetry [19–23],
- a constant superpotential.

The manifest SUSY has the advantage of a straightforward addition of quantum corrections. The Goldstino as the superpartner of the inflaton is the minimal option. The DBI structure introduces the new (BI) scale into our model, that is arguably between the Grand Unification (GUT) scale and Planck scale.

The use of a constant (field independent) FI term [24] is highly restrictive in supergravity, because its (old) standard construction (via Noether procedure) required gauging of the R-symmetry [25, 26]. However, when assuming a nonvanishing vacuum expectation value

[1] We assume the existence of a stable vacuum after inflation, and ignore run-away solutions [5].

(VEV) of the auxiliary D-component of the vector multiplet from the very beginning, one can introduce other (new) FI terms [19–23] that do not require gauging the R-symmetry.

We consider only the Starobinsky-like inflationary models for definiteness and because they are most natural in our construction. As regards Starobinsky inflation and its realizations in supergravity, see e.g., [27–30].

Our paper is organized as follows. Our technical setup, based on the superconformal tensor calculus, is briefly reviewed in section 2. In section 3 we propose the new supergravity action, and compute its bosonic part that includes the scalar potential. In section 4 we apply our construction to the Starobinsky-like inflation and spontaneous SUSY breaking. Our conclusion is section 5. In appendix A we describe our supergravity actions in terms of the superfields defined in curved superspace. In appendix B we briefly study the impact of a constant superpotential.

2 Our setup

In the main body of our paper (except of appendix A) we use the conformal $N = 1$ supergravity techniques [31–35], and follow the notation and conventions of ref. [36]. In addition to the symmetries of Poincaré supergravity, one also has the gauge invariance under dilatations, conformal boosts and S-supersymmetry, as well as under $\mathrm{U}(1)_A$ rotations. The gauge fields of dilatations and $\mathrm{U}(1)_A$ rotations are denoted by b_μ and A_μ, respectively. A multiplet of conformal supergravity has charges with respect to dilatations and $\mathrm{U}(1)_A$ rotations, called Weyl and chiral weights, respectively, which are denoted by pairs (Weyl weight, chiral weight) in what follows.

A chiral multiplet has field components

$$S = \{S, P_L\chi, F\}, \tag{2.1}$$

where S and F are complex scalars, and $P_L\chi$ is a left-handed Weyl fermion (P_L is the chiral projection operator). As regards a general multiplet, it has

$$\Phi = \{\mathcal{C}, \mathcal{Z}, \mathcal{H}, \mathcal{K}, \mathcal{B}_a, \Lambda, \mathcal{D}\}, \tag{2.2}$$

where $\mathcal{Z}$ and Λ are fermions, and the other fields are complex scalars.

The (gauge) field strength multiplet W has the weights $(3/2, 3/2)$ and the following components:

$$\bar{\eta}W = \left\{\bar{\eta}P_L\lambda, \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\gamma_{ab}F^{ab} + iD\right)P_L\eta, \bar{\eta}P_L \not{D}\lambda\right\}, \tag{2.3}$$

where η is the dummy spinor. $F_{ab} = \partial_a B_b - \partial_b B_a$ is the Abelian field strength, and λ and D are Majorana fermion and the real scalar, respectively. The related expressions of the multiplets W^2 and $W^2\bar{W}^2$, which are embedded into the chiral multiplet (2.1) and the general multiplet (2.2), respectively, are

$$W^2 = \left\{\cdots, \cdots, \cdots + \frac{1}{2}(FF - F\tilde{F}) - D^2\right\}, \tag{2.4}$$

$$W^2\bar{W}^2 = \left\{\cdots, \cdots, \cdots, \cdots, \cdots, \cdots, \cdots + \frac{1}{2}|(FF - F\tilde{F}) - 2D^2|^2\right\}, \tag{2.5}$$

where we have omitted the fermionic terms (denoted by dots) for simplicity.

In addition, we use the book-keeping notation $FF = F_{ab}F^{ab}$ and $\tilde{F}^{ab} \equiv -\frac{i}{2}\varepsilon^{abcd}F_{cd}$. We also need another chiral multiplet

$$\Sigma\left(\bar{W}^2/|S_0|^4\right) = \left\{-\frac{(\frac{1}{2}FF + \frac{1}{2}F\tilde{F} - D^2)}{|S_0|^4} + \cdots, \cdots, \frac{F_0}{|S_0|^4 S_0}(FF + F\tilde{F} - 2D^2) + \cdots\right\}, \tag{2.6}$$

where Σ is the chiral projection operator [34, 35]. The argument of Σ requires the specific Weyl and chiral weights: in order for $\Sigma\Phi$ to make sense, Φ must satisfy $w - n = 2$, where (w, n) are Weyl and chiral weights of Φ. We get the correct weights by inserting the factor $|S_0|^4$, where S_0 is the chiral compensator of weights $(1, 1)$.[2] Equation (2.6) is the conformal supergravity counterpart of the superfield $\bar{D}^2\bar{W}^2$.

The covariant derivative of W is given by [35]

$$\mathcal{D}W = \{-2D, \cdots, \cdots, \cdots, \cdots, \cdots, \cdots\} \tag{2.7}$$

and has weights $(2, 0)$. The dots in the higher components also include some bosonic terms, but we do not write them here for simplicity (see ref. [19] for their explicit expressions).

A massive vector multiplet V has field components

$$V = \{C, Z, H, K, B_a, \lambda, D\}, \tag{2.8}$$

while all of them are either real (bosonic) or Majorana (fermionic). The weights of V are $(0, 0)$.

The bosonic part of the F-term invariant action

$$[S]_F = \int d^4x\sqrt{-g}\frac{1}{2}\left(F + \bar{F}\right), \tag{2.9}$$

can be only applied when the S has weights $(3, 3)$. The bosonic part of the D-term of a real multiplet ϕ of weights $(2, 0)$ reads

$$[\phi]_D = \int d^4x\sqrt{-g}\left(D_\phi - \frac{1}{3}C_\phi R(\omega)\right), \tag{2.10}$$

where $R(w)$ is (superconformal) Ricci scalar in terms of spacetime metric and b_μ [36]. The C_ϕ and D_ϕ are the first and the last components of ϕ, respectively.

3 Our action

Having defined the multiplets and the compensators in section 2, we propose the following action:

$$S = S_V + S_{\mathrm{DBI}} + S_{\mathrm{FI}}, \tag{3.1}$$

[2] The conformal supergravity compensators are distinguished from the physical matter supermultiplets in our notation by attaching the subscript 0 to the former.

