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MATHEMATICAL MODEL FOR CONSIDERING THE CHANGE IN HEATER CAPACITY FOR PROPORTIONAL TEMPERATURE REGULATOR OF THE HYBRID-FILM MICROTHERMOSTAT

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The numerical modeling of two-dimensional non-stationary temperature fields of thermostatically controlled substrate for proportional temperature regulator of the hybrid-film microthermostat in view of change in heater capacity caused by instability of feeding currents has been carried out. The method of final differences is used together with the scheme of splitting along the coordinates and the trial run method. The mathematical model of definition in change of heater capacity for the proportional temperature regulator of hybrid-integrated schemes is offered, allowing providing stability of work of the automatic temperature control system.

Introduction

The thermostatting system implemented in the form of the heated hybrid-film microthermostat (MT) functions as temperature stabilizer of thermostatically controlled object (substrates of hybrid-integral circuits). The efficiency of MT functioning and, as a result, temperature stability of operation of the whole radio-electronic device, is considerably determined by the efficiency of functioning of automatic control system (ACS) of the substrate temperature and, in particular, the selected type of regulation.

Proportional, astatic, isodromic and proportional-integral-differential control is applied in microthermostates. Each of the mentioned control types has certain features and applied depending on the made demands to control: stability to destabilizing actions, accuracy of thermostatting, process stabilization time etc.

It should be noted that complication of control type is not always efficient. On the contrary, at changing to microthermostatting, i.e. to decreasing the volume of thermostatting, the advantages of more complex types of regulators (the latter three types may be referred to such types) should appear more seldom.

Designing the radio-electronic device (RED) of special purpose, one should have an idea, in each concrete case, of the levels of changing destabilizing factors, at which steady operation of ACS and the whole RED is provided. Investigation in determining the specified levels by natural experiment is difficult as a great number of brassboards should be made at its implementation considering stability dependence on:

- design-process parameters of MT: overall sizes and base material, relative spatial arrangement of heater and sensor, overall sizes and heater capacity, material of filling material of MT chamber etc.;
- nature of destabilizing action: supply voltage instability, environment temperature change, faulty operations etc.

Therefore, application of mathematical modeling of thermophysical control processes of temperature in the considered MT systems is of special interest.

Taking into account the abovementioned, the author examines the mathematical model of considering the change of heater capacity caused by instability of supply voltages for proportional temperature controller in the system of hybrid-film MT.

Convergence of control process of thermostatically controlled substrate temperature is meant by stability of denoted controller operation.

Mathematical statement of the problem

The generalized physical model of the studied MT type is introduced [1. P. 227]. The geometry of decision region is given in Fig. 1.

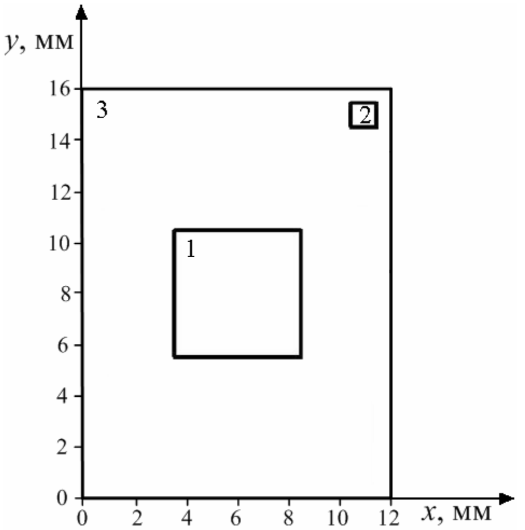


Fig. 1. *Geometry of decision region (top view): 1) heater; 2) temperature sensor; 3) thermostatically controlled substrate*

The main assumptions used at problem statement:

1. Heat generation of thermostatically controlled elements on substrate in comparison with heater capacity may be neglected;
2. The substrate represents a homogeneous isotropic body the thermophysical parameters of which do not depend on coordinates and temperature;
3. Thermal contact on the boundary between bodies (regions) is considered to be ideal;
4. Heat sink from lower and upper surfaces of thermostatically controlled substrate into environment due to radiation heat transfer (the substrate is in vacu-

um) is taken into account in the heat conduction equation by additional heat generation source;

5. Heat exchange from lateral faces is taken into account in heat conduction equation due to increasing the power of additional sources of heat generation (assumption 4).

