Conclusion

The suggested approach allows arranging localization regions of system dominant poles with interval coefficients of characteristic polynomial in specified truncated sector with any angle Θ_0, that is supporting guaranteed oscillation and degree of stability of IS.

The suggested technique is not highly computational and allows carrying out parametric synthesis of low order regulator.

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PARAMETRIC SYNTHESIS OF LINEAR REGULATOR IN INTERVAL SYSTEM WITH GUARANTEED ROOT QUALITY INDICES

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Automatic control system containing proportional-plus-integral action regulator and control object which has interval specified parameters has been considered. Using robust expansion of root-locus method the technique of synthesis of proportional-plus-integral action regulator parameters guaranteeing minimal degree of stability and maximal degree of system oscillation was developed. The technique is based on vertex analysis of root quality indices applying the equation of Theodorchik-Evans. The numeric illustration is given.

Introduction

In real systems of automatic control there are cases when some their parameters are not known exactly or change in the system maintenance process by laws unknown beforehand and their values can not be available for measuring. If the ranges of possible values of constant parameters or unstable parameters are known then it is said about parametric interval uncertainty. Systems having interval-indefinite parameters were called interval ones.

Designing interval system the main task is in supporting the desired quality of its functioning at any possible values of interval-indefinite parameters. Let us introduce root indices of system quality: degree of stability α and oscillation φ. It is obvious that at system parameter instability these quality indices may be changed. Therefore, the task of supporting the guaranteed minimal degree of stability and maximal oscillation in interval system is of interest.

To specify the desired quality of the system corresponding to these root indices the sector ABCD (Fig. 1) specifying the boundary of localization region of roots Γ may be used.

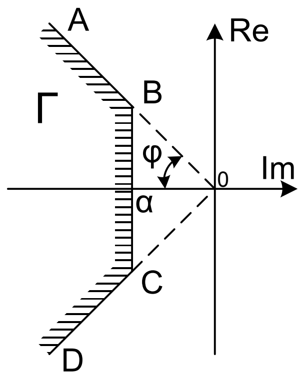

Fig. 1. *The region of desired root arrangement*

1. Problem statement

Let us consider the system of automatic control, Fig. 2.

Let linear regulator has transfer function of proportional-integral (PI) action regulator:

$$W_P(s) = \frac{K_П \cdot s + K_И}{s}, \qquad (1)$$

where $K_П$, $K_И$ are the adjustments of regulator and transfer function of control object has the form:

$$W_{OY}(s) = \frac{A(s)}{B(s)}, \qquad (2)$$

where $A(s) = \sum_{i=0}^{m} a_i \cdot s^i$, $B(s) = \sum_{j=0}^{n} b_j \cdot s^j$, $\underline{b_j} \le b_j \le \overline{b_j}$.

Then interval characteristic polynomial of the system may be written down in the form:

$$(K_И + K_П \cdot s) \cdot A(s) + s \cdot B(s) = 0. \qquad (3)$$

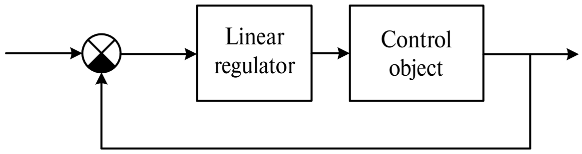

Fig. 2. *Diagram of automatic control system*

The region of possible values of interval-indefinite parameters of the system (polyhedron P_n, being a rectangular hyperparallelepiped) images to the root complex plane in the form of localization regions of complex-conjugate roots and real line cuts where real roots locate (Fig. 3).

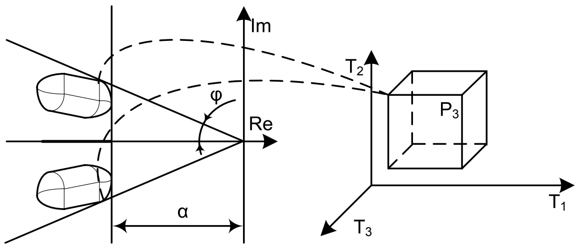

Fig. 3. *Imaging parametric polyhedron P_3*

It is necessary to determine adjustments of PI regulator supporting arrangement of root regions of interval characteristic polynomial in specified sector Γ at any values of control object interval parameters.

