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FILM FLOWING OF NONLINEAR VISCOELASTIC LIQUID ON LATERAL SURFACE OF CIRCULAR CYLINDER

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On the basis of V.N. Pokrovskiy's rheological model describing behaviour of filled polymer system the problem of film sintering of nonlinear viscoelastic liquid on lateral surface of vertical cylinder is considered. Mathematical problem statement, methods of its solution is presented, calculation results are discussed, factors influencing the film sintering process are analysed.

Introduction

Film flows of rheology of complex liquid occur in many branches of modern industry – chemical technology, power engineering, petroleum derivatives transportation and others. Methods of dipping and immersion are used for coating components and products with different covers and lubrications. At quick emptying of bulbs and reservoirs of different functions, liquid films, flowing down on their walls remain. Therefore, it is important to determine such films characteristics (thickness, consumption, flow time, covering, remained on a wall) subject to rheological liquid characteristics [1].

In most cases, real materials, being in viscoelastic state, possess pronounced non-Newtonian properties, such as nonlinear viscosity, plasticity and viscoelasticity. Wrong accounting of rheological factors influence on films motion at various technological devices maintenance results in significant miscalculations.

At theoretical description of technological processes of pigmented polymers conversion the question about flow local laws, taking into consideration significant rheological material characteristics arises. Existing flow theories are often based on the model of nonlinear viscosity or viscoplastic fluid. Such description is often not too complete and does not include very significant nonlinear effects, discovered experimentally and connected with presence of relaxation processes in liquid, such as normal stresses appearance at shearing and residual stress occurrence at flow stop, stream swelling at outflow from formative cap and liquid winding on rotating rod (Weisenberg-effect) and others [2–5].

1. Statement of problem

There are no universal determining equations for describing rheology of complex liquid behavior, suitable for all nonlinear materials, on the one hand and, on the other hand, working satisfactorily in wide range of rates of shear. Nonlinear effects are determined by material specific character, and it should be taken into account when stating rheological equations of liquid dynamics. One of the rheological models, describing rather well behavior of extended polymer systems on the basis of linear polymers, is the model, introduced by V.N. Pokrovskiy [6].

According to this model, the system of linear polymers motion equations, except general stress equations of motion and continuity equation, contains system of determining equations, consisting of deviator stress tensor definition

$$\sigma_{ij} = 3\frac{\eta_0}{\tau_0}\left(\xi_{ij} - \frac{1}{3}\delta_{ij}\right) \qquad (1)$$

and relaxation equation

$$\frac{d\,\xi_{ij}}{d\,t} = -\frac{1}{\tau}\left(\xi_{ij} - \frac{1}{3}\delta_{ij}\right) + \frac{\partial v_i}{\partial x_k}\xi_{kj} + \frac{\partial v_j}{\partial x_k}\xi_{kj}\,, \qquad (2)$$

where x_i is the Cartesian coordinates, ξ_{ij} is the internal parameter, determining deformation degree of macromolecular balls, t is the time, v_i is the rate, δ_{ij} is the identity tensor, σ_{ij} is the deviator stress tensor, k is the index current value, according to which summation is performed, τ and η is the relaxation time and shear viscosity, which are supposed to be dependent on applied voltages, and ratio [6] is performed

$$\frac{\eta}{\eta_0} = \frac{\tau}{\tau_0} = f(D), \;\; D = \sigma_{ii} + 3p, \qquad (3)$$

where τ_0 and η_0 is the initial viscosity and relaxation time at D=0, p is the pressure. Rheological properties of fluid in this model are determined by parameters τ_0 and η_0 values and also, a certain steadily decreasing function $f(D)$, which is determined by experimental data and for linear polymers it may be approximated, for example, by function [6]

$$f(D) = 1/(1 + kD)^n, \qquad (4)$$

where k is the coefficient, changing in the range 0,1...0,2; n is the index of nonlinearity.

Let us consider a problem of draining of nonlinear viscoelastic fluid film, satisfying the model of V.N. Pokrovskiy, on lateral surface of vertically installed straight circular cylinder (fig. 1).

We suppose that film flow is considered to be determined, laminar and waveless. Lateral surface of cylinder is retained at constant temperature t=t_0, and environment temperature is t=t_e. In this case, sprinkled wall curvature should be taken into account. Obviously, capillary forces, stipulated by crosscut curvature, are small for major radius cylinders. Van Rossum [7] ascertained that if Gouger number, that is determined as

$$\mathrm{Go} = R_c(\rho\, g/(2\sigma))^{\frac{1}{2}} \qquad (5)$$

larger than 1,8, capillary forces may be neglected in calculations.

