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Gravitino condensate in $N = 1$ supergravity coupled to the $N = 1$ supersymmetric Born–Infeld theory

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The $N = 1$ supersymmetric Born–Infeld theory coupled to $N = 1$ supergravity in four spacetime dimensions is studied in the presence of a cosmological term with spontaneous supersymmetry breaking. The consistency is achieved by compensating a negative contribution to the cosmological term from the Born–Infeld theory by a positive contribution originating from the gravitino condensate. This leads to an identification of the Born–Infeld scale with the supersymmetry-breaking scale. The dynamical formation of the gravitino condensate in supergravity is reconsidered and the induced one-loop effective potential is derived. Slow-roll cosmological inflation with the gravitino condensate as the inflaton (near the maximum of the effective potential) is viable against the Planck 2018 data and can lead to the inflationary (Hubble) scale as high as 10^{12} GeV. Uplifting the Minkowski vacuum (after inflation) to a de Sitter vacuum (dark energy) is possible by the use of the alternative Fayet–Iliopoulos term. Some major physical consequences of our scenario for reheating are also briefly discussed.

Subject Index B11, B12, B16, B32, E81

1. Introduction

The gravitino condensate and the gravitino mass gap in $N = 1$ supergravity [1] coupled to the Volkov–Akulov field [2] in four spacetime dimensions arise as the one-loop effect due to the quartic gravitino interaction coming from the gravitino contribution to the spacetime (con)torsion [3,4]. This is similar to the Nambu–Jona-Lasinio model [5] of the dynamical generation of electron mass and the formation of Cooper pairs near the Fermi surface in superconductivity. The dynamical gravitino mass also leads to a positive contribution to the vacuum energy and, hence, the dynamical supersymmetry breaking too [6]. Given the standard (reduced) Planck mass as the only (dimensional) coupling constant, the gravitino mass gap should be of the order of the Planck scale also, which prevents phenomenological applications of the gravitino condensate to physics under the Planck scale.

However, the effective scale of quantum gravity may be considerably lower than its standard value associated with the (reduced) Planck mass $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \approx 2.4 \times 10^{18}$ GeV. This may happen

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because the effective strength of gravity can depend upon either large or warped extra dimensions in the braneworld, or the dilaton expectation value in string theory, or both these factors together [7–9].[1] The negative results of the Large Hadron Collider (LHC) searches for copious production of black holes imply that the low-scale gravity models may have to be replaced by high-scale gravity (or supergravity) models, whose effective Planck scale $\tilde{M}_{\rm Pl}$ is much higher than the TeV scale but is still under the standard scale $M_{\rm Pl}$, i.e.

$$1\ \text{TeV} \ll \tilde{M}_{\rm Pl} \ll M_{\rm Pl}. \tag{1}$$

This can be of particular importance to the early Universe cosmology, where the Newtonian limit does not apply, as well as for high-energy particle physics well above the electroweak scale.

In this scenario, supergravity may play the crucial role in the description of cosmological inflation, reheating, dark energy, and dark matter; see, e.g., Ref. [11] and the references therein. For instance, it is unknown which physical degrees of freedom were present during inflation, while supergravity may be the answer. Describing inflation and a positive cosmological constant (dark energy) in supergravity is non-trivial, especially when one insists on the minimalistic hidden sector. Inflation is driven by positive energy so that it breaks supersymmetry (SUSY) spontaneously. As a (model-independent) consequence, the goldstino should be present during inflation in supergravity cosmology. The goldstino effective action is *universal* and is given by the Akulov–Volkov (AV) action up to field redefinition [12,13]. As was demonstrated in Refs. [14,15], the viable description of inflation and dark energy in supergravity can be achieved by employing an $N = 1$ *vector* multiplet with its $N = 1$ supersymmetric Born–Infeld (BI) action [16] in the presence of the alternative Fayet–Iliopoulos (FI) term [17–21] without gauging the R-symmetry.[2]

In this paper we also employ an $N = 1$ vector multiplet with its $N = 1$ supersymmetric BI action that automatically contains the goldstino (AV) action, but we choose the gravitino condensate as the inflaton. A dynamical SUSY breaking is achieved at the very high scale with the vanishing cosmological constant. The extra (FI) mechanism of spontaneous SUSY breaking is then used to uplift a Minkowski vacuum to a de Sitter (dS) vacuum.

The BI theory has solid motivation. It is expected that Maxwell electrodynamics does not remain unchanged up to the Planck scale, because of its internal problems related to the Coulomb singularity and the unlimited values of electromagnetic field. This motivated Born and Infeld [26] to propose the non-linear vacuum electrodynamics with the Lagrangian (in flat spacetime)

$$\mathcal{L}_{\rm BI} = -M_{\rm BI}^4\sqrt{-\det\left(\eta_{\mu\nu} + M_{\rm BI}^{-2}F_{\mu\nu}\right)} = -M_{\rm BI}^4 - \tfrac{1}{4}F^2 + \mathcal{O}(F^4), \tag{2}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $F^2 = F^{\mu\nu}F_{\mu\nu}$. The constant term on the right-hand side of Eq. (2) can be ignored in flat spacetime. The BI theory has the new scale $M_{\rm BI}$ whose value cannot exceed the GUT scale where electromagnetic interactions merge with strong and weak interactions. On the other hand, we need $M_{\rm BI} < \tilde{M}_{\rm Pl}$ in order to ignore quantum gravity corrections. The BI theory naturally emerges (i) in the bosonic part of the open superstring effective action [27],

[1] The effective Planck scale may also be dynamically generated [10].

