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SIMULATION OF NON-STATIONARY PROCESSES IN INDUSTRIAL CENTRIFUGAL CASCADES OF URANIUM ENRICHMENT

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The mathematical model of non-stationary dividing processes of uranium enrichment in industrial centrifugal cascades which can be used in a computer simulator to prepare experts in dividing production and application as an expert system in the automated control system of technological circuit has been developed and realized.

Introduction

It is known that generally non-stationary processes in dividing processes are described by non-linear equation of the second order in partial derivatives (with respect to time and the number of step) of parabolic type with variable coefficient, the analytical solutions of which are constructed only for a limited number of problems [1]. Much more perspective methods of researching non-stationary processes are those of numerical solution of equation describing these processes. Many works are devoted to the questions of numerical research of non-stationary processes in dividing cascades, e. g. [2]. However, mathematical methods investigated in these works either have a limited practical application or take much time to solve them numerically or do not take into consideration the features of process flowsheet construction of centrifugal production. In this case the simplest symmetrical counterflow rectangular-sectional cascade [3] in which the sections of the steps operate in the same hydraulic conditions. Meanwhile, the effect of centrifuge enrichment is significantly higher (about 100 times) than that of diffusive devices and the length of centrifugal cascades is essentially less. As a rule, steps operate in dissimilar hydraulic modes. Under these conditions the contribution of each step to the cascade operation increases, and, therefore, investigating the non-stationary processes it is important to know the state of every step.

In the work the mathematical model describing nearly all types of non-stationary processes both in a separate step and in cascades of optional construction differing from traditional symmetrical counterflow cascades has been proposed. Besides, all the multicascade technological circuit presenting a combination of cascades with their connecting lines of multicascade communications (MCC).

The model is an addition to the developed earlier mathematical model of non-stationary hydraulic processes in multicascade technological circuit (TC) of centrifugal plant [4, 5].

1. Description of mathematical model of non-stationary dividing processes

Multicascade TC of dividing plant consisting of K-cascades (fig. 1) is considered. Each cascade m ($1 \le m \le K$) consists of N_m steps with numbering from dump to selection.

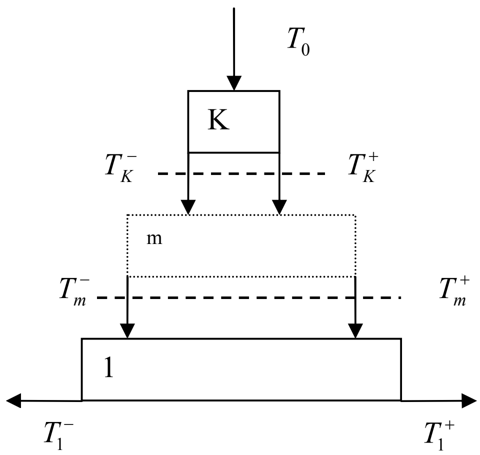

Рис.1. *Многокаскадная ТС*

The circuit of step connection in cascade can be optional, e.g. the so-called parallel-series (fig. 2).

The step of number n consists of S_{nm} parallel connected sections; section i_{nm} ($1 \le i_{nm} \le S_{nm}$) consists of J_{inm} parallel connected gas centrifuges. Gas centrifuges in the section are considered to be identical; the sections can be various (different number of GC, different geometrical sizes of communication, etc.). The design factor K (of order), N_m (of order 4...25 are defined by the nomenclature of the production, change of bundling and technological characteristics of GC.

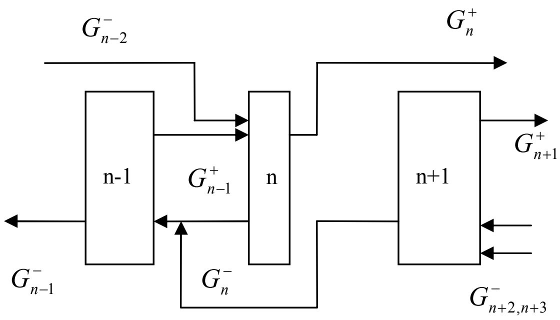

Fig. 2. *Parallel-series circuit of step connection in cascade*

In the work involved a one-volume model of dividing step. Each step of cascades of multicascade technological circuit of centrifugal production is considered to be filled with binary mixture with the same concentration of isotope ^{235}U equal to its concentration in the supply collector of this step. Gas content of dividing step is considered to be equal to the sum of gas contents of all the volumes forming the step and taken into consideration in calculation of TC non-stationary hydraulics [4]. At the output from the step (in the collectors of selection and dump) of concentration determined by the dividing properties of the steps, correspondingly higher and lower the supply concentration.