2. Specification of minimal degree of stability of interval system

Let us reduce interval polynomial (3) to the form

$$\sum_{i=0}^{k} c_i \cdot s^i = 0, \qquad (4)$$

where coefficients c_i may be functions of interval parameters of control object and regulator adjustments. Interval coefficients c_i form parametric polyhedron the vertexes of which are determined by their extreme.

It is known from root locus theory that if the root is on a real line then the angle of root locus output from this root amounts to 0° or 180°. It is obvious that for real root of interval polynomial to move to the left at any interval coefficient change it is necessary for outlet angles by all these coefficients to amount to 180°. The set of interval coefficients that supports the given requirement is determined on the basis of the following statement.

Statement

If polynomial interval coefficients are specified by alternating ranges $\overline{c_0}\underline{c_1}\overline{c_2}\underline{c_3}$..., starting with maximal for c_0, then the given set of coefficients determines the real root the outlet angles of root locus of which by all interval coefficients amount to 180°.

Argument

It is known from the main phase condition of root locus theory that root outlet angle at c_i increase may be found by the formula:

$$\Theta_i^q = 180° - \sum_{p=1}^{n} \Theta_p + i\Theta_0, \qquad (5)$$

where Θ_p and Θ_0 are the angles between the real line and vectors directed from the root respectively to *g* pole and *i* function zeroes.

Let polynomial (4) has right real root S^*. Then for any pair of complex-conjugated roots and for any real root being to the left from S^* on the basis of root locus properties it may be concluded that $\Theta_{P1}+\Theta_{P2}=360°$, $\Theta_{P3}=0°$, $\Theta_0=180°$ (Fig. 4).

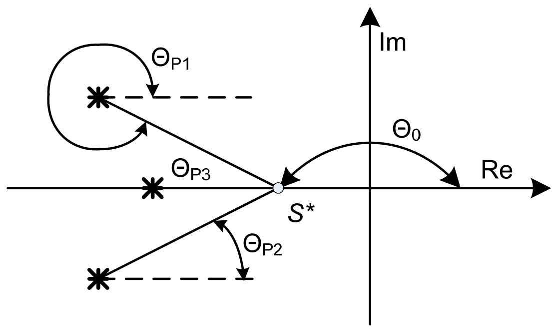

Fig. 4. *Example of arrangement of characteristic polynomial roots*

Therefore, the outlet angle of root S^* at c_i increase may be found by the formula: $\Theta_i^q=180°+i\Theta_0$, and at decrease $\Theta_i^q=i\Theta_0$.

Thus, if interval polynomial has alternating ranges of coefficients $\overline{c_0}\underline{c_1}\overline{c_2}\underline{c_3}$...,, then outlet angles form the image of vertex of parametric polyhedron P_n corresponding to this set equal to 180°.

On the basis of the given statement the conclusion may be made that minimal degree of stability of the interval system may be specified by a vertical straight line passing through the real root corresponding to alternating ranges of interval polynomial coefficients.

Let the minimal degree of stability is specified by the root $s^*=\alpha^*$. Let us introduce the information about this root to the polynomial (4) specifying alternating values b_i and value $s^*=\alpha^*$ in it. Expressing $K_И$ in terms of $K_П$ we obtain the expression:

$$K_И = f(K_П, \alpha^*, b_j^v), \qquad (6)$$

where b_j^v are the boundary values of polynomial $B(s)$ coefficients supporting alternation of ranges c_i.