[2] In Refs. [22–24], the $N = 1$ massive vector multiplet, unifying the Starobinsky inflaton (scalaron) [25] and the goldstino (photino), was used together with the BI action, the FI term, the chiral (Polonyi) multiplet representing the hidden SUSY-breaking sector, and the massive gravitino as the lightest SUSY particle (LSP) for dark matter.

(ii) as part of the Dirac–Born–Infeld (DBI) effective action of a D3-brane [28], and (iii) as part of the Maxwell–Goldstone action describing partial supersymmetry breaking of $N = 2$ supersymmetry to $N = 1$ supersymmetry [29,30].[3] The peculiar non-linear structure of the BI theory is responsible for its electricmagnetic (Dirac) self-duality, taming the Coulomb self-energy of a point-like electric charge, and causal wave propagation (no shock waves and no superluminal propagation)—see, e.g., Refs. [39,40] and the references therein for a review and non-Abelian extensions of BI theory. All this adds more reasons for using the BI structure.

In a curved spacetime with metric $g_{\mu\nu}$ the BI action is usually defined as the difference between two spacetime densities,

$$S_{\text{BI,standard}} = M_{\text{BI}}^4 \int d^4x \left[\sqrt{-\det(g_{\mu\nu})} - \sqrt{-\det\left(g_{\mu\nu} + M_{\text{BI}}^{-2} F_{\mu\nu}\right)} \right], \tag{3}$$

where the first term has been added "by hand" in order to eliminate the cosmological constant arising from the second term and in Eq. (2). In this paper we propose the gravitino condensation as the origin and the mechanism of such cancellation in the supergravity extension of the BI theory with spontaneously broken SUSY.

The $N = 1$ (rigid) supersymmetric extension of BI theory is also self-dual [41]. The supersymmetric BI theory coupled to $N = 1$ supergravity [i.e. the locally supersymmetric extension of Eq. (3)] was constructed in Ref. [42].

Our paper is organized as follows. In Sect. 2 we provide more details on how to deal with a cosmological constant and spontaneous supersymmetry breaking in the context of a supersymmetric BI theory coupled to supergravity, and relate the BI scale to the spontaneous SUSY-breaking scale. Most of the comments in Sect. 2 are known in the literature and are recalled to justify the consistency of our approach. In Sect. 3 we study the dynamical gravitino condensate arising from the one-loop effective action of pure supergravity, and investigate the induced scalar potential. Slow-roll inflation with the gravitino condensate playing the role of inflaton is studied numerically in Sect. 4. Uplifting the Minkowski vacuum to a de Sitter vacuum using the alternative FI term is proposed in Sect. 5. Our conclusion is presented in Sect. 6. We use the supergravity notation of Ref. [1].

2. Spontaneous SUSY breaking, AV and BI actions, and their coupling to supergravity

We recall that the AV Lagrangian in flat spacetime is given by [2]

$$\mathcal{L}_{\text{AV}} = -M_{\text{susy}}^4 \det\left(\delta_b^a + \frac{i}{2M_{\text{susy}}^4}\bar{\lambda}\gamma^a\partial_b\lambda\right) = -M_{\text{susy}}^4 - \tfrac{i}{2}\bar{\lambda}\gamma\cdot\partial\lambda + \mathcal{O}(\lambda^4), \tag{4}$$

where $\lambda(x)$ is a Majorana fermion field of spin 1/2. This fermionic field is called the *goldstino* because the AV action has spontaneously broken non-linearly realized rigid SUSY under the transformations

$$\delta\lambda = M_{\text{susy}}^2\varepsilon + \frac{i}{M_{\text{susy}}^2}(\bar{\varepsilon}\gamma^a\lambda)\partial_a\lambda \tag{5}$$

[3] See Refs. [31–34] for the extensions of BI theory to extended supersymmetry and higher dimensions.

with the infinitesimal Majorana spinor parameter ε, so that the goldstino is indeed a Nambu–Goldstone fermion. The AV theory of Eq. (4) has the spontaneous SUSY-breaking scale M_{susy}.

