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Determination of Geometric Parameter of Cycloidal Transmission from Contact Strength Condition for Design of Heavy Loading Mechanisms

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Abstract. In the article, the contact strength condition is considered for transmission with intermediate rolling bodies. The components of contact strength condition equation are expressed through initial parameters of the transmission. The expression is obtained for determination the minimum permissible value of the geometric parameter r_c of transmission with IRBFIR. The expression can be used in preliminary engineering calculations of the hard loaded transmission.

1. Introduction

In modern machines and mechanisms, transmissions with intermediate rolling bodies (IRB) are increasingly in demand. The transmissions are used in the oil and gas industry, thermal power plants, transport systems [1], mining [2]. In addition, IRB transmissions are used in drilling mud agitator drives, mining harvesters and other heavy loaded mechanisms. For highly loaded machines, the most important parameter is the high load capacity of the gears included in them and the ability to hold significant overloads while operability remain unchanged. Transmission with intermediate rolling bodies and free iron ring (IRBFIR) allows such characteristics to be provided to the mining machine.

Transmission with IRBFIR (figure 1) has complex of high technical characteristics provided by multi-pair engagement and sliding friction is reduced in engagement. In the design of mechanical power transmission, the cross section area of the transmission loading parts is the defining measure of strength, including for mining machines. And the cross section dimensions are determined based on the contact strength condition and affect the transmission dimensions itself and vice versa. Therefore, the determination of the transmission with IRBFIR geometric characteristic from the contact strength condition is relevant for the design of heavy loading mechanisms for mining operations and engineering calculations.

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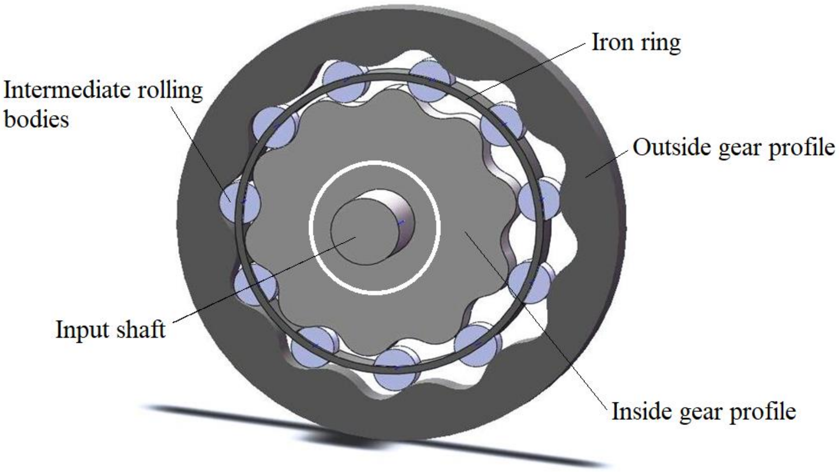

Figure 1. The scheme of transmission with intermediates rolling elements and free iron ring.

Transmissions with IRB have been studied for quite a long time [1, 3], but so far their using is limited. One reason is the lack of the gear sizes valid choice based on the contact strength condition. Scientists in Russia, Belarus [3-7] are engaged in research of transmissions with IRB and calculation of contact stresses. Beyond the borders of the Soviet Union basic information about cycloidal gearing, geometry and force calculation procedure was presented by Lehmann [8], which later was extended in [9-12]. Blanche and Yang [13] investigated the influence of machining tolerance on drive performance indexes and their relationships with drive parameters. Dynamic behavior of cycloidal parts is considered in [14-16]. However, expression was not presented for determination of the transmission with IRBFIR geometric parameters based on the contact strength condition. Therefore, the purpose of the work is to obtain such an expression for determining the radius of the centres of the rolling bodies of the transmission with IRBFIR through the initial parameters. Initial parameters for the transmission are: r_2 – the radius of the making circle, Z_2 - number of bodies of swing, χ – shift factor and r_b –the radius of a rolling body [3].

2. Considering contact strength condition for transmission with IRBFIR

The contact strength condition is written as:

$$(\sigma_H)_{\max} \le [\sigma_H],$$

where $(\sigma_H)_P$ – maximum design contact stress;
$[\sigma_H]$ – permissible contact stress.

Let us consider the equation for determining the contact stresses on the *i*-th rolling body of the transmission with IRBFIR [3]:

$$(\sigma_H)_i = \left(\frac{F_i(\rho_2 \pm \rho_{1i})}{\pi \cdot l_b \cdot \rho_{1i}\rho_{2i}\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}\right)^{1/2}, \quad (1)$$

where F_i – normal force to surfaces of the toothed gear and *i*-th rolling body contact, H;
ρ_{1i}, ρ_2– curvature radii of contacting bodies (toothed gear and rolling body respectively), mm;
l_b – contact length of roller and profile, mm;
E_1, E_2 – elasticity modulus of first and second contacting bodies respectively, MPa;
μ_1, μ_2 – Poisson’s ratios for first and second contacting bodies respectively.

Sign “+” is used for biconvex contact, but sign “-” is used for convexo-concave contact.