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where we have defined

$$S_V = \Big[|S_0|^2\mathcal{H}(V)\Big]_D, \tag{3.2}$$

$$S_{\text{DBI}} = -\frac{1}{2}[W^2]_F + \left[\frac{\alpha(V)}{|S_0|^4}\frac{W^2\bar{W}^2}{1-2\alpha(V)\mathcal{A}+\sqrt{1-4\alpha(V)\mathcal{A}+4\alpha(V)^2\mathcal{B}^2}}\right]_D, \tag{3.3}$$

$$S_{\text{FI}} = \left[|S_0|^2\mathcal{I}(V)\frac{W^2\bar{W}^2}{(\mathcal{D}W)^2(\bar{\mathcal{D}}\bar{W})^2}\mathcal{D}W\right]_D, \tag{3.4}$$

in terms of three arbitrary real functions $\mathcal{H}$, $\mathcal{I}$, and α of the vector multiplet V. In addition, we have introduced

$$\mathcal{A} = \bar{\Sigma}\left(\frac{W^2}{|S_0|^4}\right) + \text{h.c.}, \quad \mathcal{B} = \bar{\Sigma}\left(\frac{W^2}{|S_0|^4}\right) - \text{h.c.}, \tag{3.5}$$

which have weights $(0,0)$.

The supergravity theory (3.1) without the S_{FI} term was proposed and studied in ref. [37]. The new FI term above (see also [20, 23]) represents its non-trivial extension. Our FI term is different from the one introduced in [19] because it has the different structure and includes arbitrary function (See appendix A for details). In [21], the FI term of [19] is applied to a D-term inflation, where the inflaton belongs to a (charged) chiral multiplet. Our FI term for a vector multiplet is also different from the other FI terms in terms of scalar multiplets [22].

It is straightforward to calculate the bosonic terms of the action (3.1). They are given by[3]

$$\begin{aligned}\mathcal{L}_V =& -\frac{1}{3}|S_0|^2\mathcal{H}R(\omega) + 2\mathcal{H}\left(|F_0|^2 - |D_aS_0|^2\right) + |S_0|^2\mathcal{H}_C D\\ &+\frac{1}{2}|S_0|^2\mathcal{H}_{CC}\left(|N|^2 - B_a^2 - (D_aC)^2\right)\\ &+\left\{-\mathcal{H}_C N S_0\bar{F}_0 + \mathcal{H}_C i\left(B_a + iD_aC\right)S_0 D^a\bar{S}_0 + \text{h.c.}\right\},\end{aligned} \tag{3.6}$$

$$\mathcal{L}_{\text{DBI}} = \frac{|S_0|^4}{8\alpha}\left[1 - \sqrt{1 - \frac{8\alpha}{|S_0|^4}\left(D^2 - \frac{1}{2}FF\right) + \frac{4\alpha^2}{|S_0|^8}(F\tilde{F})^2}\right], \tag{3.7}$$

$$\mathcal{L}_{\text{FI}} = -\mathcal{I}|S_0|^2\frac{\left(D^2 - \frac{1}{2}FF\right)^2 - \frac{1}{4}(F\tilde{F})^2}{4D^3}, \tag{3.8}$$

where $N \equiv H + iK$, and the subscript on $\mathcal{H}$ denotes the derivative with respect to C. The D_a is the superconformal covariant derivative [36],

$$D_aS_0 = \partial_aS_0 - iA_aS_0 - b_aS_0, \quad D_aC = \partial_aC - 2b_aC. \tag{3.9}$$

[3] The Lagrangian density is defined by $S = \int d^4x\sqrt{-g}\,\mathcal{L}$.

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To eliminate the extra symmetries of conformal supergravity against Poincaré supergravity, we impose the following superconformal gauge fixing conditions:

$$\mathrm{D-gauge}: \ -\frac{1}{3}|S_0|^2\mathcal{H} = \frac{1}{2} \ , \tag{3.10}$$

$$\mathrm{A-gauge}: \ S_0 = \bar{S}_0 \ , \tag{3.11}$$

$$\mathrm{K-gauge}: \ b_\mu = 0 \ , \tag{3.12}$$

which guarantee that the Ricci scalar in the supergravity action is canonically normalized.[4] Then the $R(\omega)$ becomes the usual Ricci scalar R. Under the above conditions, eq. (3.6) becomes

$$\begin{aligned}\mathcal{L}_V = &\frac{1}{2}R - \frac{3}{4\mathcal{H}^2}(\mathcal{H}_C^2 - \mathcal{H}_{CC}\mathcal{H})(\partial_a C)^2 + \frac{3\mathcal{H}_{CC}}{4\mathcal{H}}B_a^2 \\ &+ 3A_a^2 + \frac{3\mathcal{H}_C}{\mathcal{H}}A_a B^a + 2\mathcal{H}|F_0|^2 - \frac{3\mathcal{H}_{CC}}{4\mathcal{H}}|N|^2 \\ &+ \left\{-\sqrt{\frac{-3}{2\mathcal{H}}}\mathcal{H}_C \bar{F}_0 N + \mathrm{h.c.}\right\} - \frac{3\mathcal{H}_C}{2\mathcal{H}}D \ .\end{aligned} \tag{3.13}$$

Integrating out the auxiliary fields A_a, N and F_0 by using their (algebraic) equations of motion (except for the auxiliary field D) yields[5]

$$A_a = -\frac{\mathcal{H}_C}{2\mathcal{H}}B_a, \quad N = F_0 = 0 \ . \tag{3.14}$$

Substituting them into eq. (3.6), we obtain

$$\mathcal{L}_V = \frac{1}{2}R - \frac{1}{2}\mathcal{J}_{CC}(\partial_a C)^2 - \frac{1}{2}\mathcal{J}_{CC}B_a^2 + \mathcal{J}_C D \,, \tag{3.15}$$

where $\mathcal{J}(C) \equiv -\frac{3}{2}\log\left(-\frac{2}{3}\mathcal{H}\right)$.

The full bosonic Lagrangian, before integration over the auxiliary field D, is thus given by

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}R - \frac{1}{2}\mathcal{J}_{CC}(\partial_a C)^2 - \frac{1}{2}\mathcal{J}_{CC}B_a^2 + \mathcal{J}_C D \\ &+ \frac{e^{4\mathcal{J}/3}}{8\alpha}\left[1 - \sqrt{1 - 8\alpha e^{-4\mathcal{J}/3}\left(D^2 - \frac{1}{2}FF\right) + 4\alpha^2 e^{-8\mathcal{J}/3}(F\tilde{F})^2}\right] \\ &- \mathcal{I}e^{2\mathcal{J}/3}\frac{\left(D^2 - \frac{1}{2}FF\right)^2 - \frac{1}{4}(F\tilde{F})^2}{4D^3} \quad .\end{aligned} \tag{3.16}$$

Let us consider the elimination of D that is non-trivial. Its equation of motion is given by

$$\mathcal{J}_C + \frac{1}{\sqrt{f^2 - 8\alpha e^{-4\mathcal{J}/3}D^2}}D - \mathcal{I}\frac{e^{2\mathcal{J}/3}}{4}\left(1 + \frac{FF}{D^2} - \frac{3}{4}\frac{(FF)^2 - (F\tilde{F})^2}{D^4}\right) = 0, \tag{3.17}$$

[4]The gauge fixing condition of S-supersymmetry is irrelevant for bosonic terms.

