# Knowledge models in computer-aided manufacturing systems 

A A Zakharova ${ }^{1,2}$ and Y V Grebenyuk ${ }^{\mathbf{1}}$<br>${ }^{1}$ Digital Technology Department, Yurga Institute of Technology, Tomsk Polytechnic University, Yurga, Russia<br>${ }^{2}$ Automated Control Systems Department, Tomsk State University of Control Systems and Radioelectronics, Tomsk, Russia<br>E-mail: zacharovaa@mail.ru


#### Abstract

Despite of vast amounts of information and considerable opportunities to process it with computer-aided manufacturing systems, personal experience and lore of experts become a role-defining category in the issues of creating and making technical decisions. This should be factored into the process of creating instruments for decision making, which are based on computer-aided manufacturing systems. Efficient decision-making models, as well as models of factors (determinants) evaluation, should include expert knowledge data. In this connection, it is of burning importance to find ways to formalize expert knowledge into a code readable by computer-aided manufacturing systems. The research paper presents three determinant (criteria) evaluation models which help make technical decisions. The models include expert knowledge data formalized by means of linguistic variables. The research paper also presents recommendations on how to opt for a proper model, including requirements, and peculiarities of expert information collection matters.


## 1. Introduction

Each stage of manufacturing engineering products includes decision making in structure, material, operation factors, technology and other matters [1,2]. Very often, these specifications reveal lack of formalization, being multivariate and holding qualitative description of decision-making determinants (criteria), and their relations. In this situation decision making is carried out in the conditions of information incompleteness and ambiguity, poor simulation and prognostication of processes and events. This brings personal experience and lore of experts to the forefront, when choosing between alternatives, as soon experts are true bearers of deep and unformalized understanding of issues [3]. To retrieve, store and process expertise, special classes of Automated Information Systems (AIS) such as decision support systems (DSS) and expert systems (ES) have been recently developed. They are computer applications to Computer-Aided Manufacturing systems (CAM), and use data collected and stored in CAM. At the same time, recommendations generated by AIS and ES are then transferred to CAM, and are viewed as a background for technical solutions. AIS and ES are specific as soon as they contain decision making models and knowledge databases, which include instructions on how to create new knowledge, as well as information about human experience and knowledge in a definite subject. To develop these systems, it is necessary to create new methods which help elicit and formalize expert knowledge into a code readable by CAM.

Fuzzy sets and linguistic variables, formalisms most frequently used in expert systems [2-5], allow:

- modelling smooth shift in an analyzed determinant characteristics;
- processing qualitative value of a determinant development;
- formalizing expert knowledge in case of determinants which are multiple-valued in their measurement specifications;
- collecting quantitative value of determinants in a form of a comparable scale;
- finding and describing relations between determinants;
- utilizing obtained linguistic variables and fuzzy sets as input determinants to solve different tasks when making technical decisions.
It is worth mentioning that it is important to choose methods and plot membership functions for linguistic variables, when making decisions based on fuzzy sets. Authors usually do not ground, or give details to this aspect. Consequently, it is of burning importance to develop a set of universally valid models which allow formalizing expert knowledge in technical decision making.

2. Requirements, particular qualities, and methods used to collect and process expert information In this paper a determinant is any value specification (technical, economic, ecological, etc) of an object, process or event in engineering production, which influences the option, decision makers do in preference to one or more alternatives, like material strength, production labor intensity etc. Alternatives are possible technical decisions like different kinds of material, production techniques etc, and are specified by a set of criteria decision makers define.

A linguistic variable of a determinant is characterized by a triple [6]:

$$
\begin{equation*}
<\beta, T, \mathrm{X}>, \tag{1}
\end{equation*}
$$

where $\beta$ is the name of a linguistic variable; T is the collection of its linguistic values, where $T=\left\{T_{1}, T_{2} \ldots T_{s}\right\}$, and which shows a desirable (allowed, demanded etc) level of a given determinant development and/or its importance when choosing between alternatives [7]. Each value of any linguistic variable is the name of a fuzzy set $\alpha_{s}, s=\overline{, h}$, which formalizes the $s$-level of a determinant; $X$ is the domain of a linguistic variable definition.