A coupling of the AV action to supergravity is supposed to generate a gravitino mass via the so-called super-Higgs effect [1] when the gravitino "eats up" the goldstino and thus gets the right number of physical degrees of freedom. However, it is impossible to couple the AV action to supergravity in a manifestly supersymmetric way (i.e. with the linearly realized SUSY) when using standard supermultiplets or unconstrained superfields because of the mismatch in the numbers of bosonic and fermionic physical degrees of freedom.[4] We embed the goldstino into a standard *vector* supermultiplet, i.e. identify the goldstino with the photino, and use an $N=1$ supersymmetric BI action for the vector multiplet, because it is well motivated at very high energies and includes the goldstino AV action up to a field redefinition [12,13].

The supersymmetric extension of the BI action in Eq. (3) minimally coupled to supergravity in curved superspace of the (old-minimal) supergravity (in a superconformal gauge) with a vanishing cosmological constant, and the vanishing gravitino mass is given by

$$S_{\mathrm{SBI}}[V] = \tfrac{1}{4}\left(\int d^4x d^2\theta \mathcal{E} W^2 + \text{h.c.}\right) + \tfrac{1}{4}M_{\mathrm{BI}}^{-4}\int d^4x d^2\theta d^2\bar{\theta} E \frac{W^2\bar{W}^2}{1+\frac{1}{2}A+\sqrt{1+A+\frac{1}{4}B^2}}, \tag{6}$$

$$A = \tfrac{1}{8}M_{\mathrm{BI}}^{-4}\left(\mathcal{D}^2W^2 + \text{h.c.}\right), \quad B = \tfrac{1}{8}M_{\mathrm{BI}}^{-4}\left(\mathcal{D}^2W^2 - \text{h.c.}\right),$$

where $\mathcal{E}$ is the chiral (curved) superspace density, E is the full (curved) superspace density, $\mathcal{D}^\alpha$ are the covariant spinor derivatives in superspace, W^α is the chiral gauge-invariant field strength,

$$W_\alpha = -\tfrac{1}{4}\left(\bar{\mathcal{D}}^2 - 4\mathcal{R}\right)\mathcal{D}_\alpha V, \tag{7}$$

of the gauge real scalar superfield pre-potential V describing an $N=1$ vector multiplet, $\mathcal{R}$ is the chiral (scalar curvature) supergravity superfield, $W^2 = W^\alpha W_\alpha$, and $\mathcal{D}^2 = \mathcal{D}^\alpha\mathcal{D}_\alpha$ [1].

The action in Eq. (6) is obtained from the standard (Bagger–Galperin) action [29]

$$S_{\mathrm{BG}}[W,\overline{W}] = \frac{1}{4}\int d^4x d^2\theta\, X + \text{h.c.}, \quad X + \frac{1}{4M_{\mathrm{BI}}^4} X\overline{D}^2\overline{X} = W^2 \tag{8}$$

in terms of the constrained chiral superfield X after solving the constraint in Eq. (8) and then minimally coupling the resulting action with the supergravity in curved superspace [19,39], where the spacetime metric $g_{\mu\nu}$ is replaced by the vierbein e^a_μ and is extended to an off-shell supermultiplet $(e^a_\mu, \psi_\mu, M, b_\mu)$, with ψ_μ as the Majorana gravitino field, whereas the complex scalar M and the real vector field b_μ are the auxiliary fields.[5]

The gauge vector (photon) field A_μ is extended in SUSY to an off-shell (real) gauge vector multiplet (or a general real superfield) V with the field components

$$V = (C, \chi, H, A_\mu, \lambda, D), \tag{9}$$

where λ is the Majorana fermion called the photino, D is the auxiliary field, while the rest of the fields (C, χ, H) are the super-gauge degrees of freedom that are ignored in what follows.

[4] The manifestly supersymmetric description is, nevertheless, possible at low energies when embedding the goldstino into the constrained chiral superfield $\tilde{X}$ obeying the nilpotency condition $\tilde{X}^2 = 0$ [35–38]. We avoid that goldstino superfield because it is problematic at higher energies and in quantum theory.

[5] The auxiliary fields of the supergravity multiplet do not play a significant role in our investigation and are ignored below.

Disturbing the action in Eq. (6) by adding a negative cosmological constant $-M_{\rm BI}^4$ to restore the original BI action in Eq. (2) explicitly breaks SUSY, which, however, can be restored by modifying the action and the SUSY transformation laws [6,21]. As a result, it was found that the deformed (new) BI action cannot have a non-vanishing cosmological constant but can have a spontaneously broken local SUSY with a non-vanishing gravitino mass related to the SUSY-breaking scale $M_{\rm BI}$. This does not explain, however, the physical origin of the necessary compensating positive term $+M_{\rm BI}^4$. We explain its origin by gravitino condensation (Sect. 3). To illustrate those features, we add a few simple arguments below.