For further synthesis of PI regulator the interval characteristic polynomial with one varied coefficients is obtained

$$(K_{И}(K_{П},\alpha^*,b_j^v)+K_{П}\cdot s)\cdot A(s)+s\cdot B(s)=0.$$

3. Localization of interval characteristic polynomial roots in specified region on the basis of equation of Theodorchik-Evans

As it is known [1] that in the case of interval uncertainty of characteristic polynomial the minimal degree of stability and maximal oscillation are determined by images of some vertices of interval coefficient polyhedron then it is necessary to define these vertices for solving the problem set above. For this purpose the technique developed [2] is proposed to be used.

When the set of calibration vertices is determined it is necessary to define for each one the values of adjustable parameter of regulator $K_П$ at which root locus crosses with the boundary of root location region specified in Fig. 1 on the basis of equations of Theodorchik-Evans root locus [3].

Let us present the obtained polynomial (9) in the form:

$$D(p)=\Phi(p)+K_{П}\Psi(p)=0.$$

Let us introduce the following notations:

$$E(\alpha,\omega)=\mathrm{Re}(\Phi(p)),\quad F(\alpha,\omega)=\mathrm{Im}(\Phi(p)),$$
$$P(\alpha,\omega)=\mathrm{Re}(\Psi(p)),\quad R(\alpha,\omega)=\mathrm{Im}(\Psi(p)).$$

According to [3] to define the desired values of $K_П$ it is necessary to substitute the equation of region Г boundary into (10), (11) and solve the equation

$$F(\omega)P(\omega)-E(\omega)R(\omega)=0. \qquad (7)$$

The obtained real roots ω_i should be substituted into expression:

$$K_{П}=\frac{E(\omega)P(\omega)+F(\omega)R(\omega)}{P^2(\omega)+R^2(\omega)}. \qquad (8)$$

Performing this for each calibration vertex the appropriate intervals of values $K_П$ at which interval polynomial roots are in specified region Г may be obtained. To determine interval of values KП satisfying all the vertices the intersection of all intervals should be found.

To determine finally the PI regulator adjustments it is necessary to select the value $K_П$ from the obtained range and substitute it into expression (6).

4. The technique of PPI regulator synthesis

On the basis of carried out investigations the technique of parametric synthesis of PI regulator was developed. It includes:

1. Specification of the required quality indices (minimum allowed degree of stability and maximum allowed oscillation of interval system).
2. Specification of $s^*=\alpha^*$ and ranges of control object interval parameters corresponding to the vertex with coordinates $\overline{c}_0\underline{c}_1\overline{c}_2\underline{c}_3$... in characteristic polynomial.
3. Obtaining dependence (6) and reduction of characteristic polynomial with two varied parameters of PI regulator to polynomial with one varied parameter.
4. Finding possible vertices of polyhedron P_n imaged onto the boundary of sector Г.
5. Determining value interval of varied parameter $K_П$ at which root locus branches are found for each found boundary vertex on the basis of expression (7), (8).
6. Determining intersection of $K_П$ found ranges at which polynomial roots are in specified region Г for all boundary vertices.
7. Selection of value $K_П$ from interval intersection region and determination of value $K_И$ on the basis of expression (6).

5. The example of synthesis

Let transfer function of open-loop system with single feedback is specified:

$$W_p=\frac{K_p\cdot(T_p\cdot s+1)}{s}\cdot\frac{K_0}{a_2\cdot s^2+a_1\cdot s+a_0},$$

where K_p, T_p is the adjustable parameters of regulator, K_0 is the control object transfer constant, a_2, a_1, a_0 are the interval parameters of control object, ($a_0\in[0,07;0,08]$; $a_1\in[0,3;0,4]$; $a_0\in[2;3]$).

Adjustments of PPI regulator guaranteeing $\alpha^*=1$ and $\varphi=10°$ should be defined.

Let us reduce characteristic equation of the given system to the form:

$$c_3\cdot s^3+c_2\cdot s^2+c_1\cdot s+c_0=0, \qquad (9)$$

where $c_3=a_2$, $c_2=a_1$, $c_1=a_0+K_0.K_p\cdot T_p$, $c_0=K_0\cdot K_p$.