In ratio (5) ρ is the fluid density; g is the gravitation acceleration; σ is the surface tension coefficient.

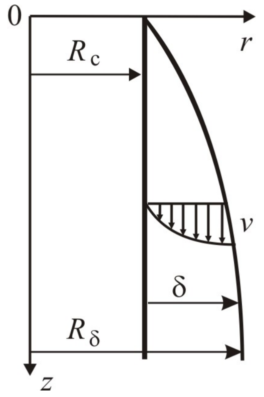

Fig. 1. *Geometric scheme of film flow: r , z – cylindrical coordinates; R_c – cylinder radius; δ – thickness of flowing fluid layer; $R_\delta=R_c+\delta$ – distance to free surface*

It follows from general considerations that at large values of generalized Prandtl number, typical for all rheology of complex liquid, near sprinkled surface efficient temperature layer should be formed, where the main change of temperature field occurs. In this layer the temperature lengthwise the film changes much slower, than in its transverse direction. Therefore, when solving the system of equations, describing the process of film flowing in the conditions of heat exchanging, let us use the hypothesis of quasi-stationarity, i.e. let us consider that every instant temperature distribution at the moment conforms to its own rates stationary distribution. Such supposition allows us to solve hydrodynamic and heat problems separately.

Thus, subject to made suppositions, for solving the formulated problem it is necessary to solve the system of equations, written in cylindrical coordinate system (z,r,φ), and consisting of motion equation

$$\frac{1}{r}\cdot\frac{d}{dr}(r\tau_{zr})+\rho g=0, \qquad (6)$$

determining equations, obtained on basis of V.N. Pokrovskiy's model eq. (1–4) and connecting components of deviator stress tensor with velocity gradient, which, in this case, take the form:

$$\sigma_{zr}=\eta_0\Big/(1+k\tau_0/\eta_0\,\sigma_{zz})^n\,\frac{\partial v_z}{\partial r}, \qquad (7)$$

$$\sigma_{zz}=2\tau_0\sigma_{zr}\Big/(1+k\tau_0/\eta_0\,\sigma_{zz})^n\,\frac{\partial v_z}{\partial r}, \qquad (8)$$

and energy equation $v_z\dfrac{\partial t}{\partial z}=a\left(\dfrac{\partial^2 t}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial t}{\partial r}\right)$, (9)

where a is the coefficient of fluid temperature conductivity.

Equations (6–9) are solved provided that adhesion conditions on cylinder lateral surface are fulfilled and wall temperature is constant

$$\left.\begin{array}{l} v_z=0\\ t=t_0\end{array}\right\}\text{at } r=R_c, \qquad (10)$$

and on a free surface of the film ($r=R_\delta$) conditions of shearing stress σ_{zr} absence and heat flow q, determined by Fourier law are given

$$\left.\begin{array}{l} \sigma_{zr}=0,\\ q=-\lambda\dfrac{\partial t}{\partial r}=0\end{array}\right\}\text{at } r=R_\delta. \qquad (11)$$

Choosing cylinder radius – $r=R_c$, as length scale, medium-consumed flowing rate – $v=U$ as rate scale and difference between cylinder wall temperature t_0 and initial fluid temperature $t_ж$ as temperature scale, let us write the initial equation system and boundary conditions in dimensionless form.

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr})+\mathrm{Re}/Fr=0, \qquad (12)$$

$$\sigma_{zr}=1/(1+k\mathrm{We}\sigma_{zz})^n\,\frac{\partial v_z}{\partial r}, \qquad (13)$$

$$\sigma_{zz}=2We\cdot\sigma_{zr}/(1+kWe\sigma_{zz})^n\,\frac{\partial v_z}{\partial r}, \qquad (14)$$

$$v_z\frac{\partial\theta}{\partial z}=\frac{1}{\mathrm{Pe}}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right), \qquad (15)$$

$$v_z=0,\quad \theta=1\ \text{at } r=1, \qquad (16)$$

$$\sigma_{zr}=0,\quad \frac{\partial\theta}{\partial r}=0\ \text{at } r=1+\Delta, \qquad (17)$$

where $\theta=\dfrac{t-t_ж}{t_0-t_ж}$ is the dimensionless temperature; $\Delta=\delta/R_c$ is the dimensionless film thickness; $\mathrm{Re}=\rho UR_c/\eta_0$ is Reynolds number; $\mathrm{Fr}=U^2/(gR_c)$ is Froude number; $\mathrm{We}=\tau_0U/R_c$ is Weissenberg number; $\mathrm{Pe}=UR_c/a$ is Peclet number.