In order to cancel the SUSY variation of the cosmological constant multiplied by $\sqrt{-\det(g_{\mu\nu})}=e$ due to $\delta_{\rm susy}e_\mu^a=-i\tilde{M}_{\rm Pl}^{-1}(\bar{\varepsilon}\gamma^a\psi_\mu)$ with the infinitesimal SUSY parameter $\varepsilon(x)$, we have to add the photino–gravitino mixing term

$$-ie\frac{M_{\rm BI}^2}{\tilde{M}_{\rm Pl}}(\bar{\lambda}\gamma^\mu\psi_\mu) \tag{10}$$

to the Lagrangian, and simultaneously demand the supersymmetric variation of the photino λ as

$$\delta_{\rm susy}\lambda=M_{\rm BI}^2\varepsilon+\cdots\,, \tag{11}$$

where the dots stand for the other field-dependent terms. The identification of the photino λ with the goldstino of the spontaneously broken local SUSY already requires

$$M_{\rm BI}=M_{\rm susy} \tag{12}$$

by comparison of Eqs. (5) and (11). This may be not surprising after taking into account that the initial (rigid) Bagger–Galperin action of Eq. (8) has a second (spontaneously broken and non-linearly realized) SUSY whose transformation law is similar to that of Eq. (5). However, our deformed super-BI action in supergravity does not respect another SUSY by construction.

The SUSY-restoring deformation comes together with the gravitino mass term having the mass parameter $m^2=\frac{1}{3}M_{\rm BI}^4/\tilde{M}_{\rm Pl}^2$, and the modification of the gravitino SUSY transformation law as $\delta_{\rm susy}\psi_\mu=-2\tilde{M}_{\rm Pl}(D_\mu\varepsilon+\frac{1}{2}m\gamma_\mu)+\cdots$. This also implies (by local SUSY) the presence of the goldstino mass term in the Lagrangian with the same mass parameter m [6]. Hence, the super-Higgs effect is in place.

The recovery of the AV action from the super-BI action is possible by identifying the goldstino λ_α with the leading field component of the superfield W_α and projecting the other fields out, $F_{\mu\nu}(A)=D=\psi_\mu=0$ in the absence of gravity, $e_\mu^a=\delta_\mu^a$. Then, the action in Eq. (8) reduces to the AV action in Eq. (4) up to a field redefinition in the higher-order terms—see Ref. [43] for details. The same conclusions are supported by the superconformal tensor calculus in supergravity [44]. In our approach, the AV action is thus the fermionic *fragment* of the supersymmetric BI theory coupled to supergravity with the spontaneously broken SUSY at the scale M_{BI}. In Sect. 3 we concentrate on the pure supergravity sector of our theory, ignoring the gravitino–photino mixing (i.e. taking into consideration only spin-3/2 gravitino components), just for simplicity. Accounting of a spin-1/2 photino contribution is beyond the scope of our investigation in this paper.

3. One-loop effective action and gravitino condensate

The classical supergravity Lagrangian $\mathcal{L}_{\rm SUGRA}$ besides the Einstein–Hilbert and Rarita–Schwinger terms,

$$\mathcal{L}_{\rm EH} = -\frac{\tilde{M}_{\rm Pl}^2}{2} eR \tag{13}$$

and

$$\mathcal{L}_{\rm RS} = -\tfrac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\lambda\psi_\rho, \tag{14}$$

respectively, also has the quartic gravitino coupling,

$$\mathcal{L}_{\rm quartic} = \tfrac{11}{16}\tilde{M}_{\rm Pl}^{-2}\left[(\bar{\psi}_\mu\psi^\mu)^2 - (\bar{\psi}_\mu\gamma_5\psi^\mu)^2\right] - \tfrac{33}{64}\tilde{M}_{\rm Pl}^{-2}(\bar{\psi}^\mu\gamma_5\gamma_\nu\psi^\mu)^2, \tag{15}$$

originating from the spacetime (con)torsion in the covariant derivative of the gravitino field in its kinetic term in the second-order formalism for supergravity [1].[6]

Since the supergravity action is invariant under the local SUSY, whose gauge field is ψ_μ, one can choose the (physical) gauge condition $\gamma^\mu\psi_\mu = 0$, which implies $(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) = -\frac{1}{2}\bar{\psi}_\mu\psi^\mu$, in the notation $\Sigma_{\mu\nu} = \frac{1}{4}[\gamma^\mu,\gamma^\nu]_-$, and rewrite the (non-chiral) quartic gravitino term in Eq. (6) as

$$\mathcal{L}_{\rm quartic} = \sqrt{11}\tilde{M}_{\rm Pl}^{-1}\rho(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) - \rho^2, \tag{16}$$

where the real scalar field ρ has been introduced. As is clear from Eq. (16), a gravitino condensate leads to a non-vanishing vacuum expectation value (VEV), $\langle\rho\rangle \equiv \rho_0 \neq 0$, whereas ρ_0 contributes to the gravitino mass.