Fuzzy sets, which specify the level of a determinant development, are characterized as follows:

$$
\begin{equation*}
<\alpha_{s}, X, \mathrm{C}_{\alpha_{s}}> \tag{2}
\end{equation*}
$$

where $\alpha$ is the name of a fuzzy set;
$C_{\alpha_{s}}=\left\{\mu_{\alpha_{s}}(x) / x\right\}$ is a fuzzy set, which specifies the value of a fuzzy set $\alpha_{s}$;
$\mu_{\alpha_{i}}(x)$ is a membership function of a fuzzy set $C_{\alpha}$. Each value $x \in X$ can obtain a degree of membership of a determinant in a fuzzy set.

Membership function of a fuzzy set should allow indicating requirements, particular qualities, as well as methods which help collect and process expert information to evaluate each definite determinant, such as:

- type of a determinant (qualitative or quantitative);
- universal measurement specifications. Quantitative determinants are essentially universal unlike qualitative determinants, which are not always universal and might exist in a form of general indices (calculated via classical methods), relative ratings of objects, etc;
- source of information (an expert, any other individual engaged in decision making, or both);
- decision makers (individuals or a group of experts);
- number of experts;
- type of a scale used to evaluate determinants;
- measurement patterns, etc.

To factor listed above matters into practice, authors opted for three basic methods which help formalize expert knowledge while evaluating determinants. They are methods based on a pair wise comparison, statistical information processing and on a base of standard functions. Each method helps create definite determinant evaluation models. Expert knowledge formalization models based on
standard (exponential) functions have already been characterized in details [5]. The model is due when a linguistic variable denotes a determinant with one or several specifications, which follow: there are universal measurement specification, which describe the notion; it is not necessary to give highly precise definitions to some separate meanings (values) of a determinant; experts can give some approximate evaluations of determinants levels through direct methods like (call membership degree or membership function parameters, which help determine its value, directly) [5]. The research also deals with two other determinant evaluation models.

## 3. Determinant evaluation model, based on pair wise comparison

Let linguistic variable $\langle\beta, T, \mathrm{X}\rangle$ describe a determinant which:

- cannot be described through universal measurement specifications, as soon as there is no any to describe the notion;
- or, it is only possible to evaluate the determiner level for a restricted number of alternatives (objects, processes, events);
- or, it might be difficult to evaluate alternatives directly, based on the level of the determinant development;
- or, it is necessary to alleviate expert's subjectivism.

Let us specify some elements of formulae (1) and (2) in this model. The image of $T$ linguistic variable includes 3-4 terms. Domain of $X$ linguistic variable includes comparison alternatives finite set $x_{i}, i=\overline{1, n}$. As a rule, the number of these alternatives does not exceed 9 [8]. A fuzzy set, which specifies a fuzzy set $\alpha_{s}$ can be shown as this $C_{\alpha_{s}}=\left\{\mu_{\alpha_{s}}\left(x_{i}\right) / x_{i}\right\}$. Herewith, each definite determiner can obtain its slevel membership degree value $\mu_{\alpha_{s}}\left(x_{i}\right)$.

1. Indices with numerical interpretation have $X$ alternative domain with definite indicator values, typical to each enterprise. For example, it is necessary to evaluate a total production determiner. To denote the indicator we use a linguistic variable $\beta$ for total production, set of basic values $\mathrm{T}=$ ("low", "middle", "high"), definition domain $X=[80,100]$ (thousand rubles). Then, we can take $\mathrm{X}=\{80,90,100,110,120\}$ (thousand rubles) as comparison alternatives;
2. When it is difficult to evaluate a determinant numerically, its development in variable conditions might be evaluated (e.g., different techniques applied, materials used, segments, items of production, etc.). Let us evaluate the competitiveness of goods produced by an enterprise. In this case, analogous goods, produced by other enterprises, and available on a market, might be taken as comparison alternatives.

Pair wise comparison technique helps divide a determiner development level evaluation, applicable to some alternatives, into several less complicated steps. Investigators carry out alternative comparison by pairs. This, in its turn, helps lower subjectivism experts evince when they produce evaluation directly [9].