The main equation of non-stationary division process is that of the balance of the enriched component supplying mixture in step n of cascade m of multicascade TC:

$$\frac{d(M_{nm}C_{0nm})}{dt} = T_{0n}(t)K_{0n}(t) + \sum_{k=1}^{K}\delta_{Rkn}T_{Rkn}K_{Rkn} + \sum_{k=1}^{K}\delta_{Wkn}T_{Wkn}K_{Wkn} + \sum_{p=n+1}^{N}\delta_{nTp}G_{Tp}C_{Tp} + \sum_{w=1}^{n-1}\delta_{nPw}G_{Pw}C_{Pw} + \delta_{OP}C_{3акр,m}(G_{ЛNm} - T_{0Pm}(t)) - \sum_{i=1}^{S}L_{01inm}C_{0nm}, \quad (1)$$

where K – a number of TC cascades; S – the quantity of technological sections in step n of cascade m; M_{nm} – gas content of step n of cascade m; C_{0nm} – concentration of supplying mixture of step n of cascade m; $T_{0n}(t)$, $K_{0n}(t)$ – consumption and concentration of the external supply flow; δ_{Rkn} – a sign of connection of selection flow T_{Rkn} with concentration K_{Rkn} of cascade k with the step supply n ($k\neq m$) of cascade m ($k\neq m$); δ_{Wkn} – a sign of connection of dump flow T_{Wkn} with concentration K_{Wkn} of cascade k with the supply of the step n ($k\neq m$) of cascade m ($k\neq m$); δ_{nTp} – a sign of connection of dump flow G_{Tp} with concentration C_{Tp} of step n of cascade m with step supply n of cascade m; δ_{nPw} – a sign of connection of selection flow G_{Pw} with concentration C_{Pw} of step w of cascade m with step supply n of cascade m; δ_{OP} – a sign of selective step (0 or 1); $G_{ЛNm}$, T_{OPm} – correspondingly selection flow of selective step N_m ($m = 1...K$) and cascade m; $C_{3акр,m}$ – flow concentration of cascade spin m; L_{01inm} – supply flow of section i of step n of cascade m.

In general case the forming the equation magnitudes (1) of gas content steps, cascade supply flows (external with respect to TC and intercascade flows being the selection and dump flows of other cascades as well as interstep flows) are the time functions. These functions can be obtained by solution of hydrodynamic equations considered in the works [4, 5].

The concentrations of intercascade supply flow K_{Rkn} and are K_{Wkn} defined through the corresponding concentrations of selection and dump flow of cascades taking into account transport delay τ_{zad}, by which the time of passing hydraulic excitement along the appropriate MCC lines is understood. The time τ_{zad} is defined by the solution of continuity equations for a steady one-dimensioned flow isolated from the environment [6] with respect to unknown velocity of gas flowing υ_i along i-area of MCC line.

$$G = S_i\rho_i\upsilon_i,$$

where G – calculated value of flow in the MCC line (the same for all areas of MCC line) g/sec; S_i – square of section of i-area of MCC line, sm^2; $\rho_i = \frac{\mu P_i 1333}{R_e T_i}$ – gas density in i-area of MCC line, g/sm^3; μ – molar mass of gas, g/mol; P_i – calculated value of gas pressure in i-area of MCC line, mmHg; R_e – universal gas constant, erg/(molK); T_i – gas temperature, K; υ_i – gas velocity in i-area of MCC line, sm/sec.

The values of pressure in i-area of MCC line are defined form the accepted turbulent character of gas flowing known from the solution of hydraulic problem of some pressure value (e.g. the pressure in supply collector of selective step) and assigned values of geometrical sizes of these areas.

Selective steps of cascades have some features connected with the fact that the part of the selection flows through the governors of spin come back as a supply of these steps. Besides, spin flow is fed to the parts of cascades after passing the purifying cascade (PC). Hence, the concentration of spin flow $C_{3акр,m}$ changes with some transport delay with respect to concentration of selection flow of selective step. The time of transport delay is also defined in the problem of non-stationary hydraulics [4, 5].

