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Aspects of geometric calculation of the planetary gear train with intermediate rollers. Part 1

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Abstract. The paper presents the geometric calculation of the planetary gear train with intermediate rollers, where the number of intermediate rollers is greater or less than that of the teeth in the annular gear by one, the profile surface of the annular gear teeth being the same. The gear ratio is changed by both a value and a sign.

1. Introduction

Planetary gear trains with intermediate rollers are currently widely used in automotive machinery and robotic machines (Figure 1) [1-3]. They possess a lot of advantages, namely: large gear ratios with small total dimensions; high loading capacity due to the multiple contact between their members; smooth running; alignment of the carrier and driven members; arrangement of the movable part in the carrier space. In some ways, this planetary gear transmission is better than that of the Fine Cyclo Japanese company ‘Sumitomo’ [4].

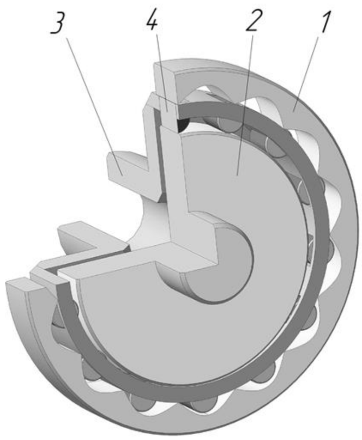

Figure 1. A planetary gear train with intermediate rollers: *1* – annular gear; *2* – carrier; *3* – separator plate; *4* – intermediate roller.

Despite a wide use of these types of planetary gear trains in different industries, especially in petroleum industry (JSC Tomsk Electric Drive Plant, LLC Sibirskii mashinostroitel', RPE Tomsk Electronic Company, LLC Siberian Machine Building Company) and a lot of publications, there are still many unsolved problems, in particular, the dependence between kinematic and geometric parameters with a certain number of intermediate rollers.

2. Calculation of the gear ratio

Figure 2 presents the schematics of the planetary rotation system. If the assemblage of intermediate rollers undergoes planetary (wave) motion by rotating about the common axis of the annular gear and the carrier, this planetary rotation system can be considered as a conventional system of a *K-H-V* type. In this type of the system, planet gears are represented by the intermediate rollers that roll in the slots of the separator plate.

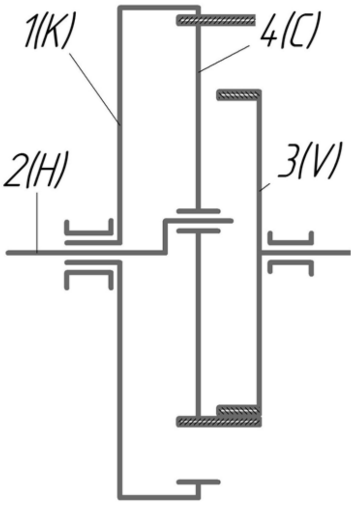

Figure 2. Schematics of the conventional gear train system of a *K-H-V* type: *1* – annular gear; *2* – carrier; *3* – separator plate; *4* – intermediate roller.

Let us consider the example of a stationary annular gear. The gear ratio between the carrier and the separation plate can be obtained using the Willis method [5] for the reversed mechanism:

$$i_{23} = \frac{Z_4}{Z_4 - Z_1},$$

where Z_1, Z_4 are the number of the teeth annular gear and intermediate rollers, respectively.

The analysis of the obtained equation is carried out in relation to the design variants, namely: $Z_1 = Z_4 + 1$ (the number of intermediate rollers is 1 less than that of the teeth annular gear) and $\boldsymbol{Z_1} = \boldsymbol{Z_4} - 1$ (the number of intermediate rollers is greater than that of the teeth annular gear by one).

In the first case, the gear ratio is

$$i_{23} = -Z_4,$$

and directions of the member rotation are opposite. In the second case the gear ratio is

$$i_{23} = Z_4.$$

Let us consider the example of a stationary separator plate. The gear ratio between the carrier and the annular gear can be obtained using the Willis method for the reversed mechanism:

$$i_{21} = \frac{Z_1}{Z_1 - Z_4}.$$

Let us suppose that $Z_1 = Z_4 + 1$; then

$$i_{21} = Z_1. \tag{1}$$

Now let us suppose that $\boldsymbol{Z_1} = \boldsymbol{Z_4} - 1$; then

$$i_{21} = -Z_1. \tag{2}$$

In the latter case, the members rotate in different directions.