The problem in such statement comes to solution of two-dimensional non-stationary non linear heat conduction equation of thermostatically controlled substrate (with proper initial and boundary conditions) together with the equation of temperature proportional controller, the model of operation of which is introduced in Fig. 2 and the equation of changing the heating element power with the course of time. Radiation heat exchange from substrate surface in boundary conditions is taken into account by Stefan-Boltzmann law. Heat sink into environment due to convection is absent; this assumption is conditioned by a distance to MT body surface less than 5 mm.

$$\begin{cases} c_p\rho\dfrac{\partial T}{\partial t}=\lambda\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}\right)+\dfrac{P_{\text{Н}}(x,y,T_{\text{Д}})}{S_{\text{Н}}h}+\\ +\dfrac{P_{\text{ВОЗМ}}(x,y,t)}{S_{\text{Н}}h}-k(x,y)\dfrac{\varepsilon_{\text{ПР}}\sigma(T^4-T_{\text{ВН}}^4)}{h};\\ P_{\text{Н}}(T_{\text{Д}})=P_{\text{Н.МАКС}},\ \text{при}\ T_{\text{Д}}\le T_{\text{СТ}};\\ P_{\text{Н}}(T_{\text{Д}})=P_{\text{Н.МАКС}}-\dfrac{P_{\text{Н.МАКС}}}{\Delta T_{\text{СТ.З.}}}(T_{\text{Д}}-T_{\text{СТ}}),\\ \qquad \text{при}\ T_{\text{СТ}}<T_{\text{Д}}<T_{\text{СТ.МАКС}};\\ P_{\text{Н}}(T_{\text{Д}})=0,\ \text{при}\ T_{\text{Д}}\ge T_{\text{СТ.МАКС}};\\ P_{\text{Н}}(x,y)=P_{\text{Н}}(T_{\text{Д}}),\ \text{при}\ x,y\in[S_{\text{Н}}];\\ P_{\text{Н}}(x,y)=0,\ \text{при}\ x,y\notin[S_{\text{Н}}];\\ P_{\text{ВОЗМ}}(x,y)=P_{\text{ВОЗМ}}(t),\ \text{при}\ x,y\in[S_{\text{Н}}];\\ P_{\text{ВОЗМ}}(x,y)=0,\ \text{при}\ x,y\notin[S_{\text{Н}}], \end{cases} \tag{*}$$

$$\begin{cases} t\in[0;t_{\text{МАКС}}],\quad x\in[0;L_x],\quad y\in[0;L_y];\quad T|_{t=0}=T_0(x,y);\\ x=0,y\in[0;L_y]:\quad -\lambda\dfrac{\partial T}{\partial x}=\varepsilon_{\text{ПР}}\sigma(T^4-T_{\text{ВН}}^4);\\ x=L_x,y\in[0;L_y]:\quad -\lambda\dfrac{\partial T}{\partial x}=\varepsilon_{\text{ПР}}\sigma(T^4-T_{\text{ВН}}^4);\\ y=0,x\in[0;L_x]:\quad -\lambda\dfrac{\partial T}{\partial y}=\varepsilon_{\text{ПР}}\sigma(T^4-T_{\text{ВН}}^4);\\ y=L_y,x\in[0;L_x]:\quad -\lambda\dfrac{\partial T}{\partial y}=\varepsilon_{\text{ПР}}\sigma(T^4-T_{\text{ВН}}^4), \end{cases}$$

where x, y are the spatial values; c_p, ρ, λ are the specific heat, density and heat conductivity coefficient of base material, respectively; t, $t_{\text{МАКС}}$ are the current and maximum time of calculation, respectively; $S_{\text{Н}}$ is the heater area; h is the substrate thickness; T, $T_{\text{ВН}}$ are the temperatures of substrate and environment; σ is the Stefan-Boltzmann constant; $\varepsilon_{\text{ПР}}$ is the reduced emittance of the surface and environment; k is the coefficient taking into account heat exchange from lateral faces; $P_{\text{Н}}$, $P_{\text{Н.МАКС}}$ are the current and maximum heat capacities; $P_{\text{ВОЗМ}}$ is the disturbance power; $\Delta T_{\text{СТ.З}}$ is the specified range of static temperature; $T_{\text{Д}}$, $T_{\text{СТ}}$ are the temperature of sensor and static temperature; T_0 is the initial substrate temperature; L_x, L_y are the sizes of substrate x and y-direction, respectively.

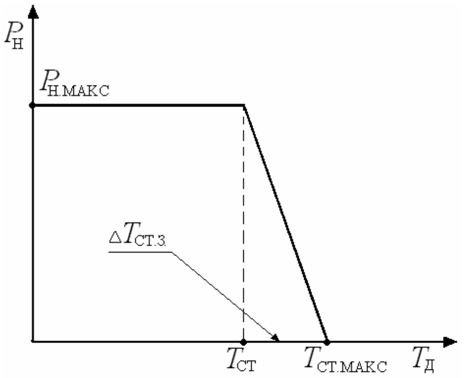


Fig. 2. *The model of operation of proportional temperature controller*

The disturbances ($P_{\text{ВОЗМ}}$) occurring in heater region caused by instability of supply voltages are modeled by the third summand in the right part of heat conduction differential equation (*); the forth summand considers heat energy sink into environment due to heat radiation [2].