Equation (1) is set up for each step of cascades of multicascade TC. The obtained set of equation consists of KN_m-differential equations with 3 KN_m unknowns, which are the concentrations of the step supply flow, of selection and dump (C_{0nm}, C_{Pnm}, C_{Tnm}). To reduce the dimension of the system let us use the equation of conservation of light component and the relation defining total coefficient of step division χ_{nm}.

$$C_{0nm} = \theta_{nm}C_{Pnm} + (\theta_{nm} - 1)C_{Tnm}, \quad (2)$$

where θ_{nm} – coefficient of flow division ($\theta_{nm} = \frac{G_{Pnm}}{G_{0nm}}$).

$$\chi_{nm} = \frac{C_{Pnm}(1 - C_{Tnm})}{C_{Tnm}(1 - C_{Pnm})}. \quad (3)$$

From equations (2) and (3) one can define C_{Tnm}:

$$C_{Tnm} = \frac{C_{Pnm}}{\chi_{nm} - C_{Pnm}(\chi_{nm} - 1)}. \quad (4)$$

By means of substitution of the expressions for C_{Tnm} and C_{0nm} set of equations (1) amounts to the set of differential expressions relatively KN_m of unknown C_{Pnm} as time functions. The solution of this set of equations at the given initial values of C_{Pnm} simulates non-stationary dividing processes in TC cascades.

2. The algorithm of equation solution of non-stationary division

Having denoted the right part of the equation (1) after substitution in it the expressions (2) and (4) through $\psi_{nm}(t)$ we obtain:

$$\frac{d[M_{nm}(\theta_{nm}C_{Pnm} - (1-\theta_{nm})f_{Ct}(C_{Pnm}))]}{dt} = \Psi_{nm}(t), \quad (5)$$

where $f_{Ct}(C_{Pnm})$ – the function of concentration of selection step n of cascade m determined by the expression (4).

The set of equations (5) presents a Cauchy problem with the given initial conditions. The Cauchy problem, with few exceptions, does not have analytical methods of solution. In the numerical method the values of the sought-for functions in discrete moment of time are calculated instead of seeking continuous functions of time, the set of equations being replaced by difference equations in this or that way. In the algorithm applied, similar to hydraulic problem [4, 5], Euler implicit model with re-computation [7], which possesses the third order of accuracy on the step and the second interval is used. According to it we have:

$$M_{nm}^{(K+1)}(\theta_{nm}^{(k+1)}C_{Pnm}^{(k+1)}-(1-\theta_{nm}^{(k+1)})f_{Ct}^{(k+1)}(C_{Pnm}^{(k+1)}))-$$
$$M_{nm}^{(k)}(\theta_{nm}^{(k)}C_{Pnm}^{(k)}-(1-\theta_{nm}^{(k)})f_{Ct}^{(k)}(C_{Pnm}^{(k)}))=$$
$$=\frac{\Delta t}{2}(\Psi_{nm}^{(k+1)}+\Psi_{nm}^{(k)}). \quad (6)$$

Having written down the equations (6) for all the steps of TC cascade, we obtain the set of algebraic nonlinear equations relatively $C_{Pnm}^{(K+1)}$. Nonlinearity is explained by the presence of terms containing the expression $f_{Ct}^{(K+1)}C_{Pnm}^{(K+1)}$. At each time step the linearization of its nonlinear terms is performed. The algorithm proposed it consists in substitution of the fuction $f_{Ct}^{(K+1)}C_{Pnm}^{(K+1)}$ the equation of tangent in point .

$$C_{Tnm}^{(K+1)}=a_{Tnm}^{(k)}+b_{Tnm}^{(k)}C_{Pnm}^{(k+1)}. \quad (7)$$

Coefficients $a_{Tnm}^{(K)}$ and $b_{Tnm}^{(K)}$ are calculated from the condition of equality of linear (7) and nonlinear (4) expressions as well as their derivatives in some point C_{0Pnm}.

$$\frac{C_{0Pnm}}{\chi_{nm}-C_{0Pnm}(\chi_{nm}-1)}=a_{Tnm}+b_{Tnm}C_{0Pnm}, \quad (8)$$

$$(C_{Tnm})'=\frac{\chi_{nm}}{[\chi_{nm}-C_{0Pnm}(\chi_{nm}-1)]^2}=b_{Tnm}. \quad (9)$$

From the equation (8, 9) we obtain:

$$a_{Tnm}^{(k)}=-\frac{(C_{Pnm}^{(K)})^2(\chi_{nm}^{(K)}-1)}{[\chi_{nm}^{(K)}-C_{Pnm}^{(K)}(\chi_{nm}^{(K)}-1)]^2},$$

$$b_{Tnm}^{(k)}=\frac{\chi_{nm}^{(K)}}{[\chi_{nm}^{(K)}-C_{Pnm}^{(K)}(\chi_{nm}^{(K)}-1)]^2}.$$

After linearization we get the set of linear equation with respect to unknowns $C_{Pnm}^{(K+1)}$, which are approximation to the magnitudes $C_{Pnm}(t+\Delta t)$. The set is solved by Gauss method of exception [7, 8]. As initial conditions the initial magnitudes $C_{Pnm}^{(0)}=C_{Pnm}(t=0)$ are taken as calculated data of initial stationary mode of the TC investigated.