Reynolds number to Froude number ratio, entering to (12), is an important similarity parameter from hydrodynamic point of view which is denoted by W=Re/Fr [8].

Let us find the solution of hydrodynamic problem.

It is not difficult to show that using equations (12–14) and the first from boundary conditions (17), we may find the expression for shearing stress

$$\sigma_{zr}=\frac{\mathrm{W}}{2r}[(1+\Delta)^2-r^2] \qquad (18)$$

and obtain connection between velocity gradient and stress tensor component σ_{zr} in form of

$$\frac{\partial v_z}{\partial r}=\sigma_{zr}(1+2k\mathrm{We}^2\sigma_{zr}^2)^n. \qquad (19)$$

Substituting expression (18) into (19) we obtain nonlinear differential equation of the first order for determining velocity profile by film thickness. Since v_z is a function only of r, partial derivative $\dfrac{\partial v_z}{\partial r}$ may be exchanged to the total one.

$$\frac{dv_z}{dr}=\frac{\mathrm{W}}{2r}[(1+\Delta)^2-r^2]\times$$
$$\times\left(1+\frac{k}{2r^2}\mathrm{We}^2\mathrm{W}^2[(1+\Delta)^2-r^2]^2\right)^n. \qquad (20)$$

Owing to nonlinearity of differential equation (20), it is not possible to obtain its analytical solution. Therefore, this equation is solved numerically using the first from boundary conditions (16). Energy equation, written in a form of (15), is also solved numerically using velocity field, obtained from equation (20) solution.

2. Results of calculation

The carried out numerical experiments allowed us to find out flow parameters and rheological fluid characteristics influence on velocity and temperatures profiles, as well as to determine their influence on flowing film thickness Δ at the given values of volume flow. To determine magnitude Δ iterative process was plotted with the use of ratio

$$Q = 2\pi \int_{1}^{1+\Delta} r v_z(r)\, dr.$$

Dependence of flowing layer thickness of vicoelastic fluid on Weisenberg number at different values of W parameter is presented in fig. 2.

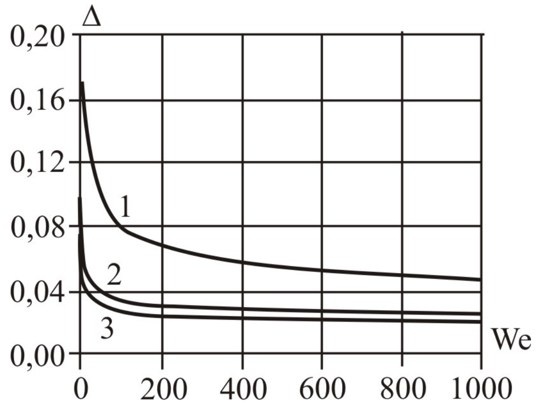

Fig. 2. *Dependence of layer thickness on We number at different values of W: 1) 5; 2) 50; 3) 100*

It is seen from presented figure that film thickness decreases both when We number rises and parameter W increases. In this case, film thickness tends to a certain stationary value while We increasing.

Dependence of maximum speed value in the same range of parameters We and W changing is presented in fig. 3.

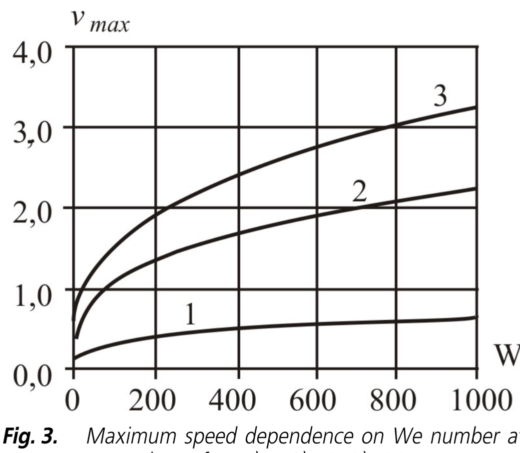

Fig. 3. *Maximum speed dependence on We number at different values of W: 1) 5; 2) 50; 3) 100*

The presented results of calculation show that if film thickness decreases with We and W parameters rise, maximal rate value, which is obtained on a free surface of film increases. Fig. 4 illustrates velocity distribution on film thickness subject to Weisenberg number (We).