To determine the normal force, let us consider the force distribution diagram in engagement of transmission with IRBFIR (figure 2). Figure 2 presents: P – the pitch point; O_1, O_2, O_3 – centers: inside gear, iron ring with rolling bodies and outside gear, respectively; r_1, r_2, r_3 – the centroid radii of the inside gear, iron ring with rolling bodies and outside gear, respectively; r_c – radius of rolling body centers position; φ_2 –angle of the iron ring rotation (together with rolling bodies); e_1 - eccentricity of engagement; e - hollow eccentricity of the transmission; F_i - force in the transmission engagement on i-th rolling body; h_i – the shortest distance from the center of the inside gear to the line of action of the i-th force.

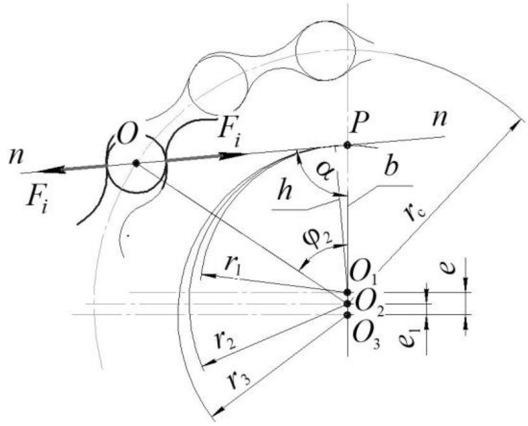

Figure 2. The calculation diagram for determination force into engagement of transmission with intermediates rolling elements and free iron ring

The torque on the inside gear of the transmission with IRBFIR is defined as

$$T_k = \sum F_i \cdot h_i.$$

It is known [8] that the maximum force $F_{\max}$, will be when α=90° (figure 2), then

$$F_{\max} = \frac{T_k \cdot b}{\sum h_i^2}$$

and

$$\frac{F_i}{h_i} = \frac{F_{\max}}{b} \rightarrow F_i = \frac{F_{\max} h_i}{b}.$$

Therefore, for normal force in contact with the i-th rolling body the expression through torque is recorded as

$$F_i = \frac{T_k \cdot b}{\sum h_i^2} \cdot \frac{h_i}{b} = \frac{T_k h_i}{\sum h_i^2}. \tag{2}$$

Assuming that the cycloidal gears and rolling bodies are made of the same material, as is generally the case. Therefore, the contact strength condition taking into account the expressions (1) and (2) will be recorded as:

$$(\sigma_H)_{\max} = \left(\frac{T_k h_i E(\rho_{2i} \pm \rho_{1i})}{2\pi \cdot l_b \cdot \rho_{1i} \rho_{2i} (1-\mu^2) \sum h_i^2}\right)^{1/2} \leq [\sigma_H]. \tag{3}$$

3. Determination of the components of contact strength condition equation

Radii of curvature are determined in [9] through initial parameters of transmission with IRBFIR. Thus, the radius of curvature of the cycloidal profile of the inside gear is expressed through initial parameters as

$$\rho_1 = r_2(1+\chi^2 - 2\chi\cos\varphi)^{1/2} - r_b - Z_2 r_2 i_{21} (1+\chi^2-2\chi\cos\varphi)^{1/2} \cdot \left(\chi Z_1 \cos\varphi + \frac{\chi^2 \cdot \sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1}, \tag{4}$$

where i_{21} is the gear ratio from the rolling bodies to the inside gear and it is defined from the expression:

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$$i_{21} = 1 - \frac{1}{Z_2}.$$

The expressions of sum and product can be represented for the radii of curvature of the transmission with IRBFIR inside cycloidal gear and rolling body through the initial parameters as follows:

$$\rho_2 + \rho_1 = r_2(1+\chi^2-2\chi\cos\varphi)^{1/2} - Z_2 r_2 i_{21}(1+\chi^2-2\chi\cos\varphi)^{1/2} \cdot \left(\chi Z_1\cos\varphi + \frac{\chi^2\cdot\sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1};$$

$$\rho_2 \cdot \rho_1 = r_b r_2(1+\chi^2-2\chi\cos\varphi)^{1/2} - r_b^2 - r_b Z_2 r_2 i_{21}(1+\chi^2-2\chi\cos\varphi)^{1/2} \cdot \left(\chi Z_1\cos\varphi + \frac{\chi^2\cdot\sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1}.$$

For brevity we will designate $a = (1+\chi^2-2\chi\cos\varphi)^{1/2}$. Then after the transformations, we will get:

$$\rho_2 + \rho_1 = r_2 a\left(1 - Z_2 i_{21}\cdot\left(\chi Z_1\cos\varphi + \frac{\chi^2\cdot\sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1}\right); \qquad (5)$$

$$\rho_2 \cdot \rho_1 = r_b r_2 a\left(1 - Z_2 i_{21}\cdot\left(\chi Z_1\cos\varphi + \frac{\chi^2\cdot\sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1} - \frac{r_b}{r_2 a}\right). \qquad (6)$$