[5]The auxiliary fields A_a, N and F_0 were not included in eqs. (3.7) and (3.8), because they do not contribute to the bosonic action.

where we have introduced the function

$$f(F) \equiv \sqrt{1+4\alpha e^{-4\mathcal{J}/3}FF+4\alpha^2 e^{-8\mathcal{J}/3}(F\tilde{F})^2}\ . \tag{3.18}$$

Hence, D is a root of the 5th order polynomial, and it is impossible to solve (3.17) explicitly.

However, when the FI term vanishes, i.e., $\mathcal{I}=0$, eq. (3.17) takes the form

$$\mathcal{J}_C + \frac{1}{\sqrt{f^2-8\alpha e^{-4\mathcal{J}/3}D^2}}D = 0\,, \tag{3.19}$$

and its solution can be found as

$$D^{(0)} = \pm K f\,, \tag{3.20}$$

where we have defined

$$K(C) \equiv \sqrt{\frac{\mathcal{J}_C^2}{1+8\alpha \mathcal{J}_C^2 e^{-4\mathcal{J}/3}}}\ , \tag{3.21}$$

and $D^{(0)}$ stands for the solution at $\mathcal{I}=0$.[6]

Since our interest is in what happens when $\mathcal{I}\neq 0$, we seek a perturbative solution to be connected to $D^{(0)}$ in the limit of $\mathcal{I}\to 0$. We assume that the perturbative solution takes the form

$$D = D^{(0)} + \mathcal{I}D^{(1)} + \mathcal{O}(\mathcal{I}^2). \tag{3.22}$$

Substituting this into eq. (3.17) and considering the coefficient of $\mathcal{I}$, we obtain the equation

$$\frac{4\mathcal{J}_C^3 e^{-2\mathcal{J}/3}}{K^2}\frac{D^{(1)}}{D^{(0)}} + 1 + \frac{FF}{D^{(0)2}} - \frac{3}{4}\frac{(FF)^2-(F\tilde{F})^2}{D^{(0)4}} = 0\,, \tag{3.23}$$

where we have neglected the terms proportional to $\mathcal{I}^n$, $(n\geq 2)$, and have used the fact that $D^{(0)}$ satisfies eq. (3.19). Note that the zero-th order equation with respect to $\mathcal{I}$ is trivially satisfied since $D^{(0)}$ is the solution when $\mathcal{I}=0$. From eq. (3.23), we find

$$D^{(1)} = \mp\frac{e^{2\mathcal{J}/3}K^3}{4\mathcal{J}_C^3} f\left(1+\frac{FF}{K^2f^2}-\frac{3}{4}\frac{(FF)^2-(F\tilde{F})^2}{K^4f^4}\right). \tag{3.24}$$

Hence, the bosonic Lagrangian up to the first order in $\mathcal{I}$ reads

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}R - \frac{1}{2}\mathcal{J}_{CC}(\partial_a C)^2 - \frac{1}{2}\mathcal{J}_{CC}B_a^2 + \frac{e^{4\mathcal{J}/3}}{8\alpha}\left(1\pm\frac{\mathcal{J}_C}{K}f\right)\\ & \mp\mathcal{I}\frac{e^{2\mathcal{J}/3}}{4}Kf\left(1-\frac{FF}{K^2f^2}+\frac{(FF)^2-(F\tilde{F})^2}{4K^4f^4}\right)+\mathcal{O}(\mathcal{I}^2).\end{aligned} \tag{3.25}$$

[6]Though the full theory is inconsistent in the limit $\xi=0$ that also implies $\mathcal{I}=0$ for our choice of this function in (4.1) below, Taylor expansion of the solution to (3.17) with respect to $\mathcal{I}$ and the $D^{(0)}$ are well defined. We always assume that $\xi\neq 0$ and $\langle D\rangle\neq 0$.

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In particular, as regards the real scalar of the vector multiplet, C, we get its Lagrangian as

$$\mathcal{L}_C = -\frac{1}{2}\mathcal{J}_{CC}(\partial_a C)^2 - V, \tag{3.26}$$

$$V = -\frac{e^{4\mathcal{J}/3}}{8\alpha}\left(1 \pm \frac{\mathcal{J}_C}{K}\right) \pm \mathcal{I}\frac{e^{2\mathcal{J}/3}}{4}K + \mathcal{O}(\mathcal{I}^2). \tag{3.27}$$

Fortunately, it is possible to compute the scalar potential $V(C)$ non-perturbatively, when ignoring the F-terms of the vector field. Indeed, when $F = 0$, the D-equation (3.17) can be solved exactly, and its solution is given by

$$D = \pm\sqrt{\frac{\left(\mathcal{J}_C - \frac{\mathcal{I}}{4}e^{2\mathcal{J}/3}\right)^2}{1 + 8\alpha e^{-4\mathcal{J}/3}\left(\mathcal{J}_C - \frac{\mathcal{I}}{4}e^{2\mathcal{J}/3}\right)^2}} \ . \tag{3.28}$$

Therefore, the full scalar Lagrangian becomes

$$\mathcal{L}_C = -\frac{1}{2}\mathcal{J}_{CC}(\partial_a C)^2 - V, \tag{3.29}$$

$$V = -\frac{e^{4\mathcal{J}/3}}{8\alpha}\left(1 \pm \sqrt{1 + 8\alpha e^{-4\mathcal{J}/3}\left(\mathcal{J}_C - \frac{\mathcal{I}}{4}e^{2\mathcal{J}/3}\right)^2}\right). \tag{3.30}$$

We choose the minus sign in eq. (3.30) because it is the only option consistent with eq. (3.17).

Some comments are in order.

First, the perturbative solution (3.25) allows us to investigate the sign in front of the vector kinetic terns F^2 in our action. In order to avoid ghosts, the sign should be negative,

$$-\frac{1}{4K}\left(\mathcal{J}_C + \mathcal{I}e^{2\mathcal{J}/3} - 2\alpha K^2\mathcal{I}e^{-2\mathcal{J}/3}\right) < 0\,, \tag{3.31}$$

which imposes the restriction on our functions.