Diagrams of function of proper changes are introduced in Fig. 3

The boundary-value problem stated in this way is solved by the finite difference method [2–6] with application of the equivalent circuit by the coordinates (locally one) and sweep method acquired the reputation for solving the heat exchange problems [2, 5–7]. The iterative process is constructed at each time step for refining coefficient values depending on solution. In this case the computational process stability is controlled by a number of iterations necessary for obtaining the required accuracy [6].

Discussion of the results

Adequacy of the implemented mathematical model and the method of solution is experimentally tested [8]. A good agreement (in the margin of error of the method of solution and errors included into mathematical model of empirical formulas) with the experimental data is obtained. It may indicate the validity of the results of numerical simulation of temperature fields in such constructions.

The main numerical results of the work (Fig. 4, 5) are given at the following values of the initial data and parameters:

- the overall sizes of thermostatically controlled substrate: 12×16×1 mm;
- base material – ceramics of the type VK-94 [10]: c_p=1888 J/(kg·K), ρ=3800 kg/m^3, λ=13,4 W/(m·K);
- heater: sizes – 5×5×1 mm and capacity – 0,5 W;
- the range of the environment temperature change is within limits of 223...323 K;

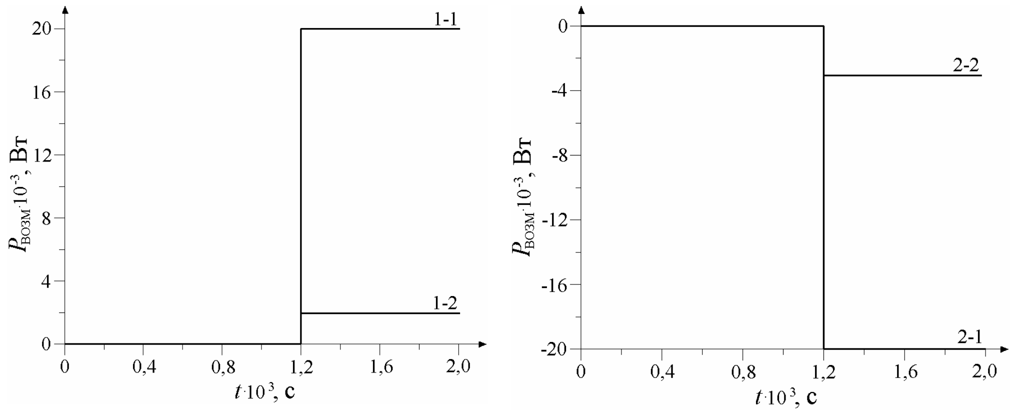


Fig. 3. *Diagrams of function of disturbing effect $P_{возм}=f(t)$ for environmental temperature: 223 K (1–1, 2–1); 323 K (1–2, 2–2); $|P_{возм}|=0,1\cdot P_{н.уст}$*