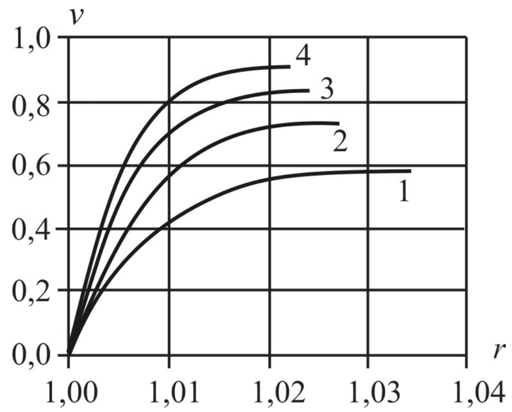

Fig. 4. *Velocity distribution on film thickness subject to We: 1) 50; 2) 100; 3) 150; 4) 200*

As it is seen from the figure, Weisenberg number increasing results in appreciable velocity increase. At the same time the presented figure shows again that draining layer thickness decreases at We increasing. Velocity distribution for visoelastic fluid and its value also depend on geometry or cylinder radius, on which liquid flows. As numeric experiments showed, cylinder radius increase at the condition of retaining of volume flow constant value results in maximum speed falling and draining film thickness decreasing.

Unlike nonlinear viscous and viscoplastic fluid, normal component of deviator stress tensor is not equal to zero at viscoelastic fluid flowing. In this case, stress value does not depend essentially on We number.

$$\sigma_{zz} = 2\mathrm{We}\sigma_{zr}^2.$$

The results of analysis of normal stress by film thickness subject to We number are presented in fig. 5. From the given figure it follows that normal stress distribution by film thickness is of nonlinear character and with We number increase, maximal value of normal stress, which is obtained on cylinder wall, rises.

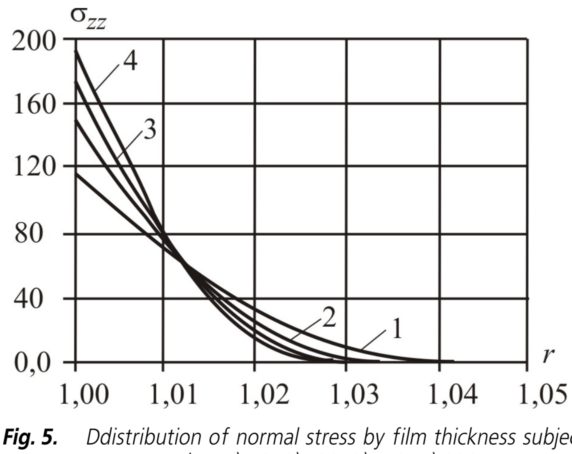

Fig. 5. *Ddistribution of normal stress by film thickness subject to We number: 1) 50; 2) 100; 3) 150; 4) 200*

In this case, since layer thickness of flowing fluid rises when We decreasing, distribution curve of normal stress assumes more flat character.

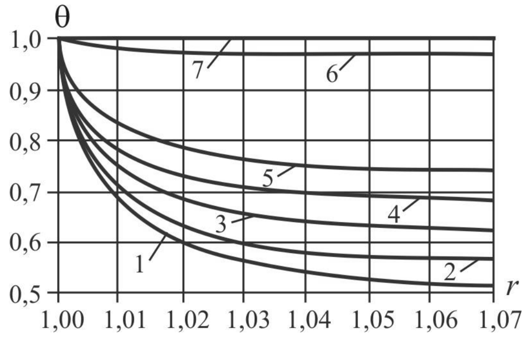

Fig. 6. *Distribution of temperature by film thickness subject to Pe number in sections: z = 0,14 – 1) 0,0001; 2) 5; 3) 10; 4) 20; 5) 25; z = 0,75; 6) 0,0001; 7) 20*

Numerical investigation of temperature field showed that at chosen values of numbers Re=$5\cdot10^{-4}$, Fr=$3\cdot10^{-7}$ and Peclet number (Pe), indicated in fig. 6, Prandl number possesses really large values. From the presented figure we can see that near sprinkled surface temperature layer is formed, where the main change of temperature field occurs. In this case, this layer thickness occupies approximately one-third fraction of the thickness of flowing layer of viscoelastic fluid. In this layer temperature lengthwise the film changes more slowly than in its transverse direction and with Pe number increasing decreas of temperature gradient on film thickness is observed.

3. Conclusion

Numerical solution of the problem on nonlinear viscoelastic liquid flowing on lateral surface of vertical cylinder is obtained. The influence of flow parameter and fluid rheological characteristics on velocity and temperature profiles is showed, and also their influence on draining film thickness at given values of volume flow is determined. It is stated that near sprinkled surface thermal boundary layer is formed in which the main change of temperature field occurs. Thickness of this layer occupies approximately one-third fraction of the thickness of flowing layer of viscoelastic fluid.

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