Second, we can generalize our action (3.1) even further by adding a constant superpotential w as the additional term

$$S_w = 2[S_0^3 w]_F \ , \tag{3.32}$$

because there is no gauged R-symmetry in our approach. Then the extra bosonic part is

$$\mathcal{L}_w = 3wS_0^2F_0 + \text{h.c.}, \tag{3.33}$$

and the superconformal gauge conditions lead to

$$\mathcal{L}_w = -\frac{9}{2\mathcal{H}}F_0 w + \text{h.c.} \tag{3.34}$$

Hence, the auxiliary fields equations of motion for N and F_0 — see eq. (3.14) — change as

$$N = \sqrt{\frac{-8\mathcal{H}}{3}}\frac{\mathcal{H}_C}{\mathcal{H}_{CC}}F_0 \quad \text{and} \quad F_0 = \frac{9}{4}\bar{w}\frac{\mathcal{H}_{CC}}{\mathcal{H}^2\mathcal{H}_{CC} - \mathcal{H}\mathcal{H}_C^2} \ . \tag{3.35}$$

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Substituting them into the total Lagrangian, we obtain the following correction:

$$\Delta\mathcal{L} = -\frac{81}{8}|w|^2 \frac{\mathcal{H}_{CC}}{\mathcal{H}^3\mathcal{H}_{CC} - \mathcal{H}^2\mathcal{H}_C^2} = 3|w|^2 e^{2\mathcal{J}}\left(1 - \frac{2}{3}\frac{\mathcal{J}_C^2}{\mathcal{J}_{CC}}\right). \tag{3.36}$$

Therefore, the only effect of adding eq. (3.32) on the scalar potential (3.30) is its modification as

$$\begin{aligned} V = &-\frac{e^{4\mathcal{J}/3}}{8\alpha}\left(1 - \sqrt{1 + 8\alpha e^{-4\mathcal{J}/3}\left(\mathcal{J}_C - \frac{\mathcal{I}}{4}e^{2\mathcal{J}/3}\right)^2}\right) \\ &- 3|w|^2 e^{2\mathcal{J}}\left(1 - \frac{2}{3}\frac{\mathcal{J}_C^2}{\mathcal{J}_{CC}}\right). \end{aligned} \tag{3.37}$$

4 Starobinsky inflation and SUSY breaking

In this section we apply our model, introduced in the previous section 3, to a description of cosmological inflation in supergravity, without using chiral matter supermultiplets. To be specific, we are looking for viable supergravity-based extensions of Starobinsky inflation,[7] with a supersymmetry breaking vacuum after inflation.

We take the following parameterization of the functions α and $\mathcal{I}$:

$$\alpha(C) = \frac{e^{4\mathcal{J}/3M_{\mathrm{P}}^2}}{8M_{\mathrm{BI}}^4}, \quad \mathcal{I}(C) = \xi e^{-2\mathcal{J}/3M_{\mathrm{P}}^2}, \tag{4.1}$$

where ξ is also C-dependent in general, and we have introduced the mass scale M_{BI} of the DBI structure [37], in addition to the (reduced) Planck scale M_{P}. Furthermore, we restore the gauge coupling constant g via the substitution $\mathcal{J}_C \to g\mathcal{J}_C$ in eq. (3.30).

The Starobinsky-type inflation is known to be described by the following function [12]:

$$\mathcal{J} = -\frac{3}{2}M_{\mathrm{P}}^2 \log\left(-\frac{C}{M_{\mathrm{P}}}e^{C/M_{\mathrm{P}}}\right) . \tag{4.2}$$

Substituting eqs. (4.1) and (4.2) into the Lagrangian (3.29), we find

$$\mathcal{L}_C = -\frac{3M_{\mathrm{P}}^2}{4C^2}(\partial_a C)^2 - V, \tag{4.3}$$

$$V = M_{\mathrm{BI}}^4\left(\sqrt{1 + \frac{9g^2M_{\mathrm{P}}^4}{4M_{\mathrm{BI}}^4}\left(1 + \frac{M_{\mathrm{P}}}{C} + \frac{\xi}{6gM_{\mathrm{P}}^2}\right)^2} - 1\right) . \tag{4.4}$$

Hence, in terms of the canonically normalized scalar φ related to C as $C/M_{\mathrm{P}} = -e^{\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\mathrm{P}}}}$, the scalar Lagrangian is given by

$$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_a\varphi)^2 - V\,, \tag{4.5}$$

$$V(\varphi) = M_{\mathrm{BI}}^4\left(\sqrt{1 + \frac{9g^2M_{\mathrm{P}}^4}{4M_{\mathrm{BI}}^4}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\mathrm{P}}}} + \frac{\xi}{6gM_{\mathrm{P}}^2}\right)^2} - 1\right) . \tag{4.6}$$

[7] We do not provide details of Starobinsky inflation, see e.g., ref. [30].

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We find convenient to define the dimensionless parameters as

$$\frac{M_{\rm P}}{M_{\rm BI}} \equiv a, \quad \frac{\xi}{M_{\rm P}^2} \equiv b\ . \tag{4.7}$$

It is reasonable to assume that the DBI scale $M_{\rm BI}$ is between the GUT scale and Planck scale, so that a belongs to the interval $[1,100]$. Then the scalar potential takes the form

$$V(\varphi) = \frac{M_{\rm P}^4}{a^4}\left(\sqrt{1+\frac{9}{4}g^2a^4\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b\right)^2}-1\right), \tag{4.8}$$

where the coupling constants a and b characterize the DBI and FI corrections, respectively. In the case of $g^2a^4 \ll 1$ and $b/g \ll 1$ we recover the original Starobinsky model. Therefore, eq. (4.8) can be considered as the motivated two-parametric extension of Starobinsky inflationary potential in supergravity, by using a single (massive) vector multiplet only.

If a constant superpotential is also taken into account, the corresponding scalar potential (3.37) with the function (4.2) reads

$$\begin{aligned}V(\varphi) =&\frac{M_{\rm P}^4}{a^4}\left(\sqrt{1+\frac{9}{4}g^2a^4\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b\right)^2}-1\right)\\ &-3\frac{|w|^2}{M_{\rm P}^2}\exp\left[-3\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}+3e^{\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\right]\left(1-\frac{2}{3}e^{\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\left(1-e^{\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\right)\right)\ .\end{aligned} \tag{4.9}$$

Here we observe the factor with the double exponent of the canonical scalar in the second line, indicating the dangerous "instability" of the (Starobinsky) inflation governed by the term in the first line. This phenomenon was observed in ref. [38] in the similar context, though with a chiral (Polonyi) matter multiplet coupled to the massive vector multiplet. Because of similar "instability" (see appendix B for details), we dispose the scalar potential (4.9) and take $w = 0$ in what follows. It is worth noticing, however, that the factor with the double exponent in (4.9) may be eliminated by changing the $\mathcal{J}$-function, as in ref. [38].