Substituting boundary values of coefficients a_i corresponding to vertex $\overline{c}_0\underline{c}_1\overline{c}_2\underline{c}_3$... into (9) we obtain the dependence:

$$T_p=-\frac{\overline{a}_2\cdot s^3+\underline{a}_1\cdot s^2+\overline{a}_0\cdot s+K_0\cdot K_p}{K_0\cdot K_p\cdot s}. \qquad (10)$$

On the basis of (10) characteristic equation of the system is reduced to the form:

$$a_2s^3+a_1s^2+a_0s+$$
$$+K_0K_p\left(-\frac{\overline{a}_2\alpha^{*3}+\underline{a}_1\alpha^{*2}+\overline{a}_0\alpha^*+K_0K_p}{K_0K_p\alpha^*}\right)s+K_0K_p=0.$$

After conversion we obtain:

$$a_2\cdot s^3+a_1\cdot s^2+a_0\cdot s-(\overline{a}_2\cdot\alpha^{*2}+\underline{a}_1\cdot\alpha^{*2}+\overline{a}_0)\cdot s+$$
$$+K_p\cdot\left(K_0-\frac{K_0}{\alpha^*}\cdot s\right)=0. \qquad (11)$$

On the basis of (11) the equations of Theodorchik-Evans of the form (7), (8) are set up. According to the method [2] general range K_P at which polynomial roots are inside region Г: $K_P\in[2,4733;5,0160]$ was found for six vertices of interval coefficient polyhedron.

Let us select value K_P from the found range: K_P=5,0160 and on the basis of the expression (15) the

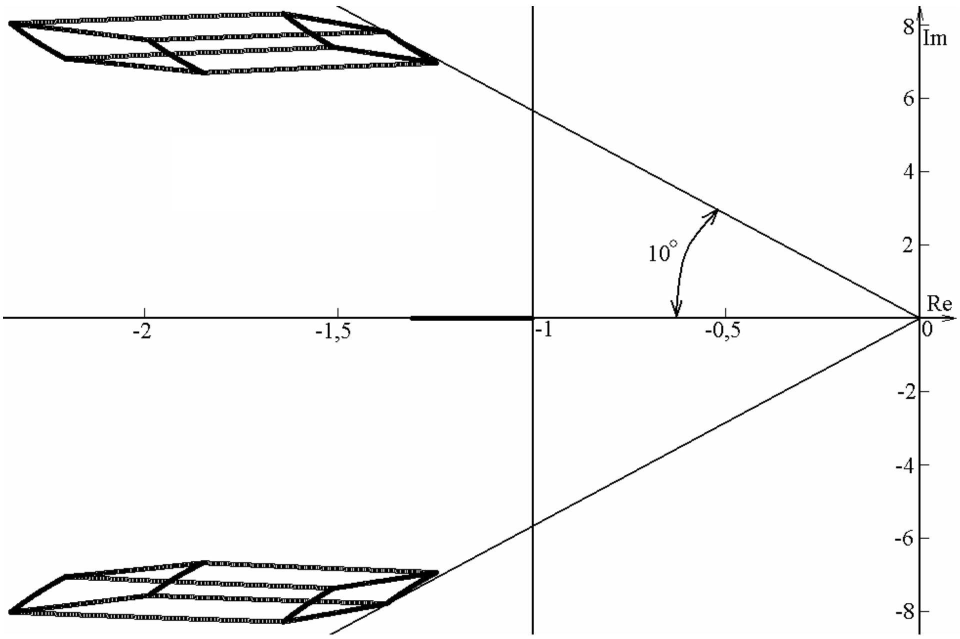

Fig. 5. *Localization regions of poles of the system with synthesized PI regulator*

value of the second required parameter T_P=0,4458 is obtained.

The regions of localization of interval characteristic polynomial roots with found adjustments of PI regulator are given in Fig. 5.

It is seen in Fig. 5 that root locus of interval characteristic polynomial is in region Γ limited by minimal degree of stability and maximal oscillation, therefore, the synthesized parameters of linear regulator guarantee specified root quality indices of the system.

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