The one-loop contribution to the effective potential $V_{\text{1-loop}}(\rho)$ of the scalar field ρ together with its kinetic term arise after quantizing the gravitino sector and taking the Gaussian integral over ψ_μ in the gauge $\gamma^\mu\psi_\mu = 0$. This yields the one-loop contribution to the quantum effective action in the standard form,

$$\Gamma_{\text{1-loop}} = -\tfrac{i}{2}{\rm Tr}\ln\Delta(\rho), \tag{17}$$

where $\Delta(\rho)$ stands for the kinetic operator in the gravitino action, and the interaction with gravity is ignored ($e^a_\mu = \delta^a_\mu$). The one-loop contribution to the ρ-scalar potential [i.e. the terms without the spacetime derivatives in Eq. (17)] was first computed in Refs. [3,4], with the result

$$V_{\text{1-loop}} = \lim_{\mathcal{V}\to\infty}\left[\frac{-1}{2\mathcal{V}}\sum_{n=1}^{\infty}\frac{(\sqrt{11}\tilde{M}_{\rm Pl})^{2n}}{2n}{\rm Tr}(P_{ab}\rho)^{2n}\right] = -\frac{4}{(2\pi)^4}\int^\Lambda d^4p\,\ln\left(1+11\tilde{M}_{\rm Pl}^{-2}\frac{\rho^2}{p^2}\right) \tag{18}$$

in terms of the standard massless gravitino propagator (in momentum space)

$$P_{ab} = -\frac{i}{2}\frac{\gamma_b\gamma^\mu p_\mu\gamma_a}{p^2}, \tag{19}$$

the spacetime four-volume regulator $\mathcal{V}$, and the ultraviolet (UV) cutoff Λ, with the trace Tr acting on all variables.

[6] We separate the quartic terms from the minimal term in Eq. (14).

The one-loop contribution in Eq. (17) expanded up to the second order in the spacetime derivatives also yields the ρ-kinetic term subject to the wave function renormalization (i.e. with the Z factor), so that the initially auxiliary scalar field ρ becomes dynamical with a mass M_c. The specific calculations can be found in the literature [3,4,45–47], and the effective potential reads[7]

$$V(\rho) \equiv V_{\text{classical}}(\rho) + V_{\text{1-loop}}(\rho) = \rho^2 - \frac{4}{(2\pi)^4} \int^{\Lambda} d^4p \, \ln\left(1 + 11\tilde{M}_{\text{Pl}}^{-2}\frac{\rho^2}{p^2}\right). \tag{20}$$

Our result of taking the four-dimensional integral in Eq. (20) is given by (cf. Refs. [3,4])

$$V(\rho) = \rho^2 + \frac{1}{8\pi^2}\left\{\frac{121\rho^4}{\tilde{M}_{\text{Pl}}^4} \ln\left(1 + \frac{\tilde{M}_{\text{Pl}}^2\Lambda^2}{11\rho^2}\right) - \frac{11\rho^2\Lambda^2}{\tilde{M}_{\text{Pl}}^2} - \Lambda^4 \ln\left(1 + \frac{11\rho^2}{\tilde{M}_{\text{Pl}}^2\Lambda^2}\right)\right\}. \tag{21}$$

The logarithmic scaling of the wave function renormalization of ρ in the one-loop approximation yields the factor proportional to $\ln\left(\frac{\Lambda^2}{\mu^2}\right)$, where μ is the renormalization scale. Hence, the canonical (physical) scalar ϕ is given by [46]

$$\phi = \text{const.}\sqrt{\ln\left(\frac{\Lambda^2}{\mu^2}\right)}\tilde{M}_{\text{Pl}}^{-1}\rho \equiv \tilde{w}M_{\text{Pl}}\sigma, \tag{22}$$

where we have introduced the dimensionless (renormalization) constant $\tilde{w}$ as the parameter. We also use the other dimensionless quantities

$$\sigma = \tilde{M}_{\text{Pl}}^{-2}\rho, \quad \tilde{M}_{\text{Pl}}^{-1}\Lambda = \tilde{\Lambda}, \quad \text{and} \quad \tilde{M}_{\text{Pl}}^{-1}M_{\text{BI}} = \alpha, \tag{23}$$

which allow us to rewrite the *full* scalar potential as

$$V(\sigma)\tilde{M}_{\text{Pl}}^{-4} = \sigma^2 - \frac{1}{8\pi^2}\left\{\tilde{\Lambda}^4 \ln\left(1 + \frac{11\sigma^2}{\tilde{\Lambda}^2}\right) - 121\sigma^4 \ln\left(1 + \frac{\tilde{\Lambda}^2}{11\sigma^2}\right) + 11\sigma^2\tilde{\Lambda}^2\right\} + \alpha^4, \tag{24}$$

where we have added the contribution of the first term on the right-hand side of Eq. (2).

The scalar potential of Eq. (24) has the double-well shape and is bounded from below, see Fig. 1, provided that

$$\tilde{\Lambda}^2 > \frac{4\pi^2}{11} \approx 3.59, \quad \text{or} \quad \tilde{\Lambda} > \frac{2\pi}{\sqrt{11}} \approx 1.89. \tag{25}$$

There is a local maximum at $\rho = \sigma = 0$ with the positive height M_{BI}^4. A similar potential near its maximum was used for describing slow-roll inflation with the inflaton field ϕ [46]; see Sect. 4 for more. There are also two stable Minkowski vacua at $\rho_c \neq 0$.