4.1 Constant FI term

Let us study the case of a constant coefficient at the FI term, $b = const$. The Starobinsky-type inflationary model can be realized when $(1+\frac{1}{6g}b) > 0$.[8] The first derivative of the scalar potential $V(\varphi)$ is given by

$$V' = \frac{9}{4}\sqrt{\frac{2}{3}}g^2M_{\rm P}^3e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\frac{1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b}{\sqrt{1+\frac{9}{4}a^4g^2\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b\right)^2}}\ , \tag{4.10}$$

[8]When $(1+\frac{1}{6g}b) < 0$, the scalar potential does not have a minimum.

where the prime denotes the derivative with respect to φ. Demanding $V' = 0$ leads to the condition

$$1 - e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}} + \frac{1}{6g}b = 0. \tag{4.11}$$

As is clear from eq. (4.8), this condition results in a Minkowski vacuum at $\varphi_0/M_{\rm P} = -\sqrt{\frac{3}{2}}\log(1+\frac{1}{6}b)$. However, in this vacuum, we have

$$\langle D\rangle = \frac{3}{2}gM_{\rm P}^2\sqrt{\frac{\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi_0}{M_{\rm P}}}+\frac{1}{6g}b\right)^2}{1+\frac{9}{4}a^4g^2\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi_0}{M_{\rm P}}}+\frac{1}{6g}b\right)}} = 0\ , \tag{4.12}$$

and therefore, SUSY is unbroken. This observation forces us to consider a *field-dependent* FI "coefficient" $b = b(C)$ or $b = b(\varphi)$.

4.2 Field-dependent FI term

When b is a function of $\varphi/M_{\rm P}$, the critical points of the scalar potential obey the equation

$$V' = \frac{9}{4}\sqrt{\frac{2}{3}}g^2M_{\rm P}^3\left(e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{M_{\rm P}}{2\sqrt{6}g}b'\right)\frac{1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b}{\sqrt{1+\frac{9}{4}a^4g^2\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}+\frac{1}{6g}b\right)^2}} = 0. \tag{4.13}$$

In this case, we have two equations

$$e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}} + \frac{M_{\rm P}}{2\sqrt{6}g}b' = 0, \tag{4.14}$$

$$1 - e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}} + \frac{1}{6g}b = 0. \tag{4.15}$$

Note that for eq. (4.14) to possess a solution, the condition $b' < 0$ is required. Moreover, even when eq. (4.14) has a solution, it cannot be a true minimum because the Starobinsky potential (4.8) is non-negative, and the solution of eq. (4.15) always leads to a Minkowski vacuum. Hence, we consider the case when eq. (4.15) does *not* have solutions.

4.2.1 Solvable case

As a simple example, where we can explicitly solve eqs. (4.14) and (4.15), let us assume that the field-dependent FI term is given by the specific function[9]

$$b/g = ke^{-2\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}. \tag{4.16}$$

In this case, $k > 0$ is required to satisfy $(1 + \frac{1}{6g}b) > 0$ and $b' < 0$, which is adopted below.

[9]The same function was introduced in the similar context in subsection 3.6 of [39].

A solution to eq. (4.14) is given by

$$\varphi_*/M_{\rm P} = -\sqrt{\frac{3}{2}}\log\left(\frac{3}{k}\right). \tag{4.17}$$

On the other hand, we have two formal solutions to eq. (4.15),

$$\varphi_\pm/M_{\rm P} = -\sqrt{\frac{3}{2}}\log\left(\frac{3}{k}\right) - \sqrt{\frac{3}{2}}\log\left(1 \pm \sqrt{1-\frac{2}{3}k}\right). \tag{4.18}$$

Hence, when $\frac{3}{2} < k$, eq. (4.15) does not have a (real) solution. Indeed, one can show that φ_* is a de Sitter minimum because of the following relations valid for $\frac{3}{2} < k$:

$$V|_{\varphi=\varphi_*} = \frac{M_{\rm P}^4}{a^4}\left(\sqrt{1+\frac{9}{4}a^4g^2\left(1-\frac{3}{2k}\right)^2} - 1\right) > 0, \tag{4.19}$$

$$V''|_{\varphi=\varphi_*} = \frac{9g^2M_{\rm P}^2}{2k}\frac{1-\frac{3}{2k}}{\sqrt{1+\frac{9}{4}a^4g^2\left(1-\frac{3}{2k}\right)^2}} > 0, \tag{4.20}$$

$$\lim_{\varphi\to\infty} V = \frac{M_{\rm P}^4}{a^4}\left(\sqrt{1+\frac{9}{4}a^4g^2} - 1\right), \quad \lim_{\varphi\to-\infty} V = \infty. \tag{4.21}$$

At $\varphi = \varphi_*$, the vacuum expectation value of D is evaluated as

$$\langle D\rangle = -\frac{3gM_{\rm P}^2}{2}\frac{1-\frac{3}{2k}}{\sqrt{1+\frac{9}{4}a^4g^2\left(1-\frac{3}{2k}\right)^2}} \neq 0. \tag{4.22}$$

Therefore, we can conclude that the minimum (vacuum) is a SUSY breaking one. As can be seen from eq. (4.19), we need $k \sim \frac{3}{2}$ to realize a tiny cosmological constant. Expanding eq. (4.19) with respect to $\delta > 0$, where $k = \frac{3}{2} + \delta$, we find the following expression:

$$V|_{\varphi=\varphi_*} = \frac{M_{\rm P}^4}{2}g^2\delta^2 + \mathcal{O}(\delta^3). \tag{4.23}$$

Thus we must tune our parameter δ in order to adjust the vacuum (dark) energy.

Two comments are in order.

First, we should check the *no-ghost* condition, eq. (3.31). During inflation, it reads

$$-\frac{3}{2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\right) + \left(1+\frac{9}{4}a^4g^2\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\right)^2\right)\left(1+\frac{3}{4}ke^{-2\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}\right) < 0\ . \tag{4.24}$$

Roughly speaking, the restriction $a^4g^2 < \frac{2}{9}$ should be imposed, when we neglect the exponential factor $e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}}$ that is truly small during inflation.

Second, let us check about *inflection* points of the scalar potential. In single-field inflationary models, an inflection point can lead to a peak in the power spectrum, that may be associated with creation of Primordial Black Holes (PBHs). In turn, the PBHs may be a (non-particle) component of dark matter [40].