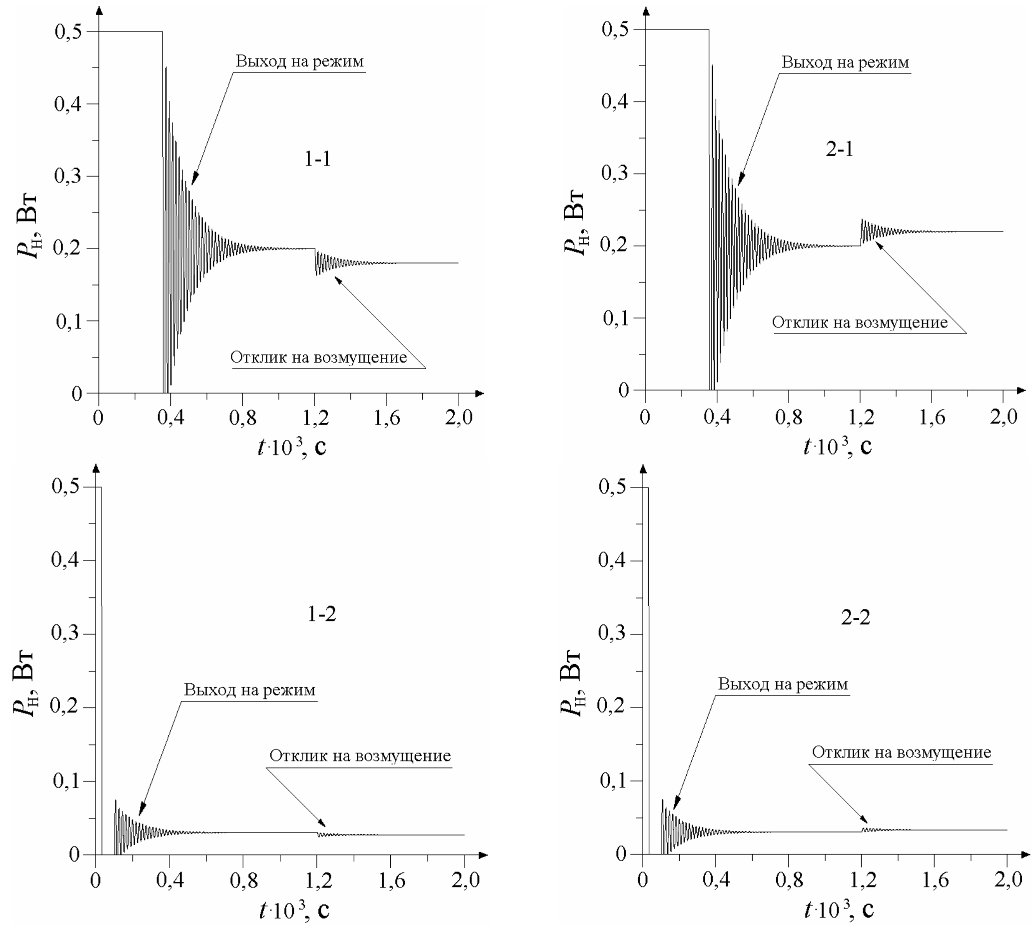


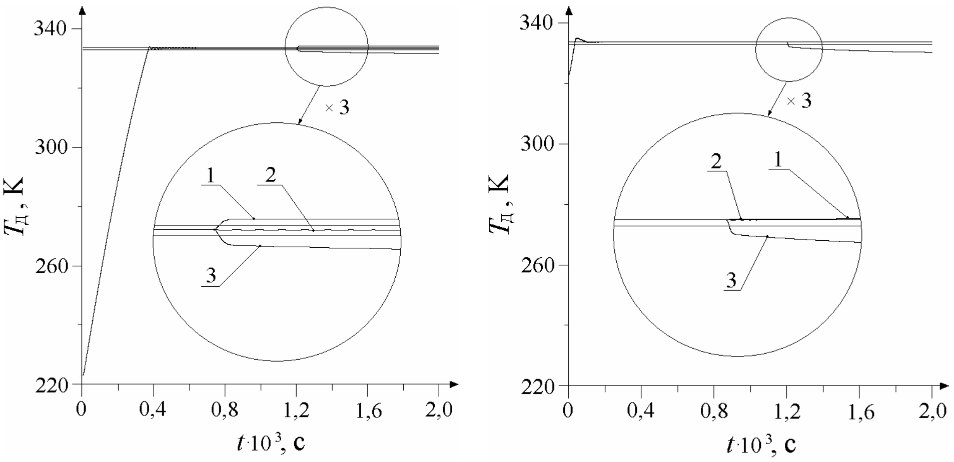
Fig. 4. *Change of heater capacity at disturbing action value $0,1\cdot P_{н.уст}$ for magnitudes of environment temperature: 223 K (1–1, 2–1); 323 K (1–2, 2–2)*

- static temperature – 333 K;
- disturbing effect value is in the range 0,1...15,7 from the heater capacity in steady state ($P_{н.уст}$).
- temperature sensor has sizes: 1×1×1 mm (Fig. 1);
- the reduced emittance of surface and environment – 0,8;
- accuracy of calculations amounts to 0,1 K;
- the given range of static temperature – 0,7 K.

The selected value $\Delta T_{ст3}$ is on the boundary of temperature control stability of thermostatically controlled substrate.

The analysis of the obtained results (Fig. 4, 5) showed that at unit change of heater capacity with value $0,1\cdot P_{н.уст}$ (Fig. 5, curve 2), caused by instability of supply voltages, the system of proportional control of substrate temperature is stable. A jump of disturbance power value:

- to positive side more than by the value $P_{н.уст}$ (Fig. 5, curve 1);



a *b*

Fig. 5. *Change of substrate temperature at various values of disturbing action for environment temperature values: a) 223 K; b) 323 K*

- to negative side by the value $1{,}5 \cdot P_{\text{Н.УСТ}}$ (Fig. 5, *a*, curve 3), and $16{,}0 \cdot P_{\text{Н.УСТ}}$ (Fig. 5, *b*, curve 3), results in the fact that the system does not reach the temperature control mode.

Conclusion

1. The numerical modeling of two-dimensional non stationary temperature fields of thermostatically controlled substrate was carried out considering the change of heater capacity caused by instability of supply voltages, for proportional temperature controller of hybrid-film microthermostate.
2. The mathematical model for determining the levels of heater capacity change for proportional temperature controller of hybrid-film circuits allowing providing stability of operation of temperature automatic control system was proposed.

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