According to the previous section, supersymmetry requires the scalar potential of Eq. (24) to vanish at the minimum, i.e. $V(\sigma_c) = 0$. In addition, according to Eq. (16), $\rho_c \neq 0$ determines the gravitino mass

$$m_{3/2} = \sqrt{11}\rho_c/\tilde{M}_{\text{Pl}} = \sqrt{11}\tilde{M}_{\text{Pl}}\sigma_c. \tag{26}$$

[7] The quantum effective action may have the *imaginary* part (sometimes lost in perturbation theory) that contributes to the decay of the gravitino condensate after inflation. Our considerations are limited to the inflationary era by assuming the scale of the imaginary part to be much less than the scale of inflation.

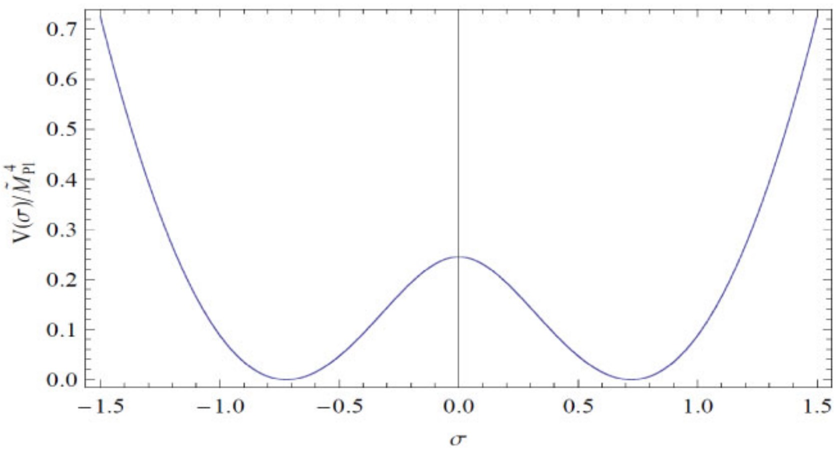

Fig. 1. The profile of the $V(\sigma)$ function in Eq. (24).

The non-vanishing values of ρ_c and σ_c are determined by the condition $dV/d(\sigma^2) = 0$, which yields the transcendental equation

$$121\sigma_c^2 \ln\left(1+\frac{\tilde{\Lambda}^2}{11\sigma_c^2}\right) = 11\tilde{\Lambda}^2 - 4\pi^2 > 0. \tag{27}$$

The hierarchy between the inflationary scale $H_{\rm inf.}$, the BI scale $M_{\rm BI}$, the SUSY-breaking scale $M_{\rm susy}$, the (super-)GUT scale $M_{\rm GUT}$, the effective gravitational scale $\tilde{M}_{\rm Pl}$, and the Planck scale $M_{\rm Pl}$ in our approach reads

$$H_{\rm inf.} \ll M_{\rm BI} = M_{\rm susy} \approx M_{\rm GUT} \approx \tilde{M}_{\rm Pl} \ll M_{\rm Pl}, \tag{28}$$

where "much less" means two to three orders of magnitude "less" (in GeV), and "approximately" means the same order of magnitude; see the next section for our numerical estimates. As regards the GUT scale, we take $M_{\rm GUT} \approx \mathcal{O}(10^{15})$ GeV.

4. Gravitino condensate as inflaton

A slow-roll inflation induced by gravitino condensation in supergravity was proposed and studied by Ellis and Mavromatos in Ref. [46]. Since our induced scalar potential differs from that of Ref. [46], we reconsider this inflation here by using $\tilde{\Lambda}$ and $\tilde{w}$ as the phenomenologically adjustable parameters.

A slow roll is possible near the maximum of the scalar potential of Eq. (24). Since the height of the potential at the maximum is related to the inflationary Hubble scale $H_{\rm inf.}$ by Friedmann equation,

$$V_{\rm max.} = 3M_{\rm Pl}^2 H_{\rm inf.}^2, \tag{29}$$

the value of $H_{\rm inf.}/M_{\rm Pl}$ is suppressed by the factor $(\tilde{M}_{\rm Pl}/M_{\rm Pl})^2$. On the other hand, the inflationary Hubble scale is related to the cosmic microwave background (CMB) tensor-to-scalar ratio r as

$$\frac{H_{\rm inf.}}{M_{\rm Pl}} = 1.06 \cdot 10^{-4}\sqrt{r}. \tag{30}$$

In turn, r is restricted by Planck (2018) measurements [49] as $r < 0.064$ (with 95% CL), which implies $H_{\rm inf.} < 6 \cdot 10^{13}$ GeV. Therefore, the ratio $(\tilde{M}_{\rm Pl}/M_{\rm Pl})$ should be of order $10^{-2} \div 10^{-3} \ll 1$ for viable inflation. This justifies our setup in Sect. 1. We define the dimensionless parameter γ as $(\tilde{M}_{\rm Pl}/M_{\rm Pl}) \equiv 10^{-3}/\gamma$, where γ is of order one.