The second derivative of our scalar potential is

$$V'' = -\frac{3}{2}g^2 M_{\rm P}^2 \left(1+\frac{9}{4}a^4g^2\left(1-x+\frac{k}{6}x^2\right)^2\right)^{-3/2} x f(x), \tag{4.25}$$

$$\begin{aligned} f(x) \equiv & \left(1-\frac{2k}{3}x\right)\left(1-x+\frac{k}{6}x^2\right)\left(1+\frac{9}{4}a^4g^2\left(1-x+\frac{k}{6}x^2\right)^2\right) \\ & -x\left(1-\frac{k}{3}x\right)^2, \end{aligned} \tag{4.26}$$

where

$$x \equiv e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm P}}} > 0. \tag{4.27}$$

Hence, we are interested in solutions to $f(x)=0$. In particular, when $a,k\to 0$ (Starobinsky case), we have one such point at

$$x=\frac{1}{2} \quad \text{or} \quad \varphi = \sqrt{\frac{3}{2}}\log 2\ . \tag{4.28}$$

For general a and k, solving the equation $f(x)=0$ is difficult, and numerical analysis may be required. However, the latter can be essentially avoided because we already assumed that a takes its values in the interval $[1,100]$, and we derived that $k\sim\frac{3}{2}$. As is demonstrated in subsection 4.3 below, the value of g is determined to be $\sim 10^{-5}$ from CMB observations. In this case, $f(x)$ becomes

$$f(x) = (1-x)\left(1-x+\frac{x^2}{4}\right)\left(1+\frac{9a^4g^2}{4}\left(1-x+\frac{x^2}{4}\right)\right) - x\left(1-\frac{x}{2}\right)^2, \tag{4.29}$$

where a^4g^2 is between 10^{-10} and 10^{-2}. Then we find two solutions to $f(x)=0$ as

$$x=1/2, \quad 2 \quad \text{or} \quad \varphi=\sqrt{\frac{3}{2}}\log 2\,, \quad -\sqrt{\frac{3}{2}}\log 2\,, \tag{4.30}$$

respectively, by neglecting the term with the factor (a^4g^2).[10] The solution $x=2$ corresponds to the vacuum, according to eq. (4.17). As regards another solution $x=1/2$, the first derivative of the potential,

$$V' = \frac{9}{4}\sqrt{\frac{2}{3}}g^2M_{\rm P}^3 \frac{x\left(1-\frac{x}{2}\right)\left(1-x+\frac{1}{4}x^2\right)}{\sqrt{1+\frac{9}{4}a^4g^2\left(1-x+\frac{1}{4}x^2\right)^2}}\ , \tag{4.31}$$

turns out to be is non-vanishing and non-negligible at this point. Numerically, we obtain

$$\epsilon = 0.59 \quad \text{for} \quad a^4g^2 \in [10^{-10}-10^{-2}]\,, \tag{4.32}$$

at $x=1/2$, where ϵ is defined in eq. (4.34), while the value of ϵ is not much affected by the value of a^4g^2.

Therefore, we conclude that our potential does not have an inflection point, and this excludes a formation of PBHs in our model.

[10] We also solved eq. (4.29) numerically under the condition of a^4g^2 between 10^{-10} and 10^{-2}, and found that there is no real solution other than eq. (4.30).

4.3 Cosmological parameters

Getting an estimate of the impact of the DBI and FI corrections during inflation on the CMB observables is non-trivial. In this subsection, we briefly consider it in the particular model of subsection 4.2.1.

The relation between the number of e-foldings and inflaton filed φ is given by

$$N \simeq \frac{1}{M_{\rm P}^2}\int_{\varphi_e}^{\varphi_N}\frac{V}{V'}d\varphi \simeq \frac{3}{2}\frac{\sqrt{1+\frac{9}{4}a^4g^2}}{1+\sqrt{1+\frac{9}{4}a^4g^2}}\exp\left(\sqrt{\frac{2}{3}}\frac{\varphi_N}{M_{\rm P}}\right), \tag{4.33}$$

where φ_N and φ_e denote the inflaton field values at the e-foldings number N and the end point of inflation, respectively. To evaluate cosmological parameters, we take the field value of φ_N for $N = 50 \div 60$, which is much larger than φ_e. Having obtained the leading contribution with respect to φ_N on the right-hand-side of eq. (4.33), we can find φ_N as a function of N.

The slow-roll parameters are defined by the standard equations:

$$\epsilon \equiv \frac{M_{\rm P}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta \equiv \frac{M_{\rm P}^2 V''}{V} \quad . \tag{4.34}$$

Using eq. (4.33), the values of the slow-roll parameters at $\varphi = \varphi_N$ can be rewritten as the functions of N as follows:

$$\epsilon_* \simeq \frac{3}{4N^2} \quad \text{and} \quad \eta_* \simeq -\frac{1}{N}, \tag{4.35}$$

where the subscript $(*)$ denotes the quantity evaluated at $\varphi = \varphi_N$. Therefore, the standard CMB observables (the spectral index and the tensor-to-scalar ratio) in our case are given by

$$n_s = 1 - 6\epsilon_* + 2\eta_* \simeq 1 - \frac{2}{N}, \tag{4.36}$$

$$r = 16\epsilon_* \simeq \frac{12}{N^2}, \tag{4.37}$$

in the leading order approximation. Hence, they are *not* affected by either of the DBI and FI parameters (a and k). Furthermore, we have confirmed that the running of the spectral index, given by

$$\alpha_{s*} = -2\xi_* + 16\epsilon_*\eta_* - 24\epsilon_*^2 \simeq -\frac{2}{N^2}, \quad \text{where} \quad \xi \equiv M_{\rm P}^4\left(\frac{V'V'''}{V^2}\right), \tag{4.38}$$

is *not* affected too, and has the same value as that in the original Starobinsky model, in the leading order approximation. The dependence upon a and k, however, appears in the subleading orders, whose study is beyond the scope of this investigation.

The coupling constant g is determined by the amplitude of the power spectrum,

$$A_s = \frac{V^3}{12\pi^2 M_{\rm P}^6 V'^2} \simeq \frac{1}{18\pi^2}\frac{1}{a^4}\left(\sqrt{1+\frac{9}{4}a^4g^2} - 1\right)N^2, \tag{4.39}$$

and it is given by $A_s \sim 2\times 10^{-9}$ by CMB observations. For example, we have

$$(a, N) = (100, 60), \quad \Rightarrow \quad g = 9.39 \times 10^{-6}, \tag{4.40}$$

$$(a, N) = (10, 60), \quad \Rightarrow \quad g = 9.34 \times 10^{-6}. \tag{4.41}$$

5 Conclusion

In this paper we studied the new supergravity model of cosmological inflation with spontaneous SUSY breaking after inflation, beyond the standard supergravity framework, i.e. with the new FI terms that do not require gauging the R-symmetry. These FI terms significantly relax the restrictions imposed on supergravity with the standard FI term and the gauged R-symmetry and, hence, lead to the new avenues for the supergravity model building.