Denote here $k = 1 - Z_2 i_{21}\cdot\left(\chi Z_1\cos\varphi + \frac{\chi^2\cdot\sin^2\varphi}{(1-\chi\cos\varphi)} + Z_2(1-\chi\cos\varphi)\right)^{-1}$, then the expressions (5) and (6) are rewritten as

$$\rho_2 + \rho_1 = k r_2 a;$$

$$\rho_2 \cdot \rho_1 = r_b r_2 a\left(k - \frac{r_b}{r_2 a}\right).$$

The ratio of the sum of the radii of curvature to their product is written as

$$\frac{\rho_2+\rho_1}{\rho_2\cdot\rho_1} = \frac{k r_2 a}{r_b r_2 a}\cdot\left(k - \frac{r_b}{r_2 a}\right)^{-1}.$$

Let us finally receive

$$\frac{\rho_2+\rho_1}{\rho_2\cdot\rho_1} = \frac{k}{r_b}\cdot\left(k - \frac{r_b}{r_2 a}\right)^{-1}. \qquad (7)$$

The expression of distance *h* is written through the initial parameters as:

$$h = \frac{i_{21} r_2 \chi\cdot\sin\varphi}{(1+\chi^2-2\chi\cos\varphi)^{1/2}}.$$

Then the ratio of distance (to the *i*-th normal) to the sum of squares of all distances is expressed as:

$$\frac{h_i}{\sum h_i^2} = \frac{i_{21} r_2 \chi\cdot\sin\varphi}{a i_{21}^2 r_2^2 \chi^2}\cdot\left(\sum\left(\frac{\sin\varphi}{a}\right)^2\right)^{-1}.$$

After conversion we finally get

$$\frac{h_i}{\sum h_i^2} = \frac{\sin\varphi}{a r_2 \chi i_{21}}\cdot\left(\sum\left(\frac{\sin\varphi}{a}\right)^2\right)^{-1}. \qquad (8)$$

By substituting the resulting expressions (6) and (7) in (2), we obtain:

$$(\sigma_H)_i = \sqrt{\frac{T_k E\cdot k\cdot\sin\varphi}{2\pi\cdot l_b(1-\mu^2) r_b a r_2 \chi i_{21}}\left(\left(k - \frac{r_b}{r_2 a}\right)\cdot\sum\left(\frac{\sin\varphi}{a}\right)^2\right)^{-1}}. \qquad (9)$$

In expression (9) the product $r_2\chi$ determines the radius of the rolling bodies centers position r_c which is one of the geometrical indicators defining overall dimensions of the transmission. We believe that the gears and rolling bodies are made from steel. Then let us remove the constant values from the root and write the contact strength condition for transmission with IRBFIR as:

$$(\sigma_{\mathrm{H}})_{\max} = 191{,}65 \cdot 10^{3}\sqrt{\frac{T_k \cdot k \cdot \sin\varphi}{l_b r_b a r_c i_{21}}\left(\left(k - \frac{r_b}{r_2 a}\right) \cdot \sum\left(\frac{\sin\varphi}{a}\right)^2\right)^{-1}} \le [\sigma_{\mathrm{H}}]. \quad (10)$$

From contact strength condition (10) we express radius of radius of the rolling bodies centers position r_c through permissible contact stress

$$r_c \ge \frac{36{,}73 \cdot 10^{9} \cdot T_k \cdot k \cdot \sin\varphi}{l_b r_b a i_{21} [\sigma_{\mathrm{H}}]^2}\left(\left(k - \frac{r_b}{r_2 a}\right) \cdot \sum\left(\frac{\sin\varphi}{a}\right)^2\right)^{-1}. \quad (11)$$

4. Example

Let us calculate the radius of the rolling bodies centers position on the basis of the following initial parameters of transmission with IRBFIR: r_2=25 mm; Z_2=25; χ=1,4; r_b=3 mm. Let torque on the inside gear T_k=200Hm and l_b=6 mm, and permissible contact stress for steel ShH15 equally [σ_H] =3000MPa.

In transmission with IRBFIR the maximum contact stress exists at angle $\varphi \approx 70°$. Therefore, all calculations will be made for this angle.

Then perform the calculation using the expressions (7), (8) and (11), obtain the following minimum allowable value of the radius of the rolling body centers position:

$$r_c \ge 0.032\ m.$$

Knowing that the radius r_c is connected to radius r_2 through shift factor χ [3], it is possible to specify centrode radius r_2

$$r_c = r_2 \cdot \chi \ge 0{,}032,$$
$$r_2 \ge \frac{0{,}032}{\chi},$$
$$r_2 \ge 0{,}023\ m.$$

Therefore, preselected initial parameters and, in particular the centroid radius r_2, satisfy the strength condition.

5. Conclusions

Thus, the expression (11) is obtained for determination the minimum permissible value of the geometric parameter r_c of transmission with IRBFIR. In the future, using this parameter it is possible to adjust initial parameters of the transmission taking into account strength. The expression obtained is basic for strength calculation of transmission with IRBFIR and mechanisms based on it and can be used in preliminary engineering calculations of the transmission.

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