3. Calculation of the profile surface teeth annular gear

Let us consider the planetary gearing design with dependence (2). The schematic layout and design parameters of gearing are shown in Figure 3. Two coordinate systems are used: fixed *S*(*Oxy*) and moving *S*1(*O*1*x*1*y*1) rigidly connected with the annular gear.

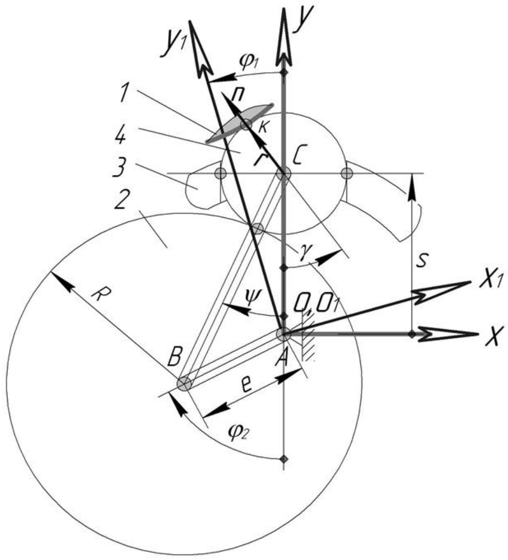

Figure 3. A schematic layout of gearing with dependence (2): *1* – annular gear; *2* – carrier; *3* – separator plate; *4* – intermediate roller**.**

Conformal rotations of the annular gear and the carrier are determined by directional angles φ_1 and φ_2 connected by (2):

$$i_{21} = \frac{\varphi_2}{\varphi_1} = -Z_1 \cdot \quad (3)$$

As shown in Figure 3, common contact normal ***n*** to point *K* is a perpendicular to vector $V^{(41)}$ of the relative velocity of the intermediate roller and the teeth annular gear at point *C*.

Fixed coordinate system *S*(*Oxy*) is used for these calculations. From triangle *ΔABC* it follows that the law of central displacement in the intermediate roller relative to the common axis of the annular gear and the carrier has the following form:

$$s = l\cos\psi - e\cos\varphi_2 \ . \quad (4)$$

Let us note that

$$\cos\psi = \sqrt{1-\sin^2\psi}\,; \quad (5)$$

$$\sin\psi = \frac{e}{l}\sin\varphi_2\,, \quad (6)$$

where *e* is the eccentricity.

$$l = R + r\ , \quad (7)$$

where *R* and *r* are the radii of the carrier and the intermediate roller, respectively.

Now we introduce unit vectors ***i*** and ***j*** for axes of fixed coordinate system *S*(*Oxy*). The velocity of point *C* of the intermediate roller can be obtained from

$$\boldsymbol{V}^{(4)} = \frac{ds}{d\varphi_2}\boldsymbol{j} = e\left(\sin\varphi_2 - \mathrm{tg}\,\psi\cos\varphi_2\right)\boldsymbol{j}\,. \quad (8)$$

The velocity of point *C* of the annular gear can be defined as follows:

$$\boldsymbol{V}^{(1)} = -\frac{s}{Z_1}\boldsymbol{i}\,. \quad (9)$$

The relative velocity vector can be found as the difference between velocities obtained in (8) and (9).

$$\boldsymbol{V}^{(41)} = e\left(\sin\varphi_2 - \operatorname{tg}\psi\cos\varphi_2\right)\boldsymbol{j} + \frac{s}{Z_1}\boldsymbol{i}\,.$$

Common contact normal $\boldsymbol{n}$ is obtained using $\boldsymbol{k} = \boldsymbol{i}\times\boldsymbol{j}$.

$$\boldsymbol{n} = \boldsymbol{k}\times\boldsymbol{V}^{(41)} = -e\left(\sin\varphi_2 - \operatorname{tg}\psi\cos\varphi_2\right)\boldsymbol{i} + \frac{s}{Z_1}\boldsymbol{j}\,. \tag{10}$$