By using the particular FI term, we constructed the explicit and very economical supergravity model of cosmological Starobinsky-type inflation, in terms of a single (massive) vector multiplet with the DBI structure of its kinetic terms, the inflaton and the Goldstino as the superpartners, and the D-type spontaneous SUSY breaking after inflation.

However, the values of the cosmological constant (the dark energy) and the SUSY breaking scale are still tightly related in our model. It may have been expected due to the D-type of SUSY breaking used in our approach. Indeed, by using eqs. (3.28) and (3.30) and defining the deformation parameter $\tilde{\alpha} = 8\alpha e^{-4J/3}$, we find the universal relation

$$V = \frac{1}{\tilde{\alpha}}\left[\frac{1}{\sqrt{1-\tilde{\alpha}D^2}} - 1\right] = \frac{1}{2}D^2 + \ldots\,, \tag{5.1}$$

so that a tiny value of the cosmological constant implies a very small value of the SUSY breaking scale. This may be resolved by combining the D-type SUSY breaking with the F-type SUSY breaking. However, this requires a separate investigation.

Acknowledgments

HA is supported in part by a JSPS (kakenhi) Grant under No. 16K05330. YA and SVK are supported in part by the Competitiveness Enhancement Program of Tomsk Polytechnic University in Russia. SA is supported in part by a Waseda University Grant for Special Research Projects (Project number: 2018S-141). SVK is also supported in part by a Grant-in-Aid of the Japanese Society for Promotion of Science (JSPS) under No. 26400252, and the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

A FI terms in curved superspace

In this appendix we formulate the new FI terms in curved superspace of supergravity, by using the standard notation and conventions of [41].

A.1 FI term I

When employing the original (new) FI term proposed in [19], whose coefficient ξ is generalized to a function $\mathcal{I}(V)$, the superspace Lagrangian reads

$$\mathcal{L}_{\mathrm{I}} = \mathcal{L}_{\mathrm{mBI}} + 2\int d^4\theta E \frac{W^2\bar{W}^2}{\mathcal{D}^2W^2\bar{\mathcal{D}}^2\bar{W}^2}\mathcal{I}\,, \tag{A.1}$$

where the massive BI Lagrangian is given by

$$
\begin{aligned}
\mathcal{L}_{\rm mBI} = & -3\int d^4\theta E e^{-2\mathcal{J}(V)/3} + \left(\frac{1}{4}\int d^2\Theta 2\mathcal{E} W^2 + \text{h.c.}\right) \\
& + \frac{1}{4}\int d^4\theta E \frac{W^2\bar{W}^2}{1+8\alpha(\omega+\bar{\omega})+\sqrt{1+8\alpha(\omega+\bar{\omega})+16\alpha^2(\omega-\bar{\omega})^2}}\ ,
\end{aligned} \tag{A.2}
$$

and $\omega = \frac{1}{8}\mathcal{D}^2 W^2$. The $\mathcal{J}(C) = \mathcal{J}(V)|$ is arbitrary real function of the real scalar C that is the lowest component of the massive vector multiplet. The α is the BI parameter, and the vector multiplet coupling is set to one for simplicity.

After eliminating the auxiliary fields and Weyl rescaling to Einstein frame, $e \to e^{4\mathcal{J}/3}e$ and $g^{mn} \to e^{-2\mathcal{J}/3}g^{mn}$, we derive the bosonic part of the Lagrangian as follows:

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\rm I} = & \frac{1}{2}R - \frac{1}{2}\mathcal{J}''\partial_a C\partial^a C - \frac{1}{2}\mathcal{J}'' B_a B^a \\
& + \frac{e^{4\mathcal{J}/3}}{8\alpha}\left[1 - \sqrt{1+8\alpha Z^2}\sqrt{1+4\alpha F^2 e^{-4\mathcal{J}/3} + 4\alpha^2 (F\tilde{F})^2}\right],
\end{aligned} \tag{A.3}
$$

where

$$
Z \equiv \frac{\mathcal{I}}{4} - \mathcal{J}' e^{-2\mathcal{J}/3}\,, \tag{A.4}
$$

$\tilde{F}_{ab} \equiv -\frac{i}{2}\epsilon_{abcd}F^{cd}$, B_a is the vector field whose field strength is F_{ab}, and the primes denote the derivatives with respect to C. The absence of ghosts requires $\mathcal{J}'' > 0$.

The auxiliary field D is eliminated via its equation of motion as

$$
D = \frac{Z}{\sqrt{1-8\alpha Z^2}}\sqrt{1+4\alpha F^2 e^{-4\mathcal{J}/3} + 4\alpha^2 (F\tilde{F})^2}\,, \tag{A.5}
$$

and it must have the non-vanishing VEV, $\langle D \rangle \neq 0$ or $\langle Z \rangle \neq 0$, that spontaneously breaks SUSY. The scalar potential in this case is given by

$$
\mathcal{V} = \frac{e^{4\mathcal{J}/3}}{8\alpha}\left(\sqrt{1+8\alpha Z^2} - 1\right)\ . \tag{A.6}
$$

A.2 FI term II

In the main text of our paper we employ the Lagrangian with the different FI term [20, 23]

$$
\mathcal{L}_{\rm II} \supset 2\int d^4\theta E \frac{W^2\bar{W}^2}{(\mathcal{D}W)^3}\mathcal{I}\,, \tag{A.7}
$$

where, similarly to the previous case, $\mathcal{I} = \mathcal{I}(V)$. Then the D-term Lagrangian in Jordan frame reads

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\rm II}(D) = & -\frac{\mathcal{I}}{16}\left[4D - \frac{4F^2}{D} + \frac{F^4 - (F\tilde{F})^2}{D^3}\right] + e^{-2\mathcal{J}/3}\mathcal{J}' D \\
& + \frac{1}{8\alpha}\left(1 - \sqrt{1+4\alpha(F^2 - 2D^2) + 4\alpha^2(F\tilde{F})^2}\right)\ .
\end{aligned} \tag{A.8}
$$