To plot a membership function $\mu_{\alpha_{1}}(x)$ for each term of a linguistic variable, it is necessary to form a matrix of alternatives $M_{s}=\left\|m_{i j_{s}}\right\|$ pair wise comparison. The matrix elements $m_{i j_{s}}(i, j=1,2, \ldots, n)$ are comparative evaluations of an investigated determinant development degree upon the given alternatives. An expert should evaluate the $x_{i} \in X$ alternative proximity to the notion, described via a $C_{\alpha_{s}}$ fuzzy set in comparison with an $x_{j} \in X$ alternative. In this regard, investigators use an interpretation scale for $m_{i j}$ indices shown in table 1 [10]. If alternative $x_{i}$ exceeds alternative $x_{j}$, index $m_{i j} \in[1,2, \ldots 9]$. If $x_{j}$ element exceeds $x_{i}$ element, $m_{j i}=\frac{1}{m_{i j}}$.

Table 1. $m_{i j}$ indices interpretation.

| Meaning | $m_{i j}$ |
| :--- | :---: |
| $\mu\left(x_{i}\right)$ approximately equal $\mu\left(x_{j}\right)$ | 1 |
| $\mu\left(x_{i}\right)$ a little bit bigger $\mu\left(x_{j}\right)$ | 3 |
| $\mu\left(x_{i}\right)$ bigger $\mu\left(x_{j}\right)$ | 5 |
| $\mu\left(x_{i}\right)$ visibly bigger $\mu\left(x_{j}\right)$ | 7 |
| $\mu\left(x_{i}\right)$ much bigger $\mu\left(x_{j}\right)$ | 9 |
| Intermediate value | $2,4,6,8$ |
| Reciprocal value - when $m_{i j} \neq 0$ | $m_{j i}=\frac{1}{m_{i j}}$ |

Membership function value $\mu_{\alpha_{s}}\left(x_{1}\right), \mu_{\alpha_{s}}\left(x_{2}\right) \ldots \mu_{\alpha_{s}}\left(x_{n}\right)$ for $x_{1}, x_{2}, \ldots x_{n}$ is calculated through the equation $M \cdot r=v_{\max } \cdot r$, where $M$ is a pair wise comparison matrix, $r=\left(r_{1}, r_{2} \ldots r_{n}\right)$ is a proper vector; $v_{\text {max }}$ is a maximal proper number M . In accordance with a simplified proper vector calculation procedure [9], values of the elements of vector $r_{s}=\left(r_{1}, \ldots, r_{j} \ldots, r_{n}\right)$ and corresponding unnormalized value of element $\mu_{\alpha_{s}}\left(x_{i}\right)_{H H}$ membership degree might be calculated through the equation (3).

$$
\begin{equation*}
r_{s_{j}}=1 / \sum_{i=1}^{n} m_{i j_{s}} ; \mu_{\alpha_{s}}\left(x_{i}\right)_{H H}=1 / \sum_{i=1}^{n} m_{i j_{s}} \tag{3}
\end{equation*}
$$

Vector $r$ can be used to assess consistency of an expert's opinion (pair wise comparison matrix). The equation is solved through $M \cdot r=v_{\max } \cdot r$ at this point the degree $v_{\max }$ deviates from $n$ is used to evaluate accuracy [9]. Index and homogeneity ratio, calculated through the equation (4) are used.

$$
\begin{equation*}
H I=\left(v_{\max }-n\right) /(n-1) ; H R=H I / M(H I), \tag{4}
\end{equation*}
$$

where $H I$ is a homogeneity index; $H R$ is a homogeneity ratio ( $H R \leq 0,10$ is admissible); $M(H I)$ is a mean value of homogeneity index, defined in accordance with [9];

$$
v_{\max }=\frac{1}{n} \sum_{i=1}^{n} v_{i}
$$

$n$ is a number of alternatives;
$v_{i}$ are values obtained as a result of element-wise division of vector $\rho_{s}$ elements by elements of vector $r_{s}$, e.g., $v_{1}=r_{s_{1}} / \rho_{s_{1}}$;
$\rho_{s}=M_{s} \cdot r_{s}$ is a vector.
In case of group expertise, individual matrices of each expert's comparisons lie in a base of an aggregated matrix of a pair wise comparison. An aggregated evaluation is calculated through the following equation (5).