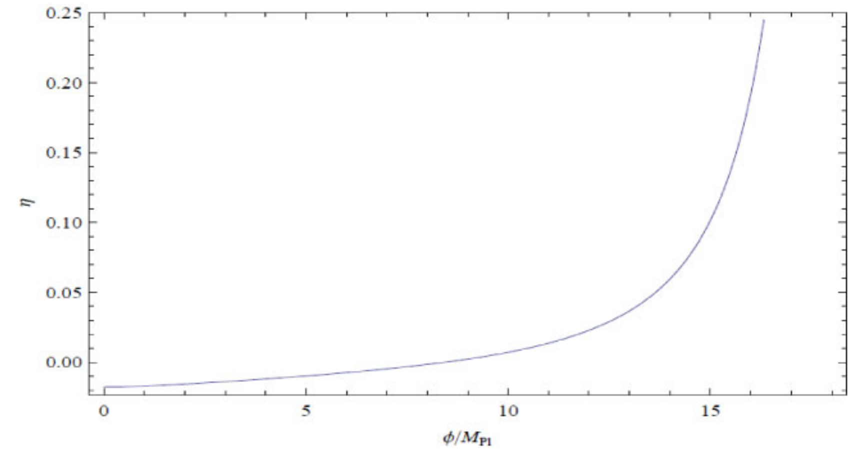

Fig. 2. The running of the slow-roll parameter η for $\gamma = 0.5$ and $\tilde{w} = 13$.

In our numerical calculations we have chosen the cutoff scale $\tilde{\Lambda} = 3$, so that the restriction in Eq. (25) is satisfied. Then, Eqs. (24) and (27) imply that

$$V_{\max}\tilde{M}_{\mathrm{Pl}}^{-4} = 0.245 \quad \text{and} \quad \sigma_{\mathrm{cr.}} = \tilde{w}^{-1}(\phi_{\mathrm{cr.}}/M_{\mathrm{Pl}}) = 0.722. \tag{31}$$

In turn, this yields the gravitino mass $m_{3/2}$ and the gravitino condensate mass $m_{\mathrm{cond.}}$ as follows:

$$m_{3/2} = 2.39\tilde{M}_{\mathrm{Pl}} \quad \text{and} \quad m_{\mathrm{cond.}} = m_\phi = \sqrt{8/11}\, m_{3/2} = 2.038\tilde{M}_{\mathrm{Pl}}. \tag{32}$$

We numerically studied the running of the slow inflationary parameters $\varepsilon = \frac{1}{2}M_{\mathrm{Pl}}^2(V'/V)^2$ and $\eta = M_{\mathrm{Pl}}^2(V''/V)$ with respect to the inflaton field ϕ for values of the parameter γ of 0.1, 0.5, and 1, and found that ε is always under $\mathcal{O}(10^{-4})$ so that it can be ignored within the errors of the Planck 2018 data. Then the value of the scalar index $n_s = 1 - 6\varepsilon + 2\eta = 0.9649 \pm 0.0042$ (with 68% CL) [49] can be reached with $\eta = -0.0177$ at the horizon crossing by using the parameter $\tilde{w}$ of order one. There are no additional constraints on the parameters γ and $\tilde{w}$ from demanding that the e-folding number,

$$N_e = -\frac{1}{M_{\mathrm{Pl}}^2}\int_{\phi_{\mathrm{ini.}}}^{\phi_{\mathrm{end}}} \frac{V}{V'}d\phi, \tag{33}$$

be between 50 and 60, as is desired for viable inflation, when assigning the inflaton field ϕ/M_{Pl} to run somewhere between 0 and 5 during inflation. The running of the slow-roll parameter η is displayed in Fig. 2.

In summary, our results qualitatively agree with those of Ref. [46], but quantitatively allow considerably higher values of ε and r up to order $\mathcal{O}(10^{-4})$, contrary to the $\mathcal{O}(10^{-8})$ of Ref. [46], with Planckian values of the inflaton ϕ during inflation, contrary to its sub-Planckian values of $\mathcal{O}(10^{-3})M_{\mathrm{Pl}}$ in Ref. [46]. Hence, the inflationary scale $H_{\mathrm{inf.}}$ can be as high as 10^{12} GeV versus the 10^{10} GeV of Ref. [46].

5. Adding the FI term

In order to uplift the Minkowski vacuum to a de Sitter vacuum (dark energy) in our approach, we need an extra tool of spontaneous SUSY breaking. In the BI theory (without chiral matter) coupled to supergravity such a tool can be provided by the (alternative) FI terms [18–21] that do not require the gauged R-symmetry, unlike the standard FI term [17] whose extension to supergravity is severely restricted [48].