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An exact solution to the (algebraic) D-equation of motion amounts to finding a zero of the 5th-degree polynomial. So, we solve it perturbatively, by ignoring the terms of $\mathcal{O}(F^4)$. Then the Lagrangian (A.8) takes the form

$$e^{-1}\mathcal{L}_{\rm II}(D) = -ZD + \frac{\mathcal{I}}{4D}F^2 + \frac{1}{8\alpha}\left(1 - \sqrt{1-8\alpha D^2} - \frac{2\alpha F^2}{\sqrt{1-8\alpha D^2}}\right) + \mathcal{O}(F^4)\ . \quad \text{(A.9)}$$

We search for a solution in the form

$$D = D_0 + D_1 F^2 + \mathcal{O}(F^4)\,, \quad \text{(A.10)}$$

and find

$$D_0 = \frac{Z}{\sqrt{1+8\alpha Z^2}} \quad \text{and} \quad D_1 = \frac{\sqrt{1+8\alpha Z^2}(\frac{\mathcal{I}}{4}e^{-2\mathcal{J}/3} + 2\alpha Z^3)}{Z^2 + 4\alpha Z^4}\ . \quad \text{(A.11)}$$

Plugging this solution into eq. (A.9) and Weyl-rescaling result in the full bosonic Lagrangian

$$e^{-1}\mathcal{L}_{\rm II} = \frac{1}{2}R - \frac{1}{2}\mathcal{J}''\partial_a C\partial^a C - \frac{1}{2}\mathcal{J}'' B_a B^a + \frac{1}{4}\sqrt{1+8\alpha Z^2}\left(\frac{\mathcal{I}}{Z} - 1\right)F^2 + \mathcal{O}(F^4) - \mathcal{V}\,, \quad \text{(A.12)}$$

where the scalar potential is

$$\mathcal{V} = \frac{e^{4\mathcal{J}/3}}{8\alpha}\left(\sqrt{1+8\alpha Z^2} - 1\right)\,, \quad \text{(A.13)}$$

i.e. *the same* as in the case I. This is the reason why we do not emphasize the differences between the two FI terms in the main text of our paper, because they lead to the same scalar potentials (but the different theories).

When using eq. (A.12), we get the no-ghosts condition for F_{ab} as

$$\frac{\mathcal{I}}{Z} \equiv \frac{4\mathcal{I}}{\mathcal{I} - 4\mathcal{J}' e^{-2\mathcal{J}/3}} < 1\ . \quad \text{(A.14)}$$

After the field definitions

$$\mathcal{I} = \xi e^{-2\mathcal{J}/3}\,, \quad \mathcal{J} = -\frac{3}{2}\log(-Ce^C)\,, \quad C = -e^{-\sqrt{2/3}\varphi}\,, \quad \text{(A.15)}$$

the condition (A.14) takes the form

$$\frac{4\xi}{\xi - 4\mathcal{J}'} = \frac{4\xi}{\xi + 6 - 6e^{\sqrt{2/3}\varphi}} < 1\ . \quad \text{(A.16)}$$

B Constant superpotential

Let us investigate the impact of a constant superpotential in eq. (4.9) on inflation and vacuum stability in our model defined in subsection 4.2.1. The scalar potential (4.9) has two parts,

$$V = V_D + V_F\,, \quad \text{(B.1)}$$

where V_D is given by eq. (4.8) and V_F stands for the contribution of the constant superpotential. The first and second derivatives of V_D are given by eq. (4.13) subject to eqs. (4.16) and (4.25). The derivatives of V_F are given by

$$V_F' = -6\sqrt{\frac{2}{3}}\frac{|w|^2}{M_{\rm P}^2}e^{3/x}\left(1-\frac{4}{3}x+\frac{13}{6}x^2-\frac{3}{2}x^3\right), \tag{B.2}$$

$$V_F'' = 16\frac{|w|^2}{M_{\rm P}^2}e^{3/x}\left(-\frac{3}{4x}+1-\frac{47}{24}x+\frac{53}{24}x^2-\frac{9}{8}x^3\right), \tag{B.3}$$

where the field x is defined by eq. (4.27).

B.1 During inflation

During (Starobinsky) inflation the value of $\sqrt{\frac{2}{3}}\varphi/M_{\rm P}$ varies between 5.5 and 0.5 [30], so that $x < e^{-0.5}$. When assuming $x \ll e^{-0.5}$, the leading contributions to V_D and V_F can be estimated as

$$V_D \sim \frac{M_{\rm P}^4}{a^4}\left(\sqrt{1+\frac{9}{4}a^4g^2}-1\right), \tag{B.4}$$

$$V_F \sim -2\frac{|w|^2}{M_{\rm P}^2}xe^{3/x}. \tag{B.5}$$

Hence, for the large inflaton field values the V_F becomes dominant, and the derivatives of the full potential can be approximated as

$$V' \sim -6\sqrt{\frac{2}{3}}\frac{|w|^2}{M_{\rm P}^3}e^{3/x} \quad \text{and} \quad V'' \sim -12\frac{|w|^2}{M_{\rm P}^4}\frac{1}{x}e^{3/x}. \tag{B.6}$$

Therefore, the slow-roll parameters,

$$\epsilon \sim \frac{3}{x^2} \quad \text{and} \quad \eta \sim \frac{6}{x^2}, \tag{B.7}$$

become large during the inflation, $\epsilon > 8$ and $\eta > 16$, and the slow-roll conditions are violated. This instability appears for any non-vanishing value of w.

B.2 After inflation

Let us examine stability of the vacuum after (Starobinsky) inflation in our model. With a non-vanishing V_F, the φ_* value deviates from that of eq. (4.17). We assume that a new solution to $V' = 0$ takes the form $\varphi_* + \delta\varphi_*$ and then find the $\delta\varphi_*$ by solving the equation $V' = 0$ in the linearized approximation. Also, for simplicity, we take $k = 3/2$. We find

$$V' \sim e^{3/2}\frac{|w|^2}{M_{\rm P}^3}\left(10\sqrt{6}-\frac{166}{3}\delta\varphi_*\right)+\mathcal{O}(\delta\varphi_*^2) = 0\,, \quad \text{so that} \quad \delta\varphi_* \sim \sqrt{\frac{3}{2}}\frac{30}{83}M_{\rm P} \sim 0.4M_{\rm P}. \tag{B.8}$$

Inserting this solution into V'' yields

$$V''|_{\varphi=\varphi_*+\delta\varphi_*} \sim g^2M_{\rm P}^2 - \frac{|w|^2}{M_{\rm P}^4}\times\mathcal{O}(10^2). \tag{B.9}$$

Therefore, we find that the vacuum instability appears only for the sufficiently large w when $|w| \geq \frac{1}{10}gM_{\rm P}^3$.

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