$$
\begin{equation*}
\left(m_{i j_{s}}\right)_{A}=\left(m_{i j_{s}}\right)_{1}{ }^{w_{1}}\left(m_{i j_{s}}\right)_{2}^{w_{2}} \ldots\left(m_{i j_{s}}\right)_{k}^{w_{k}} \tag{5}
\end{equation*}
$$

where $\left(m_{i j}\right)_{A}$ is an aggregated evaluation of the element of matrix;
$k$ is a number of individual matrices of pair wise comparisons (number of experts);
$w_{k}$ is the weight of an expert, with $w_{1}+w_{2}+\ldots+w_{k}=1$.
Next, it is necessary to normalize the membership function. For this purpose it is important to divide each $r_{s_{j}}$ element of $r_{s}$ vector (or coincident unnormalized value of element membership degree) $\left.\mu_{\alpha_{s}}\left(x_{i}\right)_{H H}\right)$ by its maximal value (equation 6).

$$
\begin{equation*}
r_{s_{j}}{ }^{\prime}=\frac{r_{s_{j}}}{r_{s_{j} \max }} ; \tag{6}
\end{equation*}
$$

The suggested model of a determinant evaluation, based on a pair wise comparison method, allows obtaining membership function values of comparison alternatives for each basic value of a linguistic variable (determinant level) within a specific domain

## 4. Determinant valuation model, which uses statistical information

Let a linguistic variable $<\beta, T, \mathrm{X}>$ describe a determinant which:

- can be described through universal measurement specifications, which characterize the notion;
- to be evaluated, needs processing a big amount of information received from many respondents.

Let us specify some elements of equation (1) and (2) of the model. The image of a $T$ linguistic variable includes 3-4 terms. The domain of an $X$ linguistic variable is divided into 1 number of intervals of equal $j=\overline{1, l}$ length. A fuzzy set, which describes $\alpha_{s}$ values, can be described as $C_{\alpha_{s}}=\left\{\mu_{\alpha_{s}}\left(x_{j}\right) / x_{j}\right\}$; where $x_{j}$ is a subset $X$ which enters a $j$ interval; $\mu_{\alpha_{s}}\left(x_{j}\right)$ is a $\mathrm{C}_{\alpha_{s}}$ fuzzy set membership function. Each $x_{j} \in X$ interval can obtain its $s$-level membership degree value.

Polled data are presented in a form of an empirical table, where lines are values (terms) of a linguistic variable, columns are intervals of a linguistic variable domain. Cells contain polled data -n number of respondents' replies where they used some definite value of a linguistic variable $\left(b_{s j}\right)$ to the definite determinant value interval (table 2).

Table 2. Statistical observation data.

| Linguistic <br> variable value | Linguistic variable domain intervals |  |  |  |  |  |
| :--- | :---: | :---: | :--- | :---: | :--- | :--- |
|  | $x_{1}$ | $x_{2}$ | $\ldots$ | $x_{j}$ | $\ldots$ | $x_{l}$ |
| $\alpha_{1}$ | $b_{11}$ | $b_{12}$ | $\ldots$ | $b_{1 j}$ | $\ldots$ | $b_{1 l}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\alpha_{s}$ | $b_{s 1}$ | $b_{s 2}$ | $\ldots$ | $b_{s j}$ | $\ldots$ | $b_{s l}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\alpha_{h}$ | $b_{h 1}$ | $b_{h 2}$ | $\ldots$ | $b_{h j}$ | $\ldots$ | $b_{h l}$ |

To find membership degree of each linguistic variable value interval, number of replies, given by respondents who used the setting linguistic variable value in the ratio of the determinant value interval, should be divided by the maximal number of replies. It is worth noticing that the number of observations within each interval can vary. That is why polled data need further processing.
Membership function can be determined through the equation (7).