The values of coefficients of total division χ_{nm} of steps are determined by dividing properties of the steps. Dividing properties of a separate centrifuge (GC) are described by empirical dependence defined as a result of mathematical treatment of experimental data obtained in testing the equipment of GC of the given type.

$$\chi_{nm}=\chi_{nm}(g,\theta), \quad (10)$$

where g, θ – supply consumption and coefficient of flow division of separate GC.

Considering magnitude χ invariant, independent on concentration of supply flows of the steps one can connect the magnitudes C_{Tnm} and C_{Pnm} by the formula (4) at operation of steps in concentration region differing from concentration of natural material.

Dividing characteristic $\chi_{nm}(g,\theta)$ is defined at GC operation in stationary conditions when there is a substance balance and light component. Application of this characteristic in non-stationary process is explained by the fact that the processes of attaining equilibrium in a separate GC proceed significantly faster than they do in TC cascades. The centrifuge is considered as one operating in stationary conditions which is slowly changing.

According to the model described in Borland Delphi 5.0 medium bundled software is realized and the calculations for the following cases of non-stationary TC excitements: change of selection and supply flow; transfer of supply delivery points; closing steps and sections of cascades; change of frequency of GC supplying current; change of equipment loading; change of external supply flow concentration. As a dividing one the approximate model characteristic differing from the real one, but showing the main tendencies of real process proceeding is used.

As an example, present the results of numerical calculation of non-stationary processes of division appearing at closing separate steps of a nine-step cascade (fig. 3).

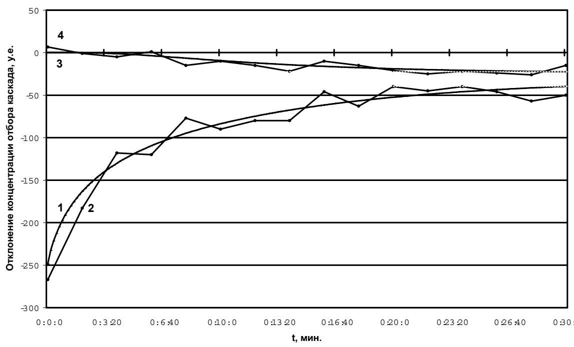

Fig. 3. *Time dependence of concentration deflection of selection cascade with respect to reference quantity in standard units (1 s.u.=0,001 %). Closing steps: 1) calculation; 2) real – selective and 3) calculation; 4) real – predump*

Comparative analysis of the pairs of curves 1, 2 and 3, 4 permits to conclude that the developed model describes the behaviour of the real object adequately.

The increase of enrichment of cascade selection with time (1, 2) is explained by the fact that from the moment of step closing the cascade passes to the hydraulic mode with high loading on the remaining steps at which the magnitude χ has great value. The broken character curves 2, 4 is explained by the technique of concentration measurement of ^{235}U based on recording of γ-radiation of working gas mixture, intensity of which has a probabilistic nature.

From the figure it is also seen that the influence of selective step on the final product is significantly higher than that of dump steps.

The calculations performed show that properties of quick action of the model permit to carry out calculations of multicascade technological circuit efficiently in the condition of real time. Thus, for example, the time of calculation of four-hour time interval of non-stationary process for a four-cascade TC with total number of steps of the order of 50 and the time step equal to 1 sec. amounts about 140 sec. In this connection the developed model has found application in ACSTC as an expert system. Besides, the developed model is used in computer simulator to train specialists if dividing production [9, 10].

Conclusion

Mathematical model of non-stationary dividing processes in the industrial centrifugal cascades taking into account the construction distinguished features if the technological circuit involved in dividing production has been proposed. It presents a set of differential equations of the first order with specified initial conditions. The algorithm of equation solution describing non-stationary processes in centrifugal cascades is developed.

The model realized allows to obtain solutions in real time conditions at optional set of exciting actions and research the non-stationary processes occurring at the change of external flow values, their disconnection; the change of frequency and current interruption; the change of supply delivery point; failure and realignment of multistep governor; switching off the part of the equipment; failure in the operation of equipment of MCC lines; at different combination of these factors.

The model is introduced and used in ACSTC of dividing production at the Electrochemical plant an expert system as well as in computer simulator to train specialists of dividing production.

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