The (Abelian) gauge vector multiplet superfield V can be decomposed into a sum of the reduced gauge superfield $\mathcal{V}$ including the gauge field A_μ, and the nilpotent gauge-invariant goldstino superfield $\mathcal{G}$ that contains only th egoldstino λ and the auxiliary field D [19],

$$V = \mathcal{V} + \mathcal{G}, \quad \mathcal{G}^2 = 0. \tag{34}$$

The simplest examples of the goldstino superfield are given by [18,19]

$$\mathcal{G}_1 = -4\frac{W^2\bar{W}^2}{\mathcal{D}^2W^2\bar{\mathcal{D}}^2\bar{W}^2}(\mathcal{D}W) \tag{35}$$

and

$$\mathcal{G}_2 = -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3}, \tag{36}$$

respectively, in terms of the standard $N = 1$ gauge superfield strength

$$W_\alpha = -\frac{1}{4}\left(\bar{\mathcal{D}}^2 - 4\mathcal{R}\right)\mathcal{D}_\alpha V, \tag{37}$$

where $\mathcal{R}$ is the chiral scalar curvature superfield. The W_α obeys the Bianchi identities

$$\bar{\mathcal{D}}_{\dot\beta} W_\alpha = 0 \quad \text{and} \quad \bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha} \equiv \bar{\mathcal{D}}\bar{W} = \mathcal{D}^\alpha W_\alpha \equiv \mathcal{D}W. \tag{38}$$

The field components are given by $W_\alpha| = \lambda_\alpha$, $\mathcal{D}W| = -2D$, and $\mathcal{D}_{(\alpha}W_{\beta)}\big| = i(\sigma^{ab})_{\alpha\beta}F_{ab} + \cdots$. The difference between the superfields $\mathcal{G}_1$ and $\mathcal{G}_2$ is only in the gauge sector, and is not essential for our purposes here.

The extra FI term with the coupling constant $\xi \neq 0$ is given by

$$S_{\mathrm{FI}} = \xi \int d^4x d^4\theta E\mathcal{G}, \tag{39}$$

where E is the supervielbein (super)determinant [1]. This FI term is manifestly SUSY- and gauge-invariant, does *not* include the higher spacetime derivatives of the field components, but leads to the inverse powers of the auxiliary field D (up to the fourth order) in the non-scalar sector of the theory.[8] Integrating out the auxiliary field D leads to a *positive* contribution to the cosmological constant,

$$V_\xi = \frac{1}{2}\xi^2 > 0. \tag{40}$$

Matching V_ξ with the observed cosmological constant allows us to include a viable description of the dark energy into our approach. The phenomenological values of the cosmological constant and the related contribution (ξ) to the VEV of the auxiliary field D are tiny, so that they do not affect our considerations of the high-scale SUSY breaking in the previous sections.

[8] The limit $\xi \to 0$ does not lead to a well-defined theory, so that $\langle D\rangle = \xi$ must be non-vanishing.

The nilpotent goldstino superfield $\mathcal{G}$ introduced above is composed of the usual (standard) superfields and, hence, is very different from the intrinsically nilpotent goldstino superfield introduced in Refs. [35–38].

As the FI term affects the quartic and higher-order terms with respect to the gauge field and its fermionic (spin-1/2) superpartner, back reaction of the FI term on the effective action should be examined (work in progress). This should be done together with quantum renormalization of those terms and, perhaps, requires a field-dependent FI parameter ξ. The D-type scalar potential and the associated dark energy are expected to be unaffected because of cancellation of (perturbative) quartic and quadratic (ultraviolet) divergences due to supersymmetry of the action.

6. Conclusion

The gravitino condensate can be considered as a viable candidate for the inflaton in supergravity, when assuming the effective (quantum) gravity scale to be close to the (super-)GUT scale that is also close to the SUSY-breaking scale in our approach, with all scales close to 10^{15} GeV. Actually, in this scenario we have the hyper-GUT where *all* fundamental interactions merge, including gravity. At the same time, it is the weak point of our calculations because we ignored (other) quantum gravity corrections.

The inflationary (Hubble) scale is well below the GUT scale, and can be as large as 10^{12} GeV. The gravitino mass is above the inflationary scale, so that there is no gravitino overproduction problem in the early Universe. The constraints from proton decay and big bang nucleosynthesis are very weak because of high-scale SUSY. Then, SUSY is not a solution to the hierarchy problem with respect to the electroweak scale. This is similar to the setup studied in Refs. [50,51]. Our scenario is consistent with the known Higgs mass of about 125 GeV after taking into account the extreme possible values of the gaugino mixing parameter $\tan\beta$ in the context of SUSY extensions of the Standard Model [52].

As regards reheating after inflation, the inflaton (gravitino condensate) field decays into other matter and radiation, which is highly model dependent, as usual. Unlike Ref. [24], the inflaton as the gravitino condensate cannot decay into gravitinos because Eq. (32) leads to the kinematical constraint $2m_{3/2} > m_{\text{cond.}}$. It also implies that the gravitino cannot be a dark matter particle in this scenario. A detailed study of reheating requires knowledge of the couplings of the gravitino and gravitino condensate to the Standard Model particles, which is beyond the scope of this paper.

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