$$
\begin{equation*}
\mu_{s j}=c_{s j} / c_{s \max }, \tag{7}
\end{equation*}
$$

where $\mu_{\mathrm{sj}}$ is the membership function value of an s-term of a linguistic variable within a $j$ interval; $c_{s j}$ are transformed elements of $b_{s j}$ from the Statistical Observation Data Table, given above;
$c_{s \text { max }}$ is the maximal $c_{s j}$ element if $j$ is taken.
$b_{s j}$ elements transformation is carried out through the equation (8):

$$
\begin{equation*}
c_{s j}=\frac{b_{s j} k_{\max }}{k_{j}}, \tag{8}
\end{equation*}
$$

where $k_{\text {max }}=\max k_{j}$;
$k_{j}$ are the prompts matrix elements, calculated to smooth the function through the equation (9).

$$
\begin{equation*}
k_{j}=\sum_{s=1}^{h} b_{s j}, \tag{9}
\end{equation*}
$$

If $k_{j}=0$, transformation of elements from the $j$-column is carried out through the approximation by linearization (equation 10).

$$
\begin{equation*}
c_{s j}=\frac{c_{s(j-1)}+c_{s(j+1)}}{2} . \tag{10}
\end{equation*}
$$

## 5. Recommendations on how to choose a determinant evaluation

Determinant evaluation models described in the paper play two important roles:

1. First of all, determinant evaluation models are efficient instruments which help formalize an expert's perception of a desirable (available, demanded, etc) level of a determinant development. The models allow collecting linguistic evaluations, as well as clear and accurate quantitative analyses of determinants with prescribed initial values. They also allow setting smooth shifts in determinants values membership intensity, and formalize experts' confidence in this or that value of a determinant.
2. Secondly, determinant evaluation models are setting determinants for other decision-making models. Membership function values are considered to be initial when evaluating alternatives for technical decision through multi criteria filter.

The research proceeds with recommendations on how to choose the most suitable and efficient model which help evaluate determinants (table 3).

## 6. Summary

The paper presents a final kit of models which help plot the membership functions for linguistic variables terms, which allow creating new expert knowledge-based models of evaluation determinants that influence the choice between alternatives when making technical decisions. All the possible requirements, methods of collecting and processing expert information, which help evaluate determinants, have been observed. Elaborated recommendations on how to choose a determinant evaluation model allow choosing the model, which can formalize expert knowledge in any type of a determinant into a code readable by computer-aided manufacturing systems.

Table 3. Recommendations to choose the model.

| Determinant specification (determinant evaluation process specification) | Determinant evaluation model (three methods) |  |  |
| :---: | :---: | :---: | :---: |
|  | Pair wise comparison | Based on statistical information | Based on standard functions |
| Source of information, which is used to evaluate determinants | Experts | Polled data/ Experts | Experts |
| Recommended number of expertsgroup participants | No more than 9 | 9 and more | Unrestricted |
| Type of a determinant | Qualitative/ Quantitative | Quantitative | Quantitative |
| Universal measurement specifications of a determinant | No/Yes | Yes | Yes |
| Number of comparison alternatives (intervals) | No more than 9 | 10 and more | Not used |
| Required degree of detailed elaboration and accuracy when describing a determinant | High | High and medium | Medium and low |

## Acknowledgments

The reported study was funded by RFBR according to the research project № 19-013-00486A.

## References

[1] Szafranko E 2018 Proceedings of Materials Today 2002-2009
[2] Gorbatova E, Emelianenko E, Zaretckii M 2017 IOP Conference Series: Earth and Environmental Science 87072001 doi:10.1088/1755-1315/87/7/072001
[3] Malyshev N, Berstein L, Bozhenuk 1998 Proceedings of Int. Conf. Fuzzy Sets in Informatics 44
[4] Yadav O, Singh Chinnam N, Goel P 2003 Reliability Engineering and System Safety 80 63-74
[5] Zakharova A, Ostanin V 2015 IOP Conf. Ser.: Mater. Sci. Eng. 91012070 doi:10.1088/1757899X/91/1/012070
[6] Zadeh L 1965 Proceedings of Inf. Control 8 338-353
[7] Zadeh L 1978 Fuzzy Sets and Systems 13-28
[8] Saaty T, Ozdemir M 2003 Mathematical and Computer Modelling 38 233-244
[9] Saaty T 2002 Scientia Iranica 9 215-229
[10] Saaty T 1978 Fuzzy Sets and Systems 1 57-68