The radius-vector of the profile surface at point K can be represented in the following form:

$$\boldsymbol{r}^{(K)} = s\,\boldsymbol{j} + \boldsymbol{r} = -r\sin\gamma\,\boldsymbol{i} + \left(s + r\cos\gamma\right)\boldsymbol{j}\,. \tag{11}$$

Using (10), we obtain the components included in (11):

$$\cos\gamma = \frac{\boldsymbol{n}\cdot\boldsymbol{j}}{n} = \frac{s}{Z_1\sqrt{e^2\left(\sin\varphi_2 - \operatorname{tg}\psi\cos\varphi_2\right)^2 + \left(\dfrac{s}{Z_1}\right)^2}}\,, \tag{12}$$

$$\sin\gamma = \sqrt{1-\cos^2\gamma}\,. \tag{13}$$

In order to obtain the profile surface of the annular gear teeth, (11) should be transformed to the moving coordinate system $S1(O1x1y1)$. The matrix technique can be used for this purpose [6].

Without going into details, let us show the final system of equations for the profile surface of the annular gear teeth:

$$\left.\begin{aligned} X_1 &= -r\sin\gamma\cos\varphi_1 + (s + r\cos\gamma)\sin\varphi_1\,,\\ Y_1 &= \quad r\cos\gamma\sin\varphi_1 + (s + r\cos\gamma)\cos\varphi_1. \end{aligned}\right\} \tag{14}$$

For the calculation of the profile surface coordinates, the system of equations (13) should be combined with (3)-(7), (12) and (13).

Let us further consider the planetary gearing design with dependence (1). The schematic layout and design parameters of gearing are presented in Figure 4.

Iterating the operations given above and considering the change of the directional angle φ_1 (1), we obtain

$$\left.\begin{aligned} X_1 &= r\sin\gamma\cos\varphi_1 - (s + r\cos\gamma)\sin\varphi_1\,,\\ Y_1 &= r\cos\gamma\sin\varphi_1 + (s + r\cos\gamma)\cos\varphi_1. \end{aligned}\right\} \tag{15}$$

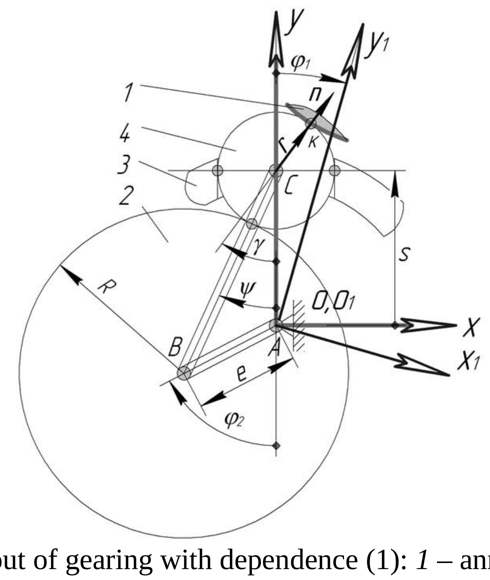

Figure 4. A schematic layout of gearing with dependence (1): *1* – annular gear; *2* – carrier; *3* – separator plate; *4* – intermediate roller.

Note that (14) and (15) describe the teeth profile surface opposite to the symmetry axis of the gear tooth. Hence, we now prove that the profile surface is similar in both cases.

4. A calculation example

Figure 5 depicts the planetary gear train with intermediate rollers designed by the obtained parameters, namely: $Z_1 = 16$; r = 5.16; R = 45; e = 2.58.

Under the condition of the stationary annular gear and the driven separator plate, the gear ratio as shown in Figure 5, is $i_{23}^{(1)} = -15$ and $i_{23}^{(1)} = 17$ according to *a*) and *b*) design variants, respectively.

It should be noted that, unlike the separator plate that has the different number of intermediate rollers and slots for them, the change in the separator plate parameters has no effect on the parameters of the annular gear and the carrier.

5. Conclusions

1. The planetary gear train with intermediate rollers may incorporate the number of intermediate rollers, which is greater or less by 1 as compared to the number of the teeth annular gear.
2. The teeth profile surfaces of the annular gear are similar in both cases described herein.
3. A change of the number of intermediate rollers provides a change of the gear ratio and the direction of rotation of the driven member.

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